Model-based Reinforcement Learning

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Finnish Center for Artificial Intelligence (FCAI)
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Slides available here

17th July 2024

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AlphaGo

Model-based reasoning for games

Silver et al. (2016). Mastering the game of Go with deep neural networks and tree search. Nature, 529(7587), 484.

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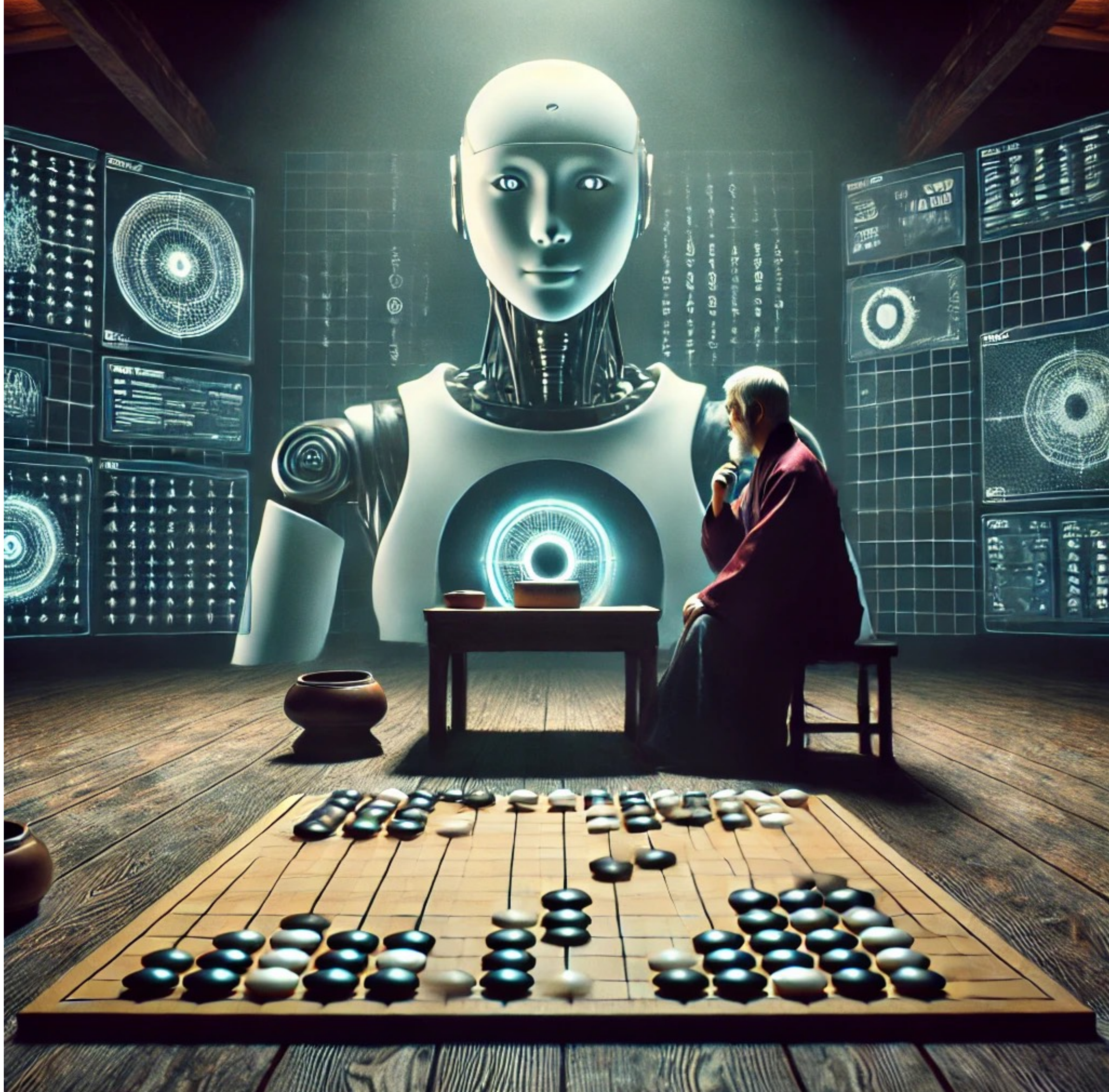

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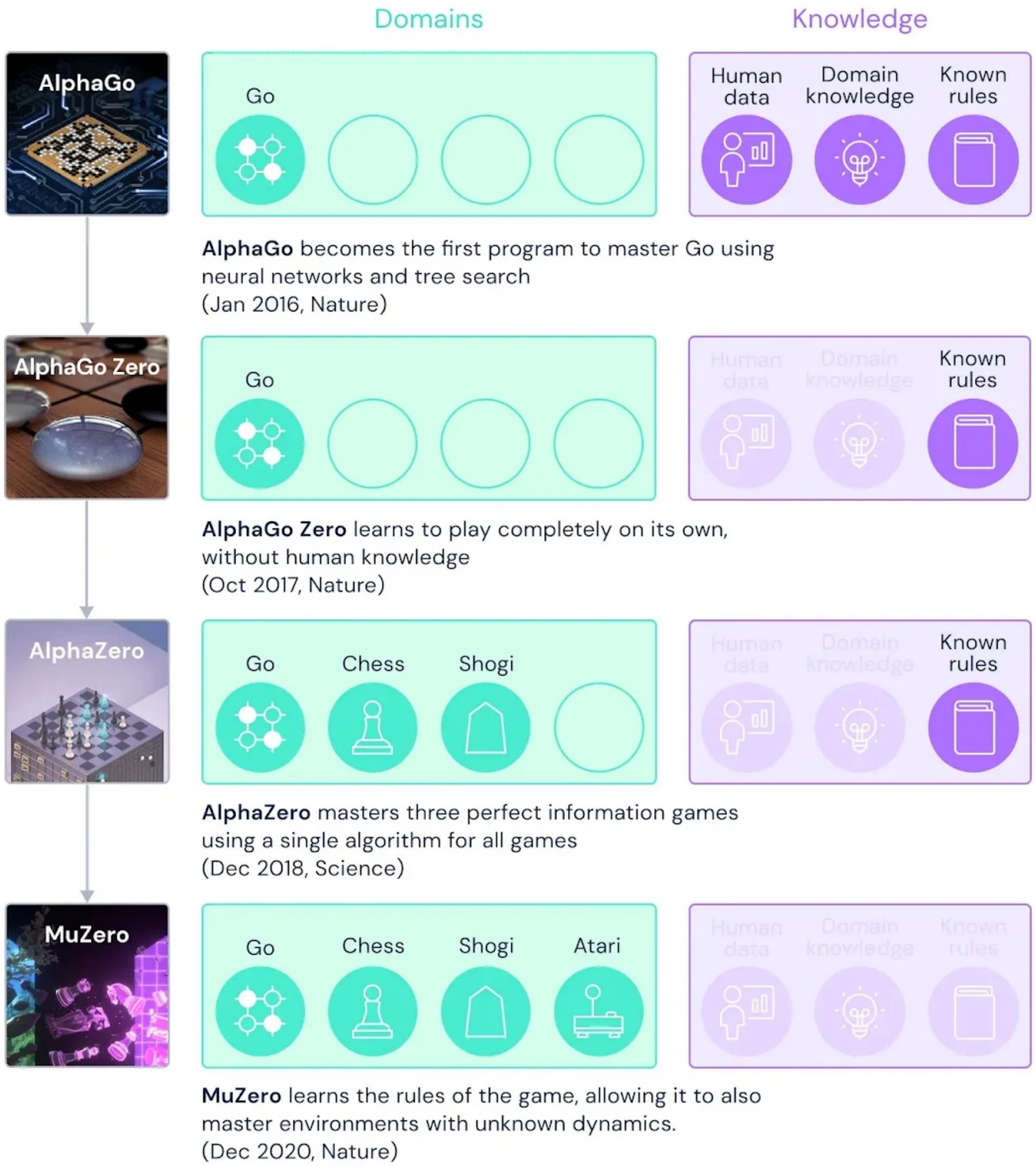

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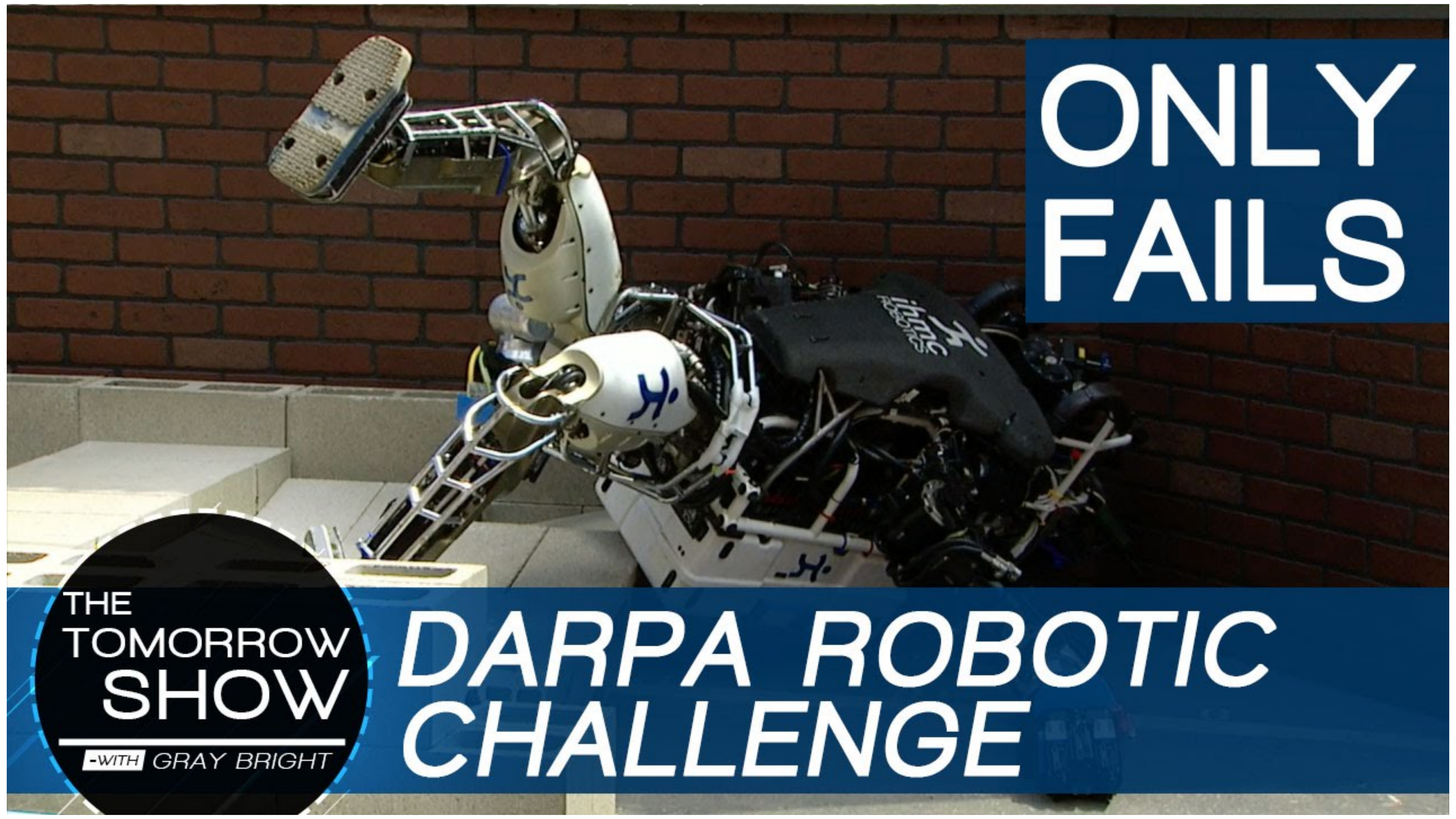
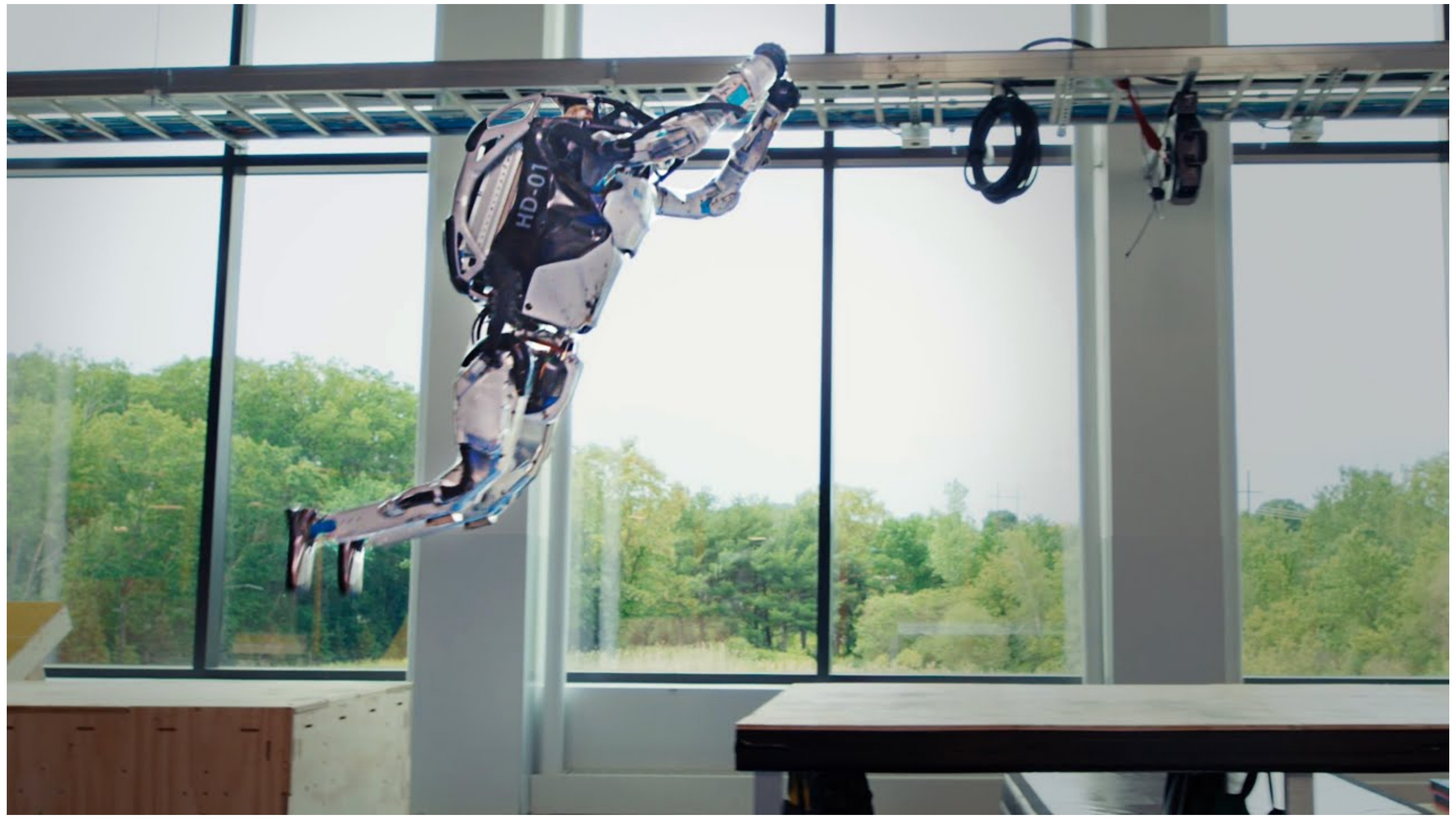
Machine Learning for Robotics

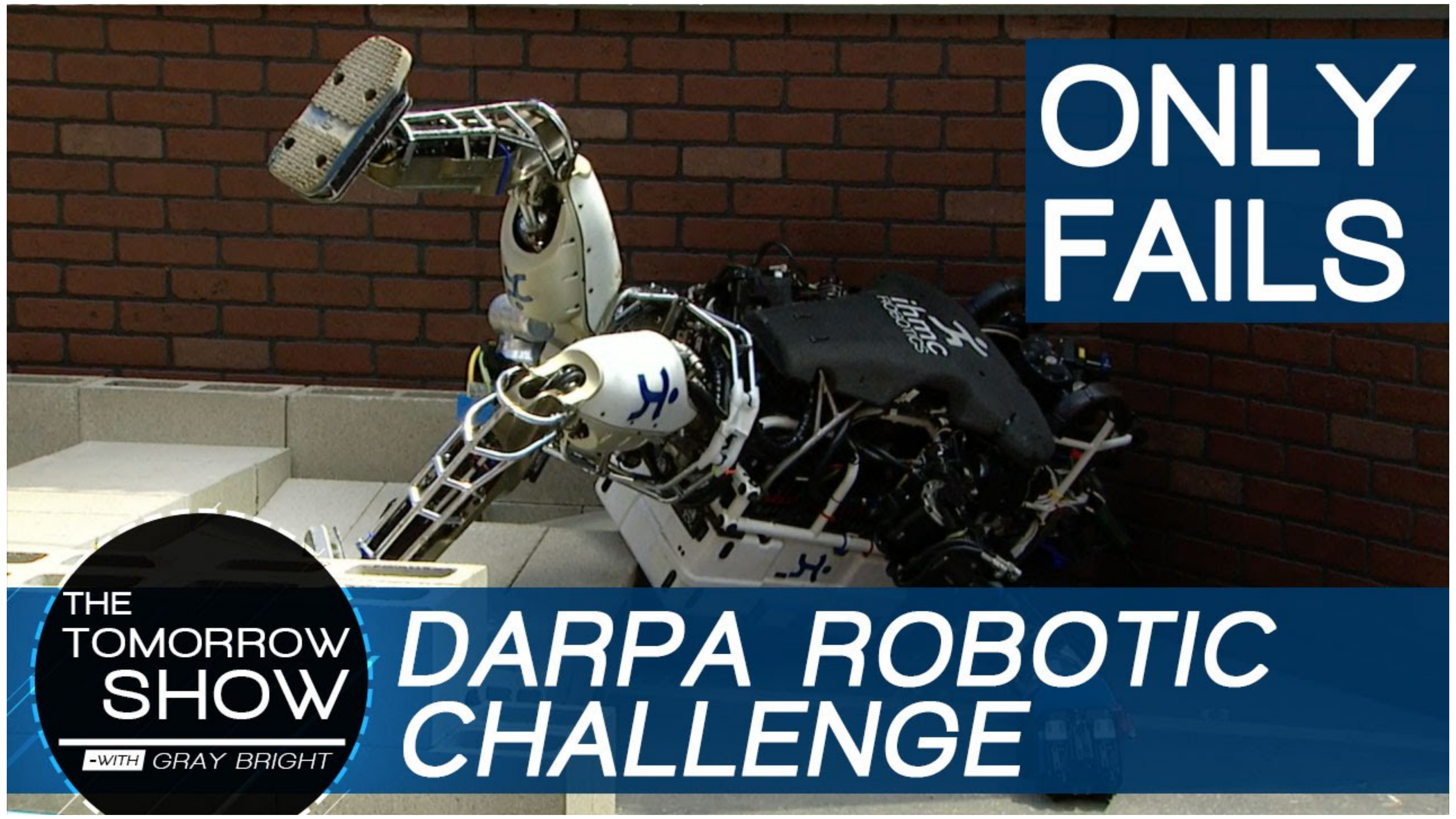


DARPA Robotics Challenge 2015



Boston Dynamics Atlas - Partners in Parkour

Machine Learning for Robotics



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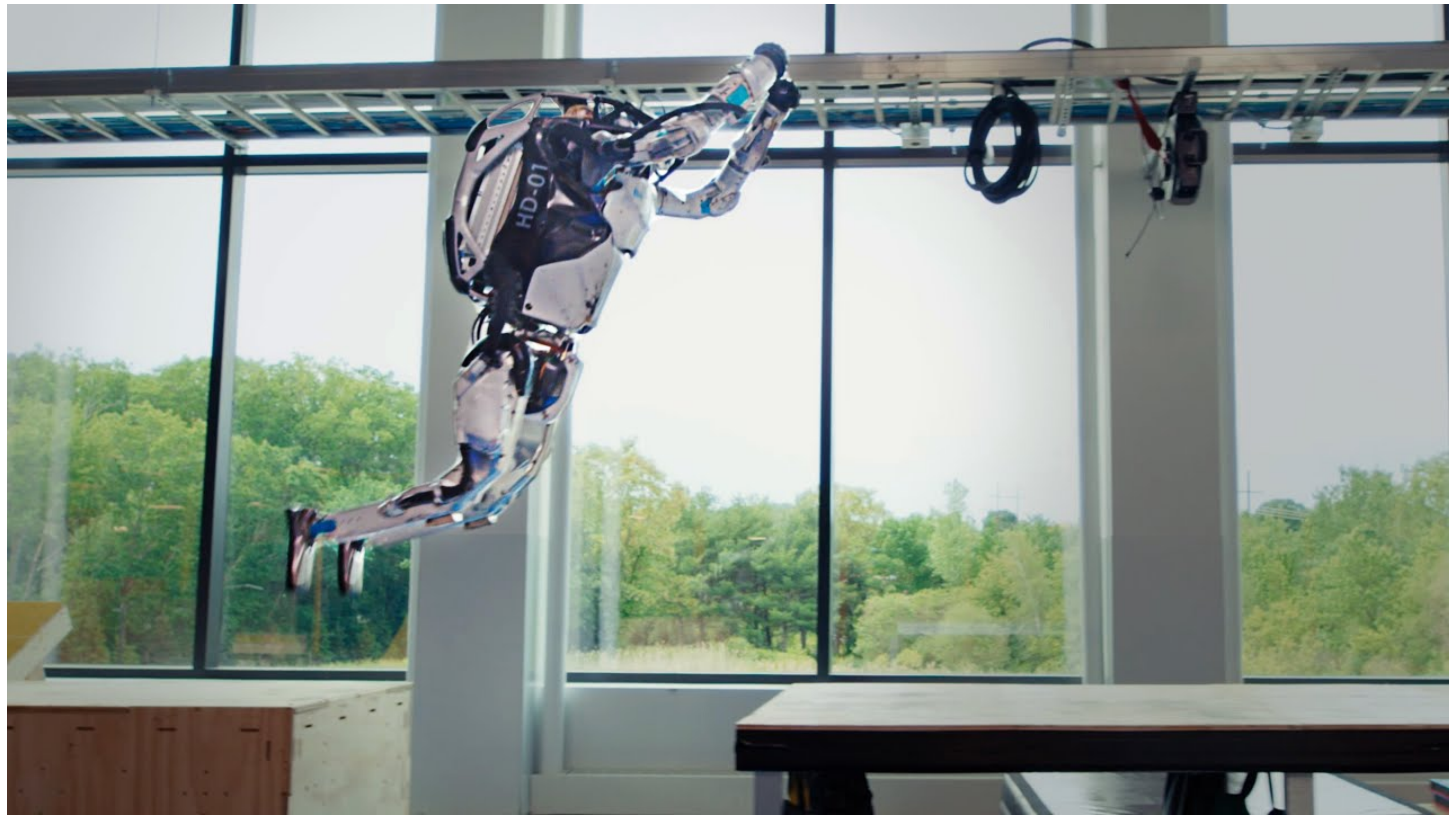

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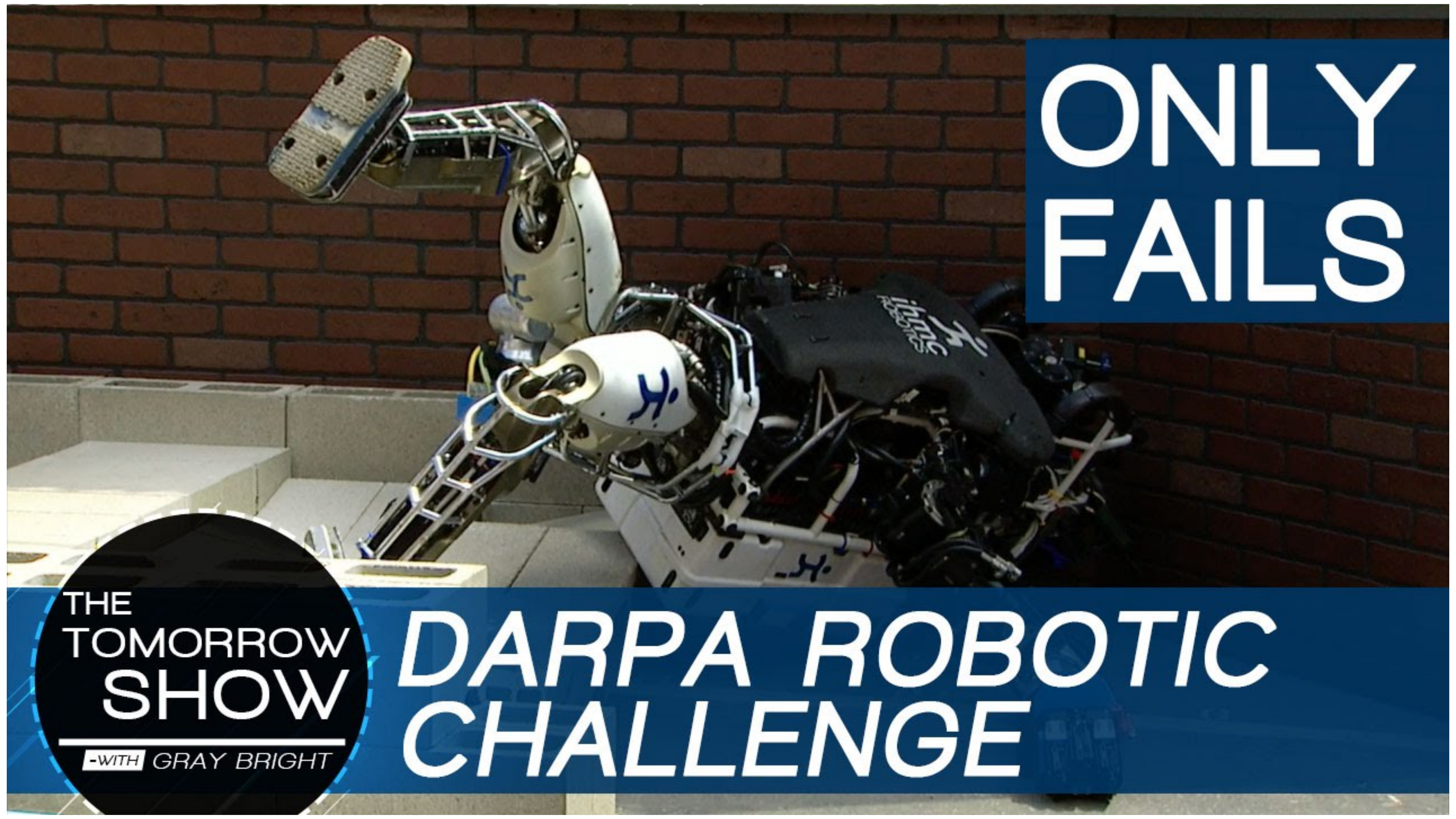

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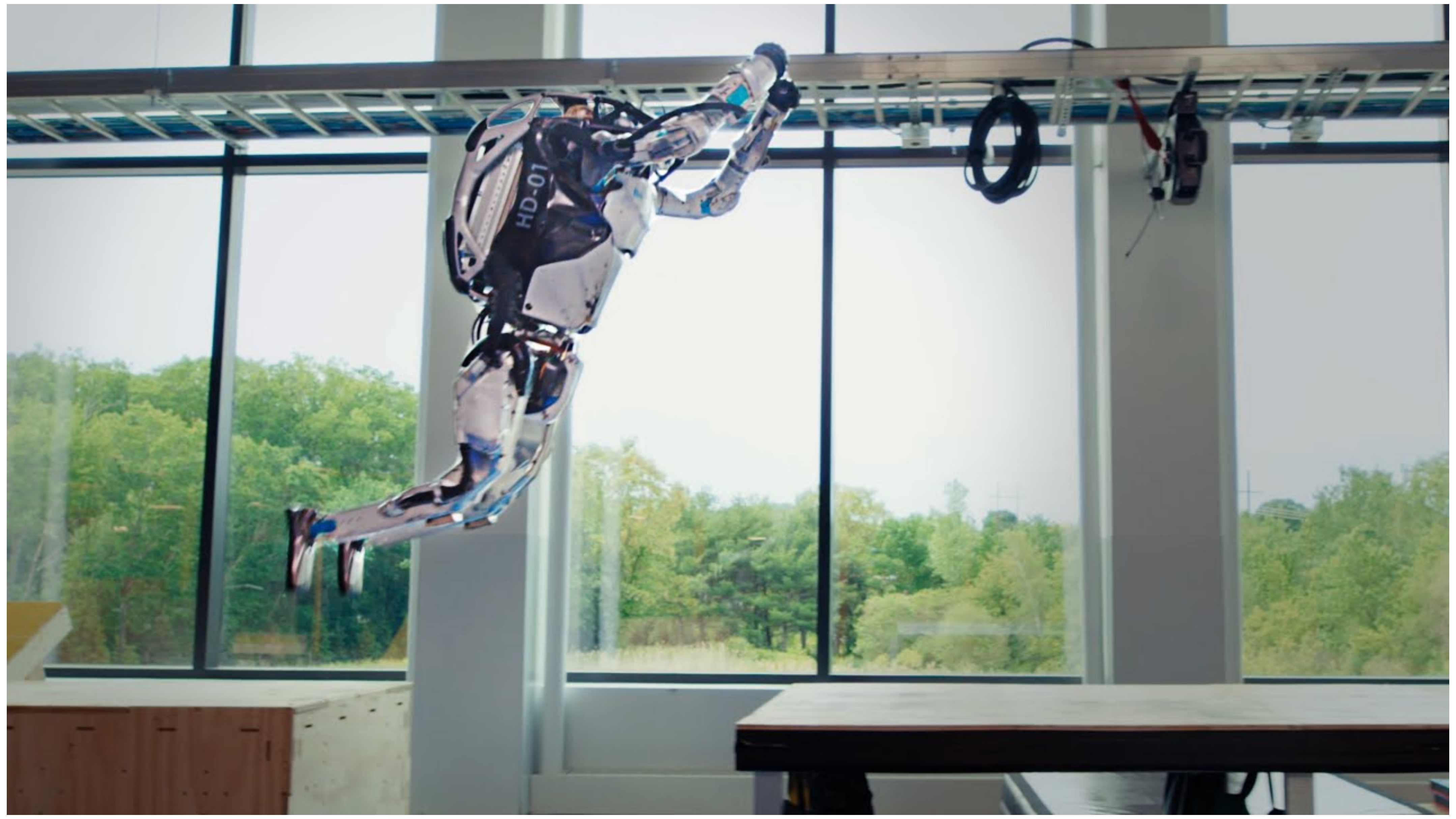
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Learning Objectives

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5. Sources of uncertainty in model-based RL

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1. What a “model” is in model-based RL
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3. Differences between background and decision-time planning
4. Decision-time planning strategies for continuous actions
5. Sources of uncertainty in model-based RL
6. Rationale and insights for decision-making under uncertainty

Reinforcement Learning

Markov Decision Process (MDP)

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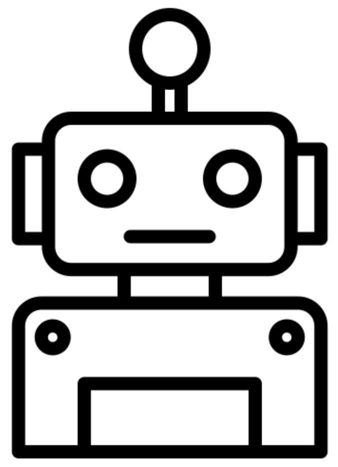

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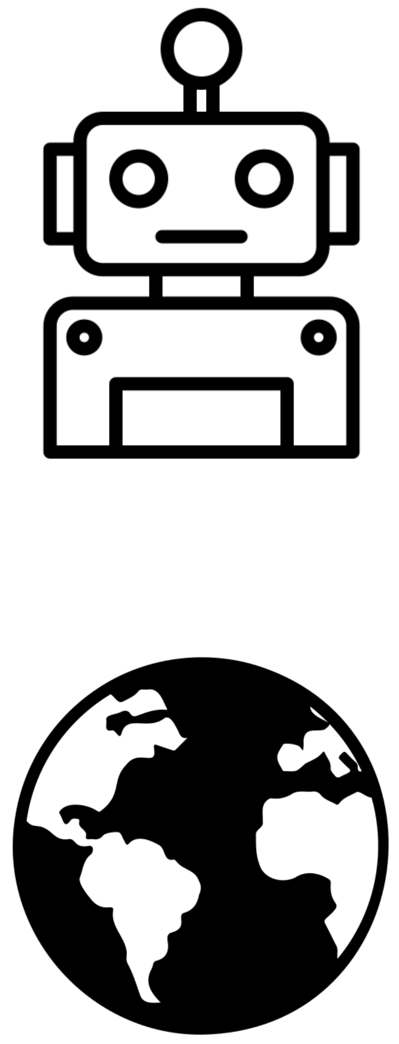

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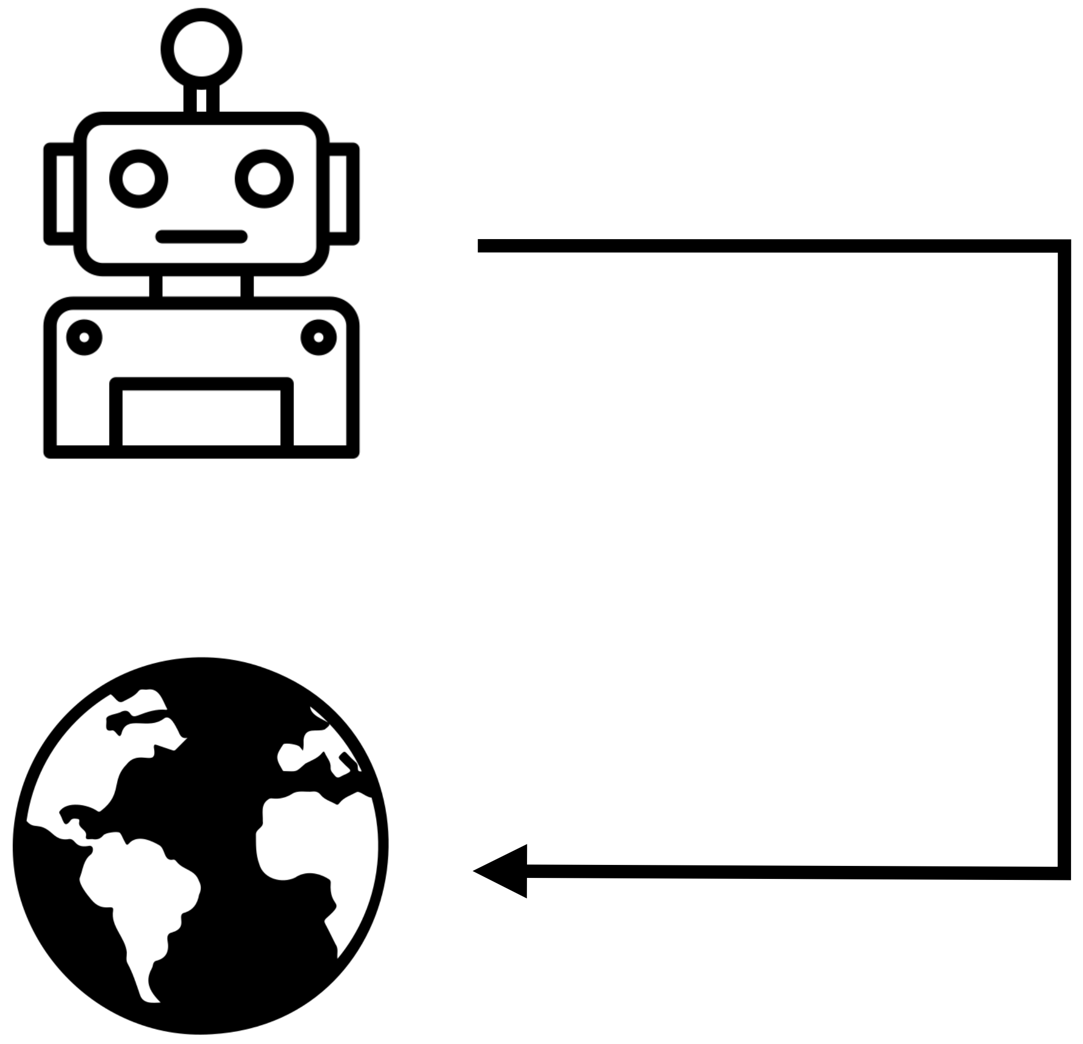

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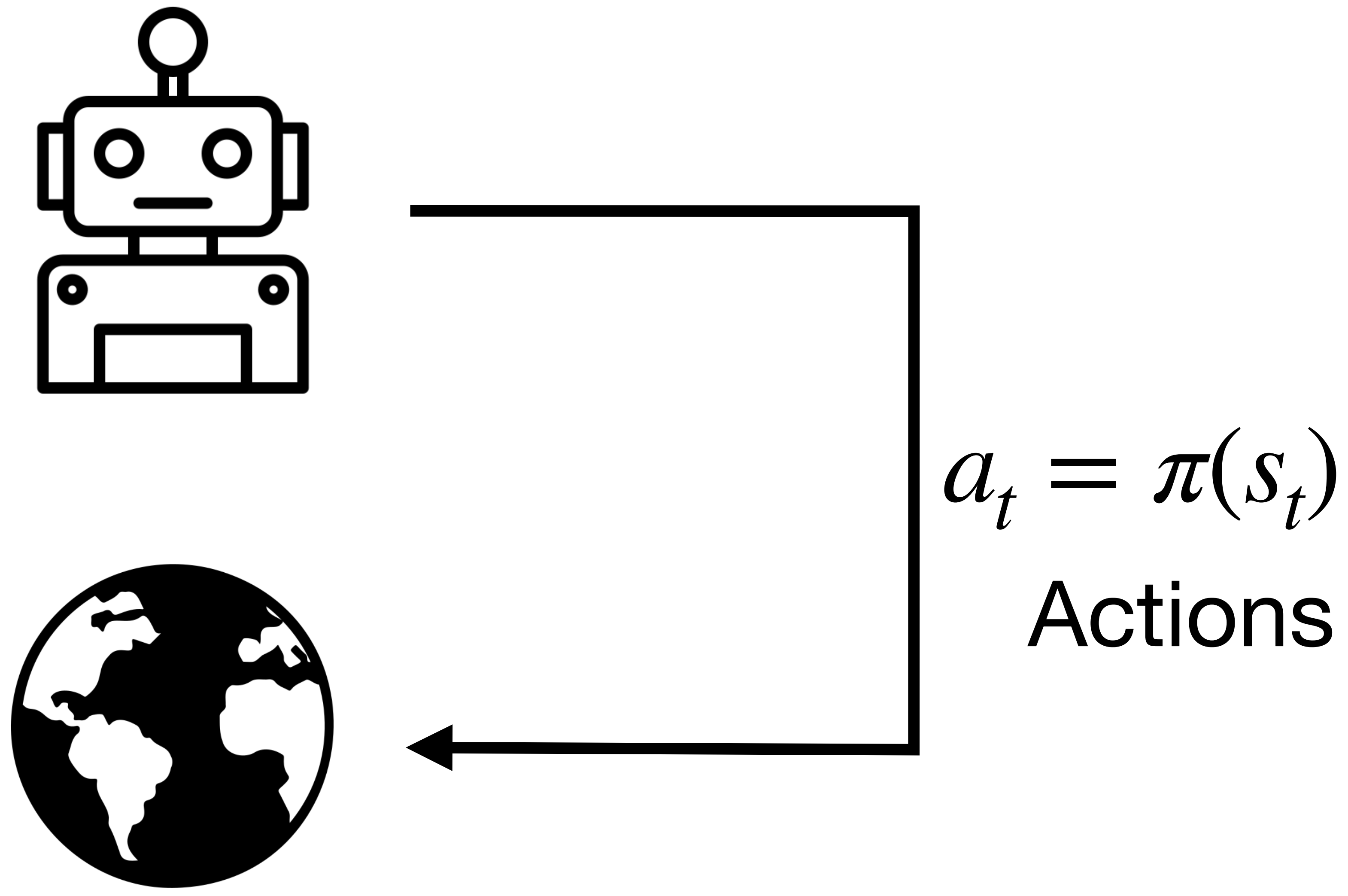

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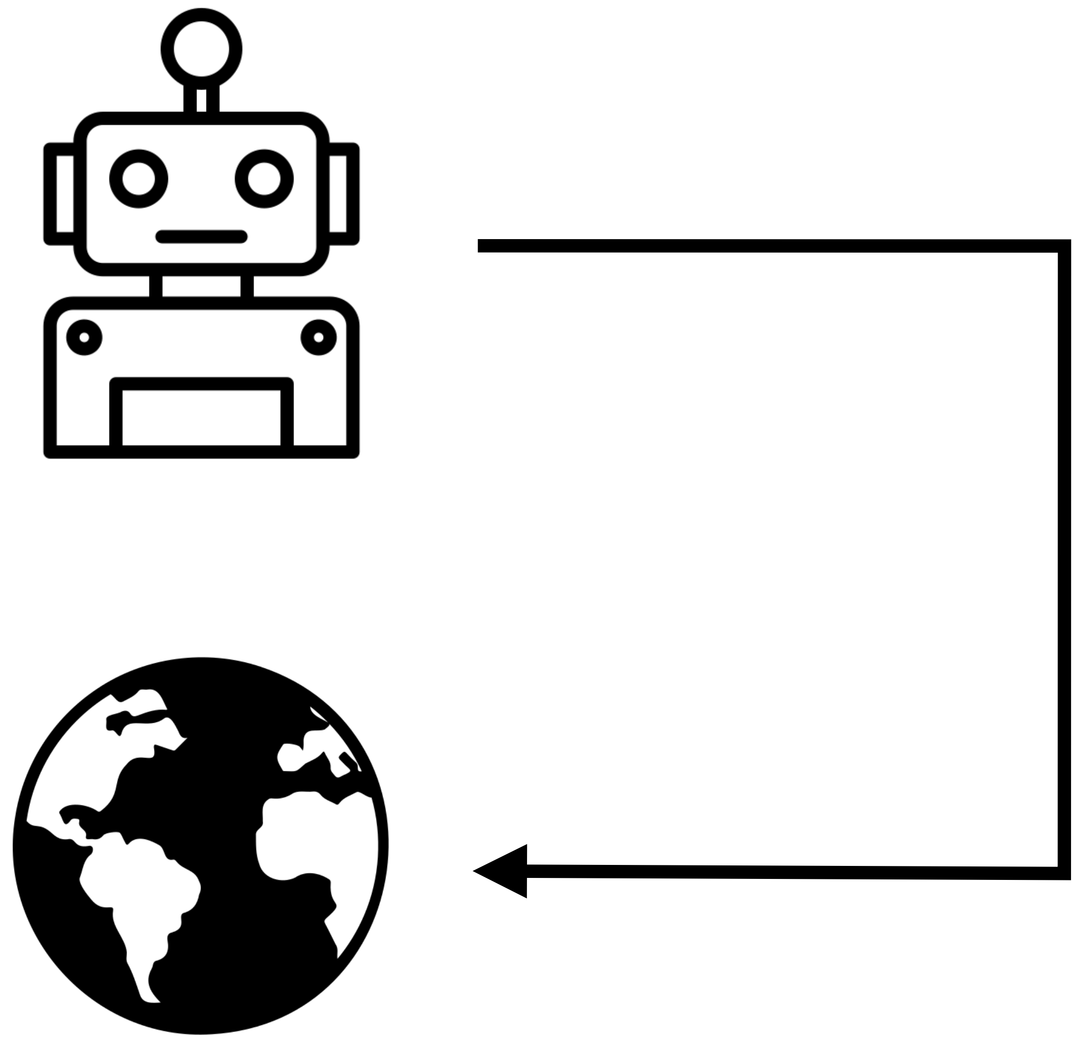

$a_t = \pi(s_t)$

Actions

$s_{t+1} \sim P(\cdot \mid s_t, a_t)$

Transition function

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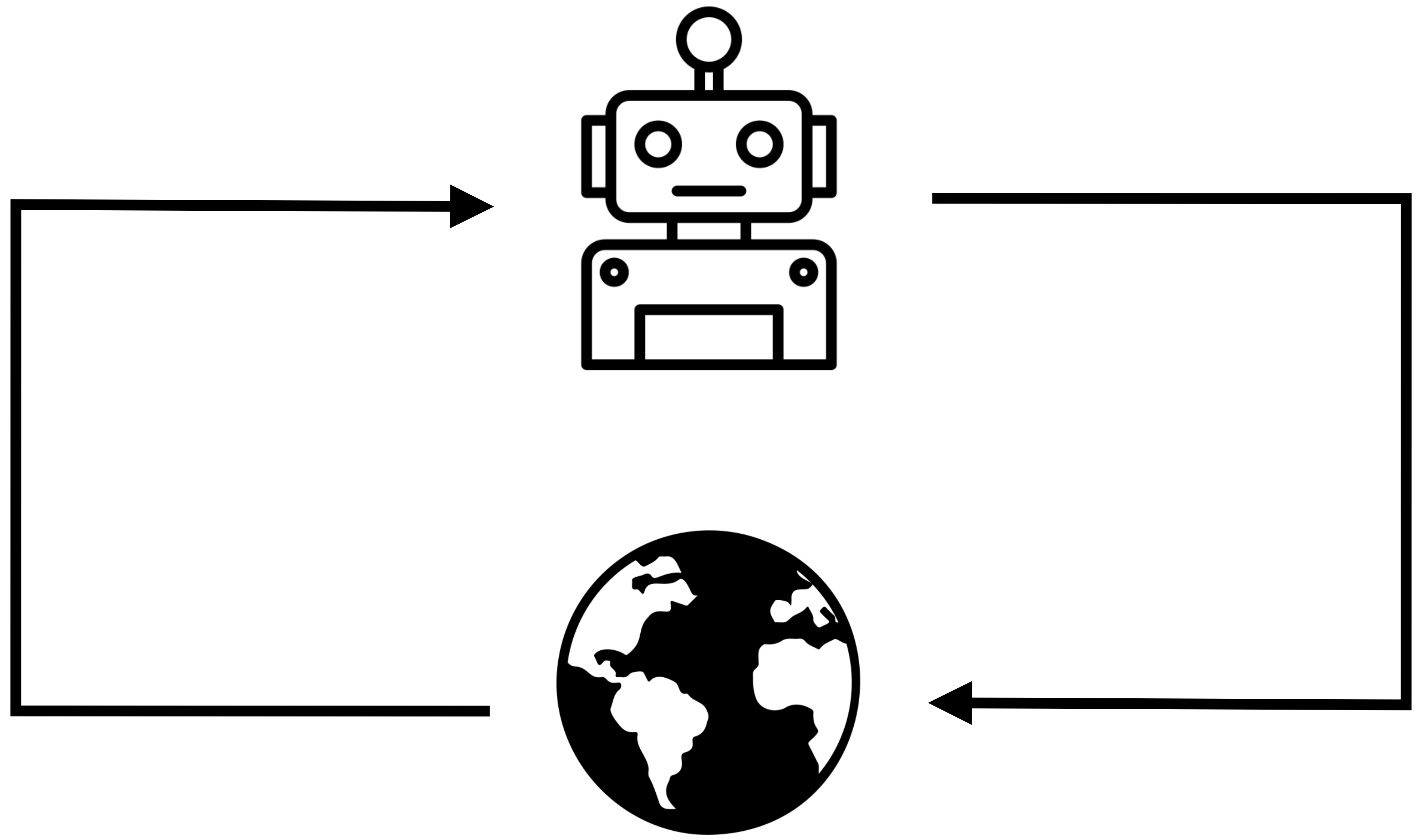

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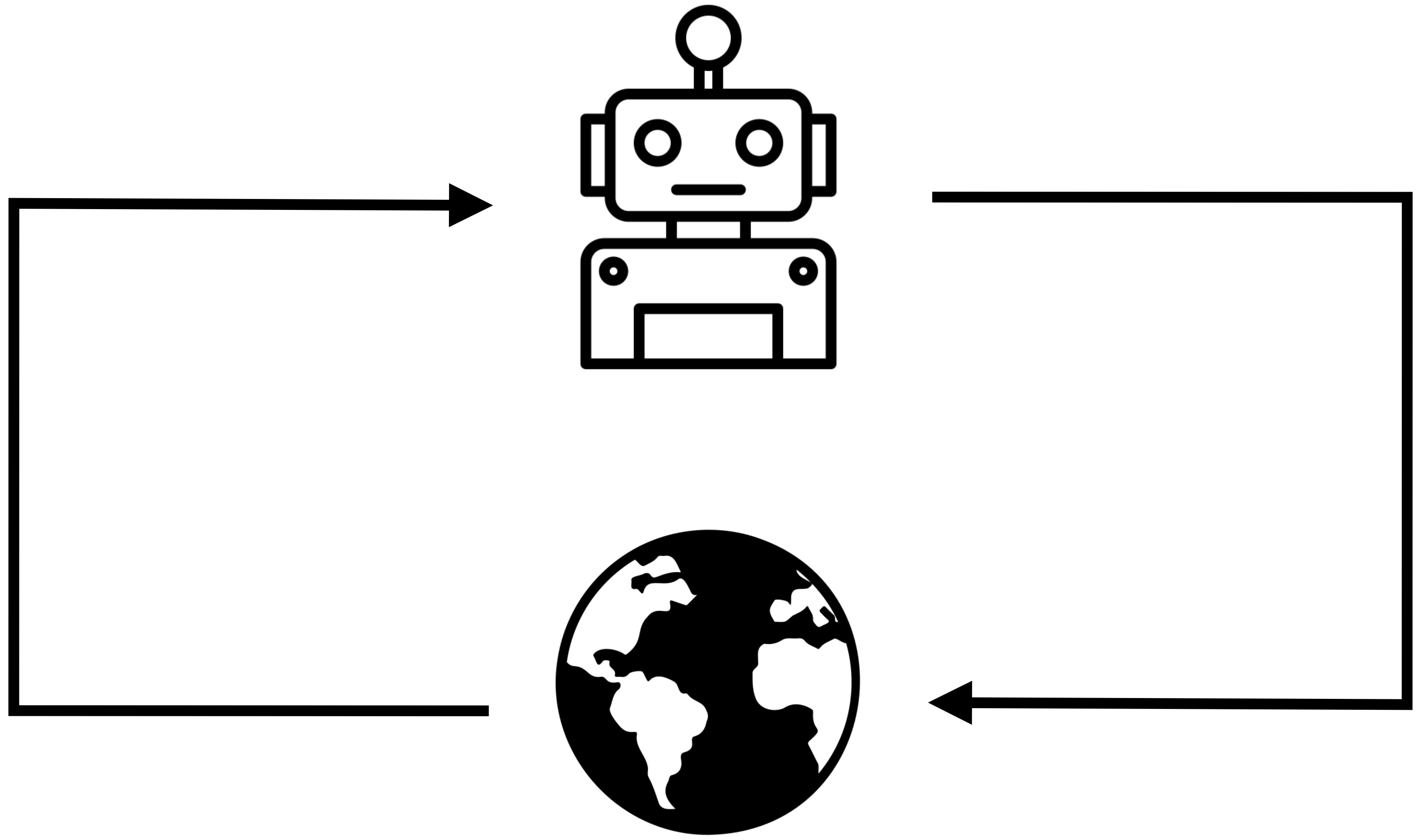

$s_{t+1}, \quad r(s_t, a_t)$

State, Reward

$a_t = \pi(s_t)$

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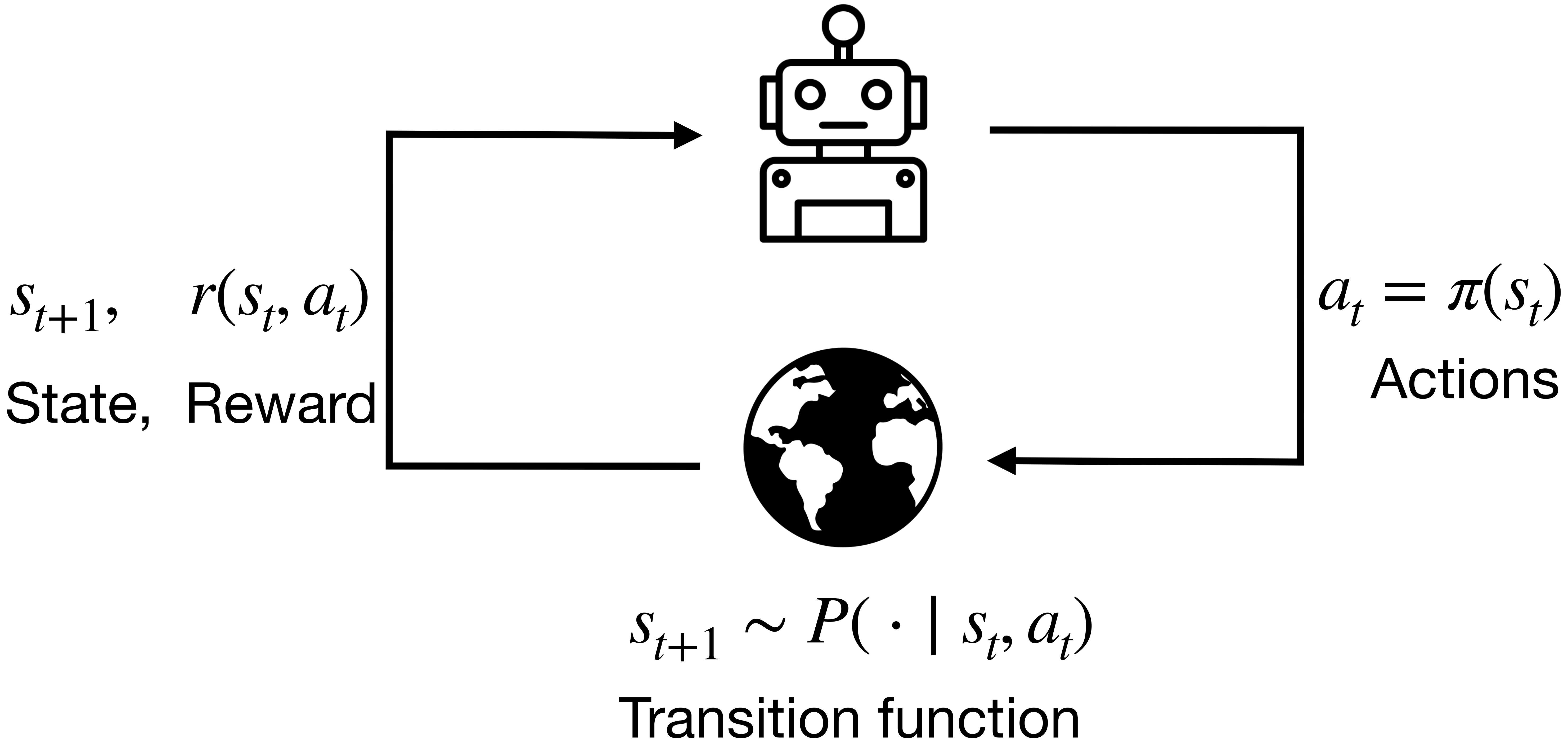
States $s \in \mathcal{S}$



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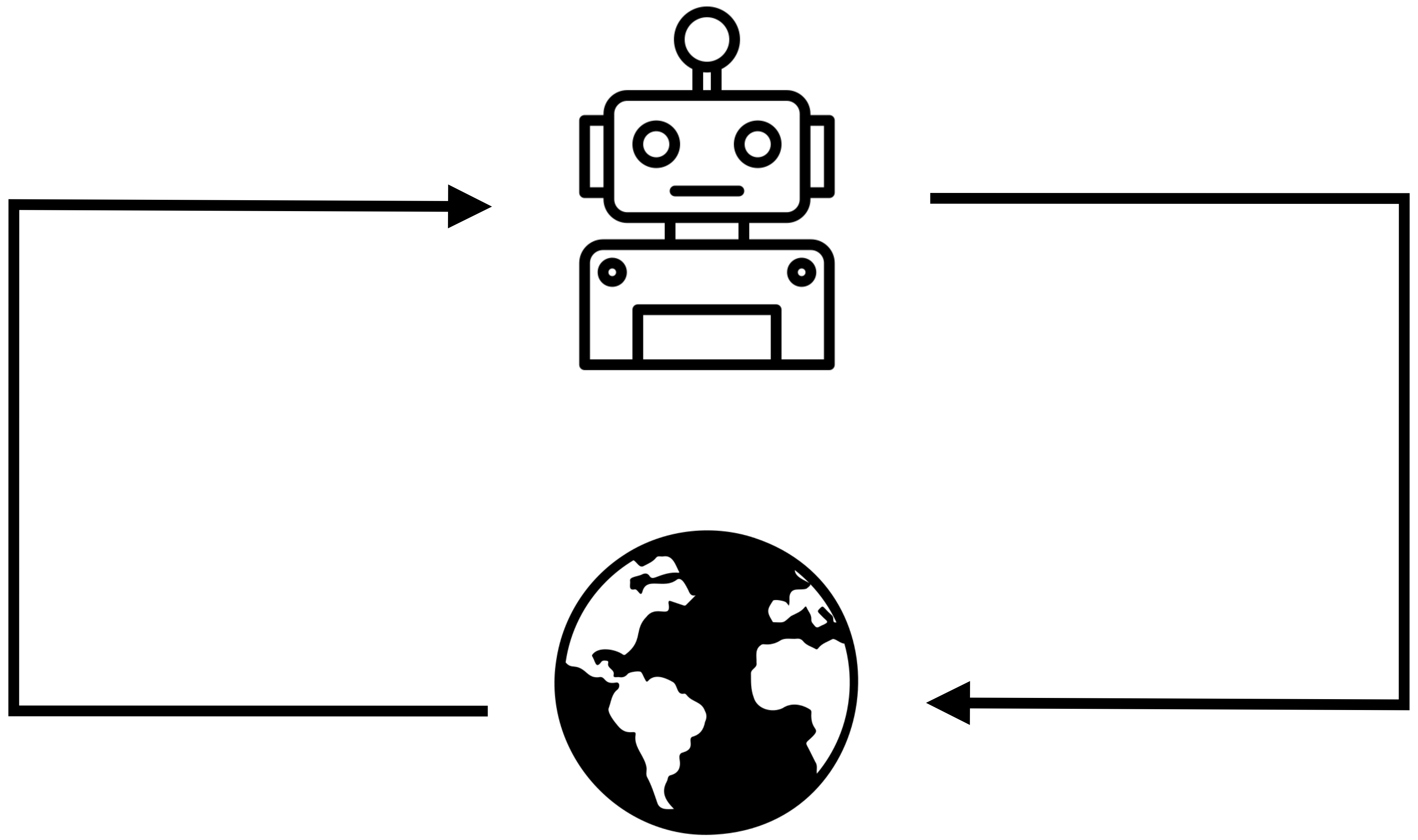

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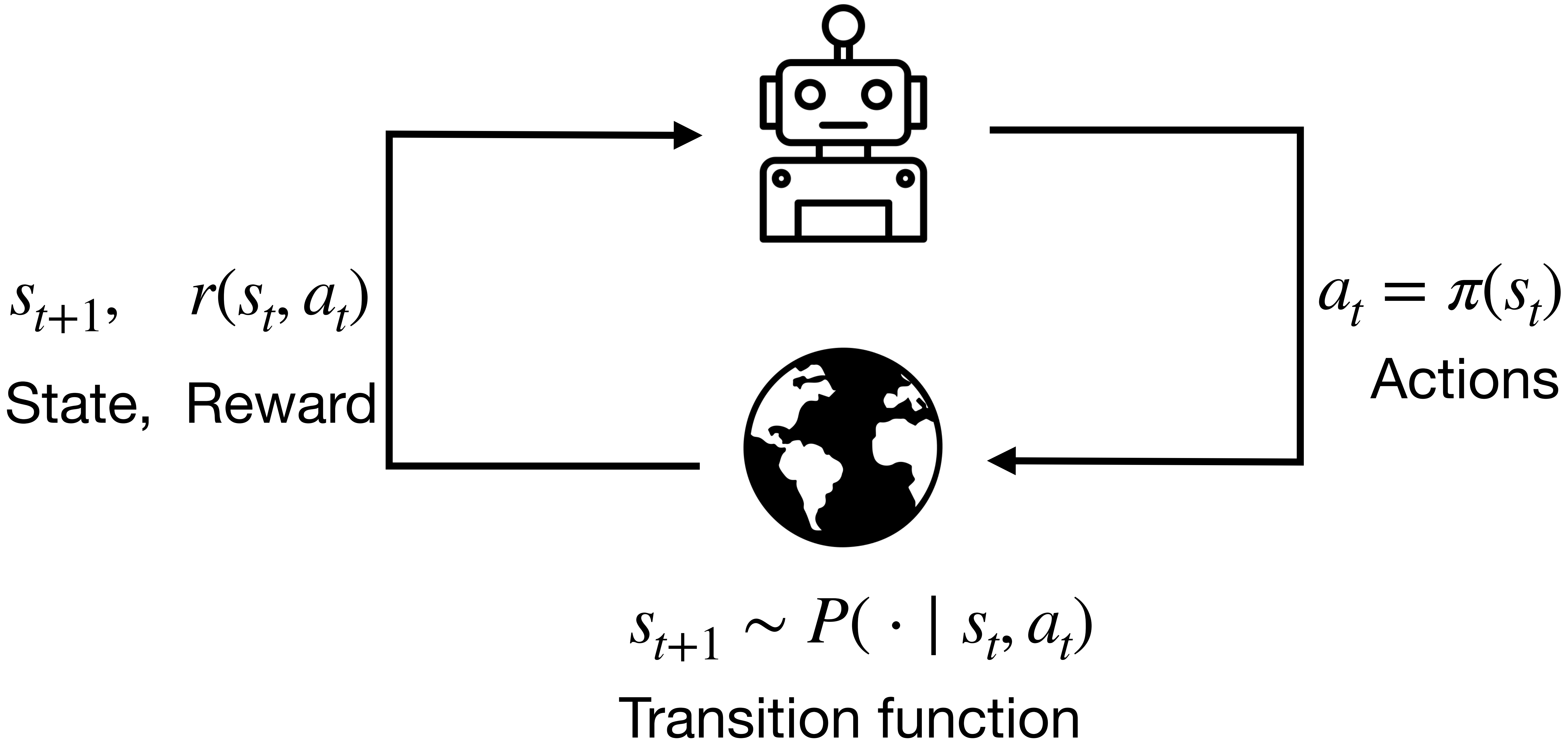

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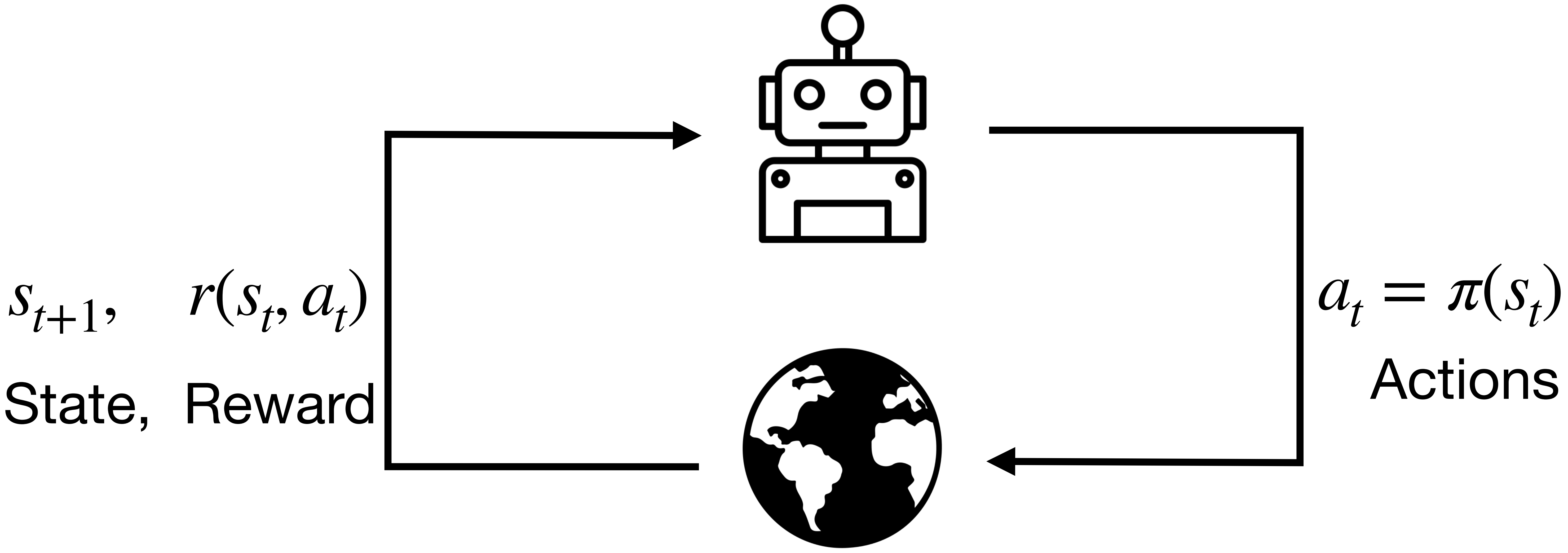

$s_{t+1} \sim P(\cdot \mid s_t, a_t)$

Transition function

$s_{t+1} = f(s_t, a_t) + \epsilon_t$

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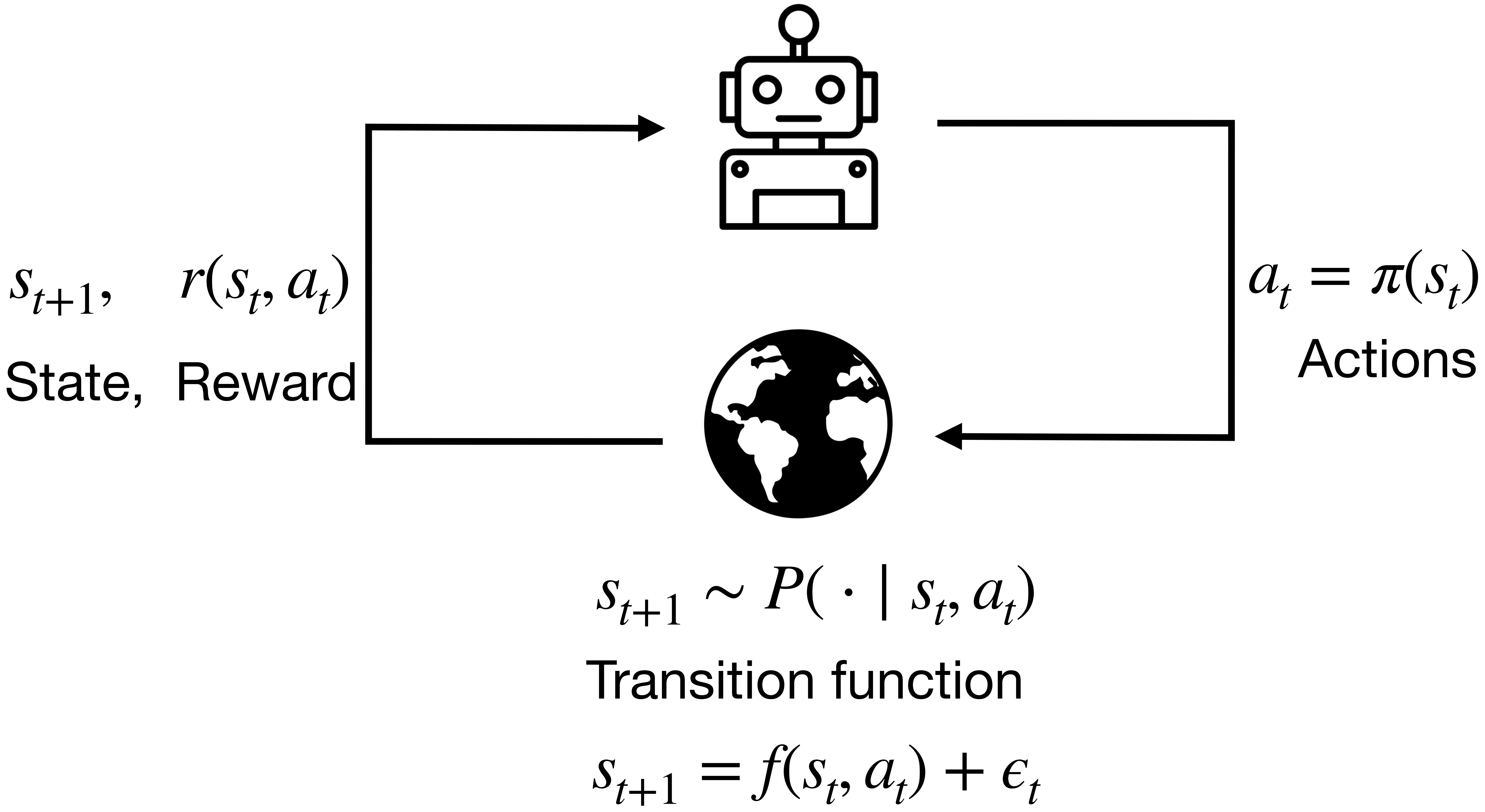

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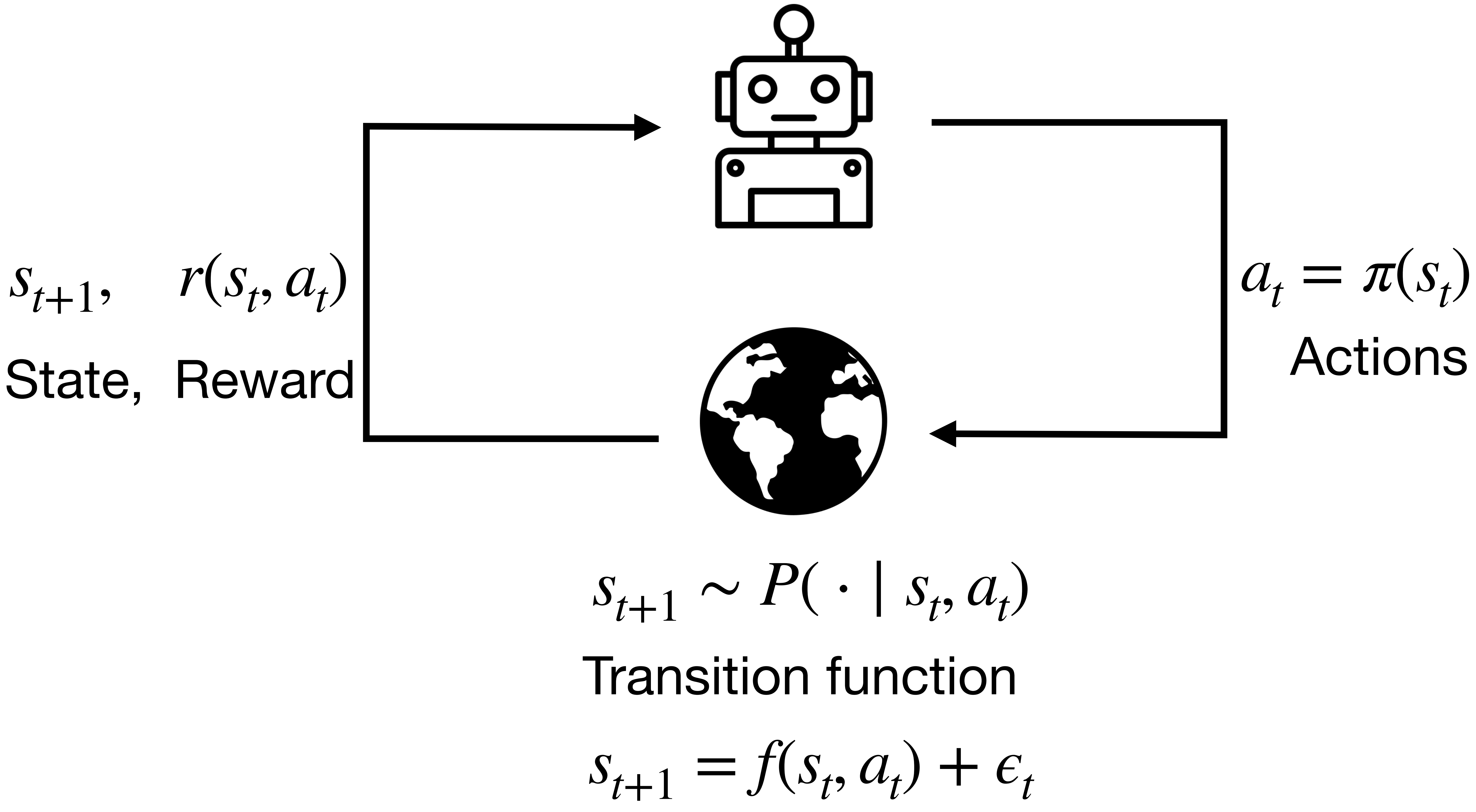

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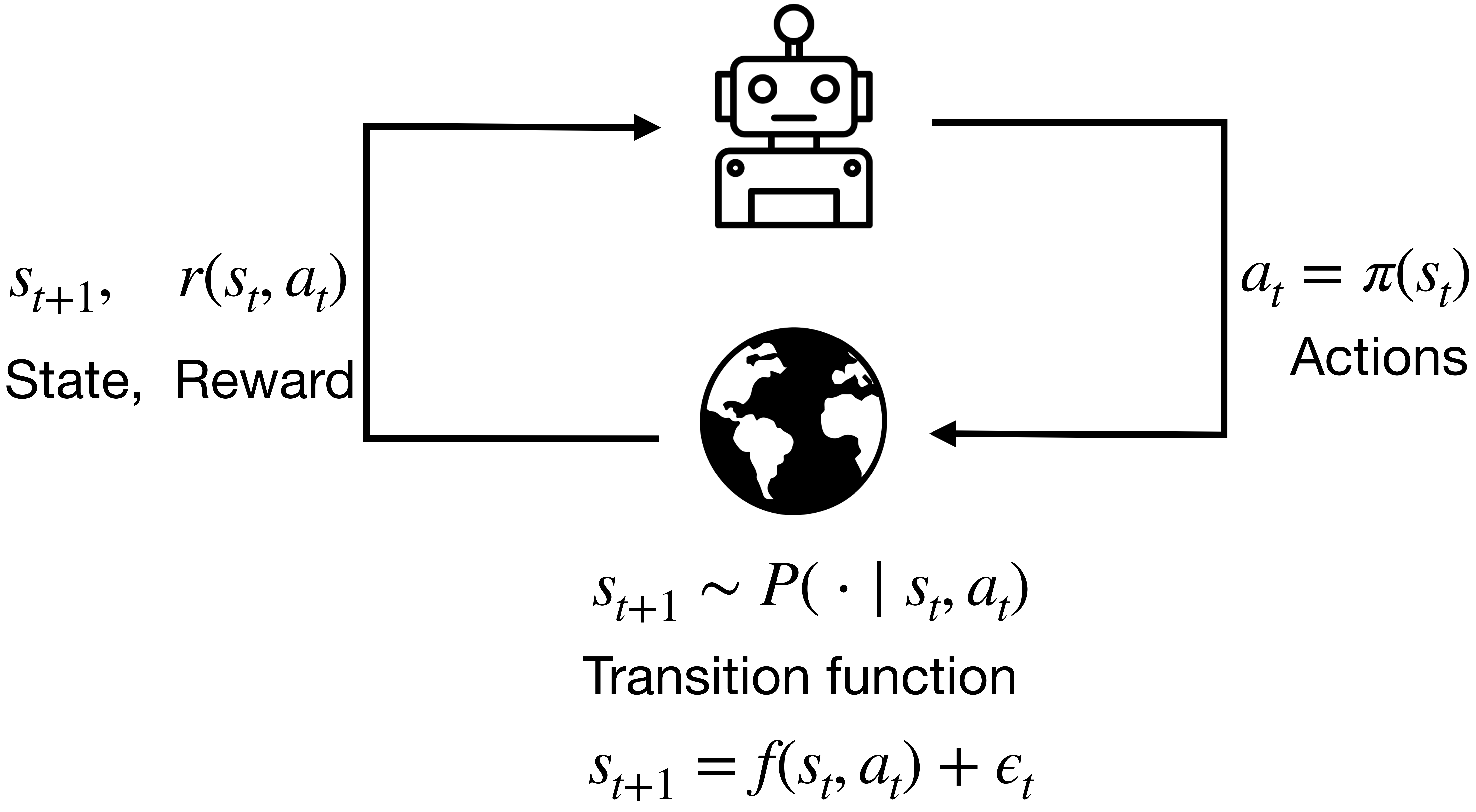

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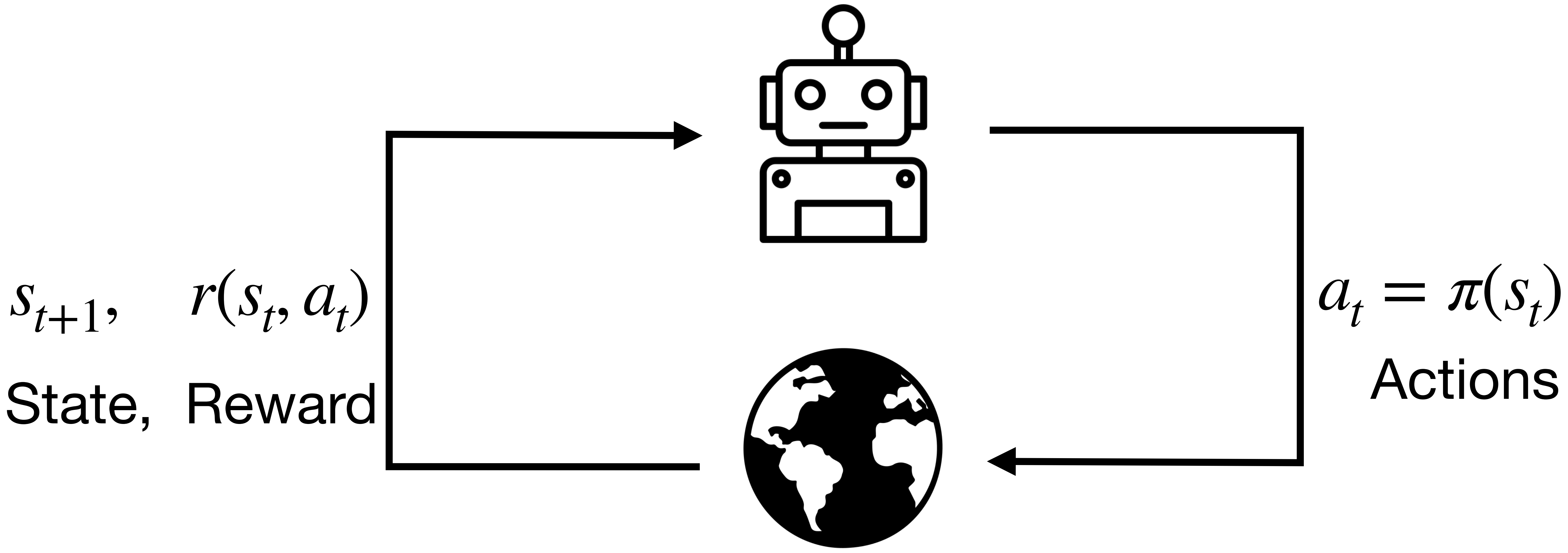
Policy $\pi : \mathcal{S} \rightarrow \mathcal{A}$



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Goal:

$$\max_{\pi} \mathbb{E}_{\pi,P}\left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \mid s_0 = s, \pi\right]$$

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Value function:

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Value function:

$$V_\pi(s) = \mathbb{E}_{\pi,P}\Big[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \mid s_0 = s, \pi\Big]$$

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Action-value function (aka Q-function):

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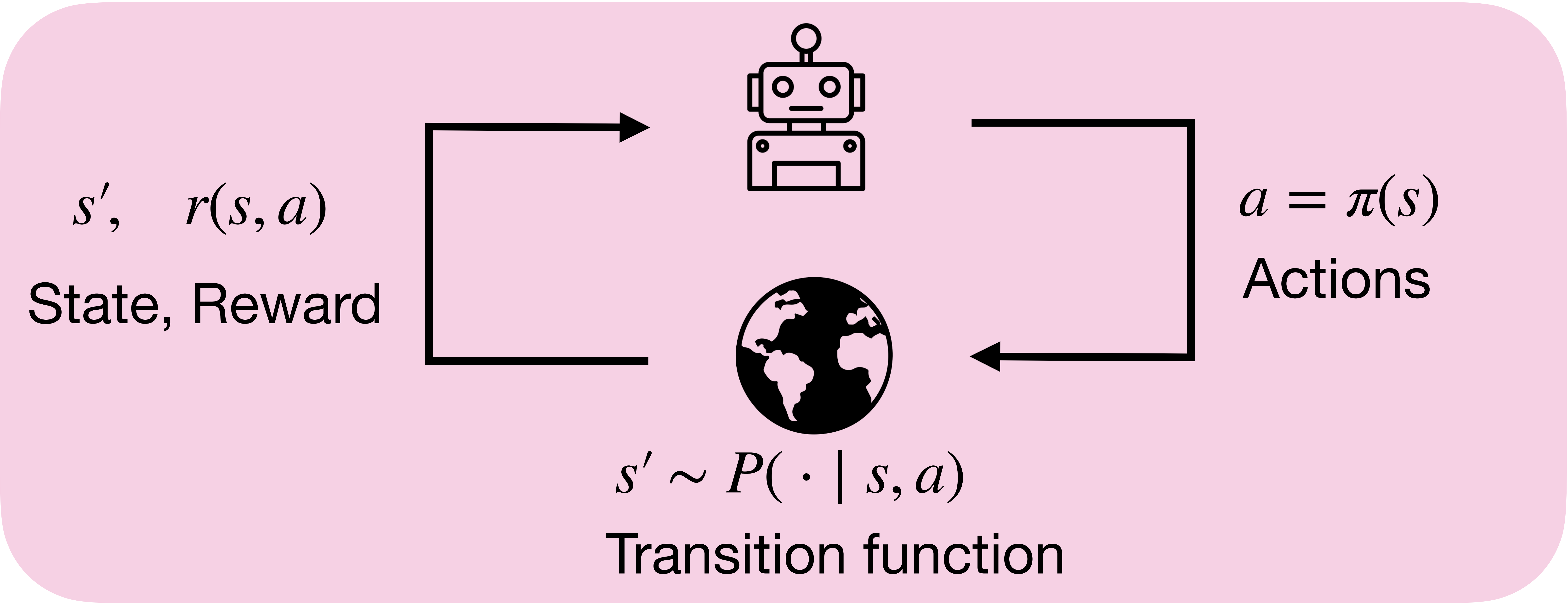
Action-value function (aka Q-function):

$$Q_\pi(s, a) = \mathbb{E}_{\pi,P}\left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \mid s_0 = s, a_0 = a, \pi\right]$$

Reinforcement Learning



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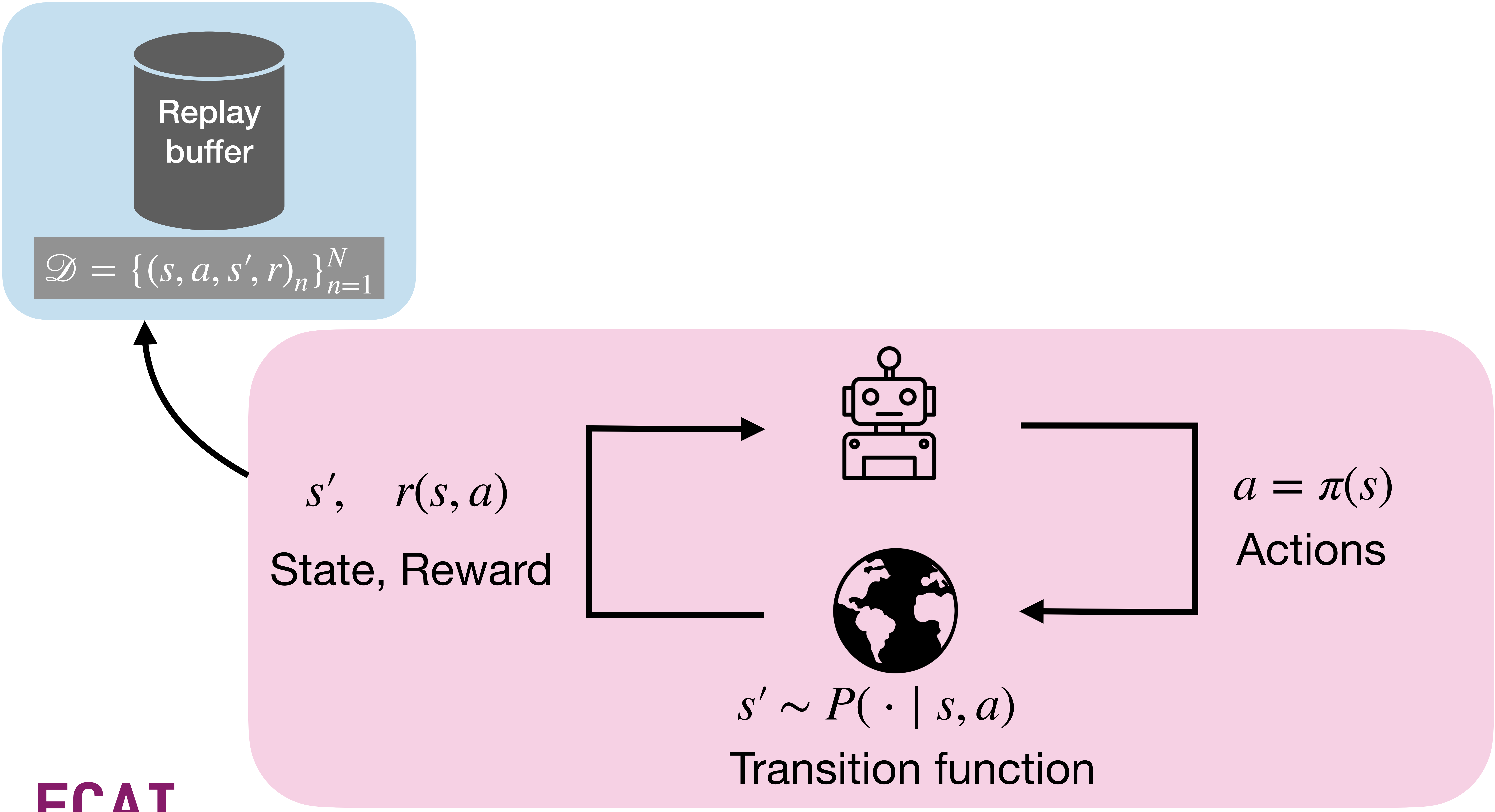

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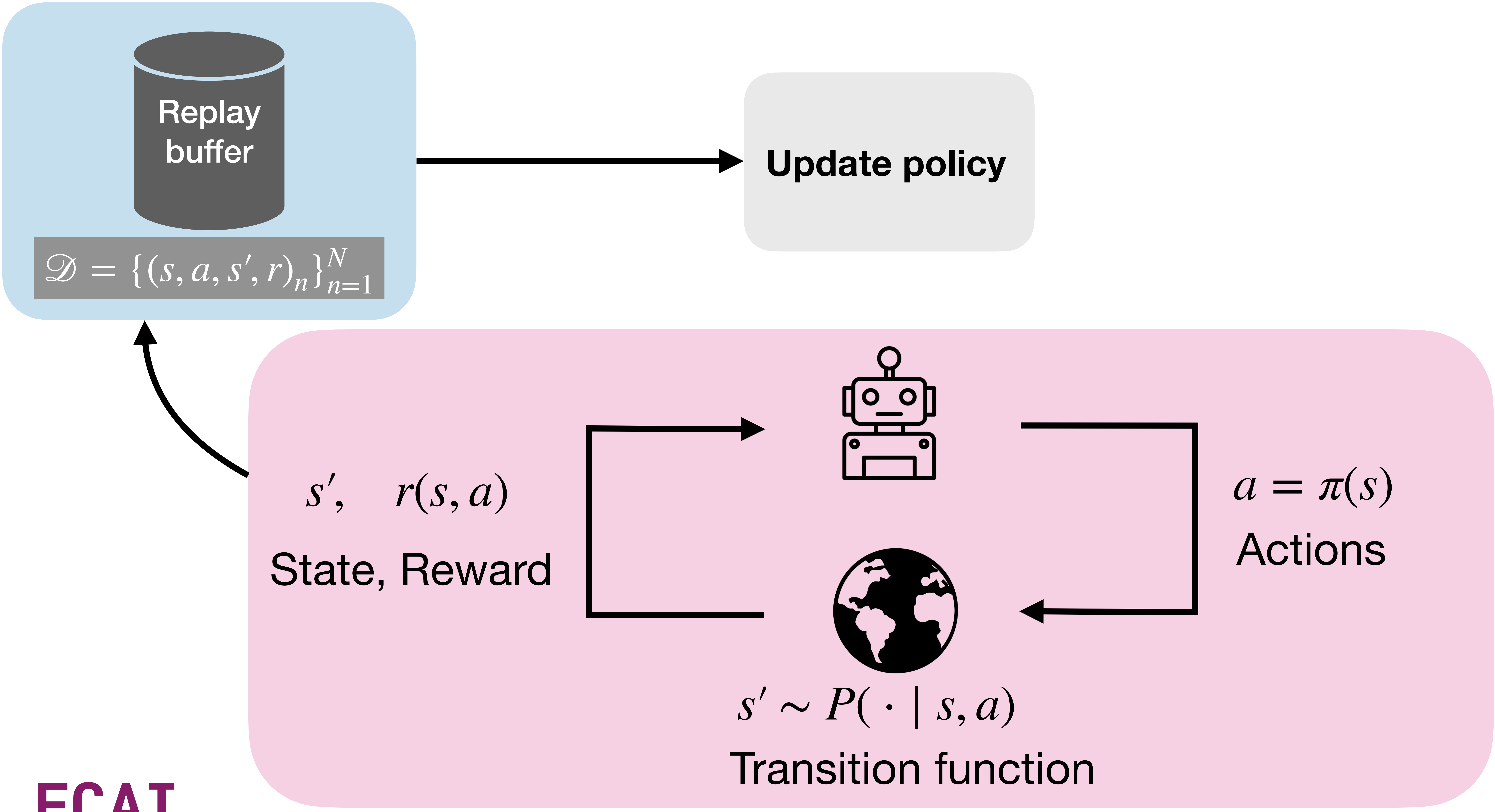

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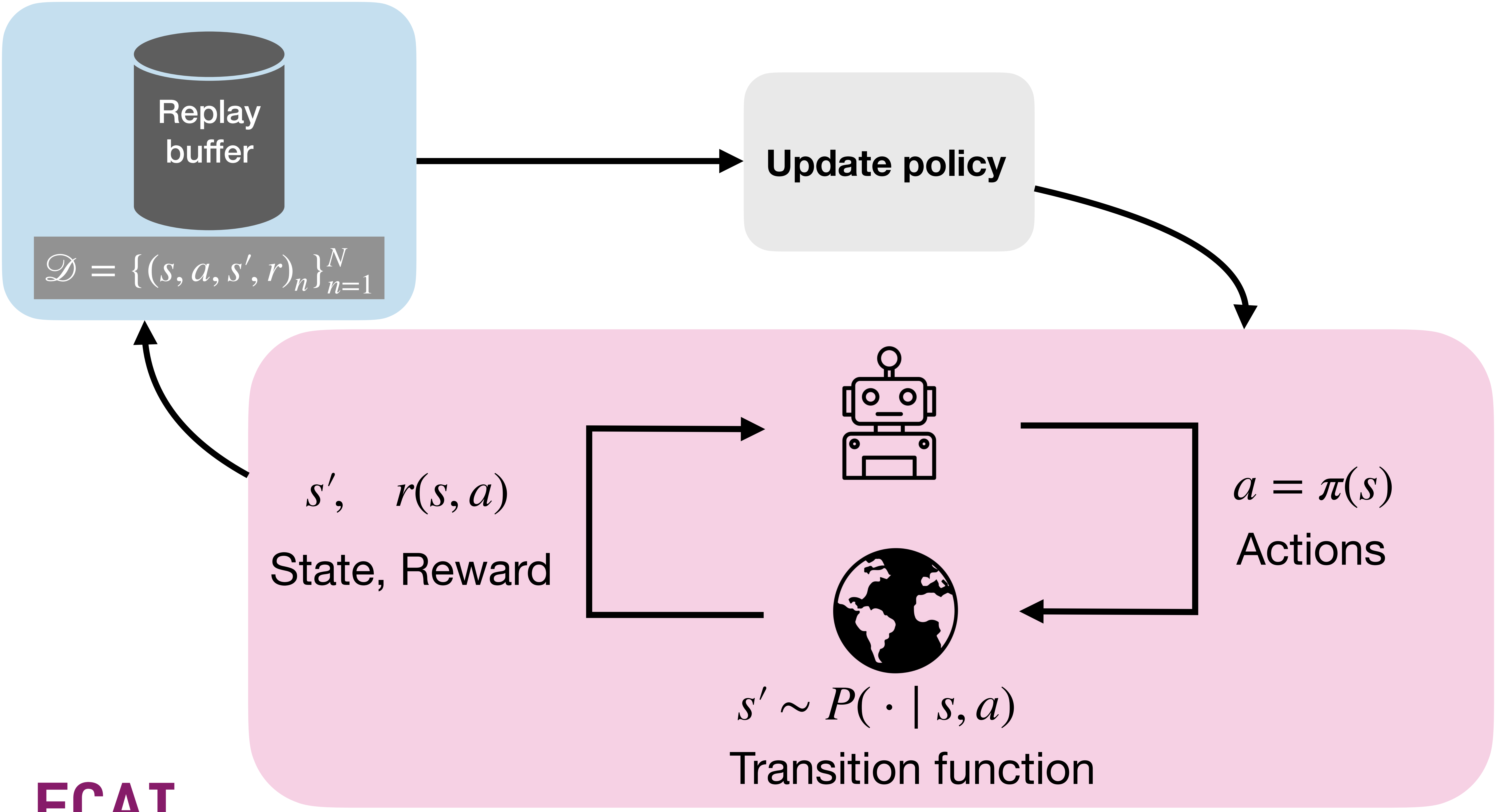
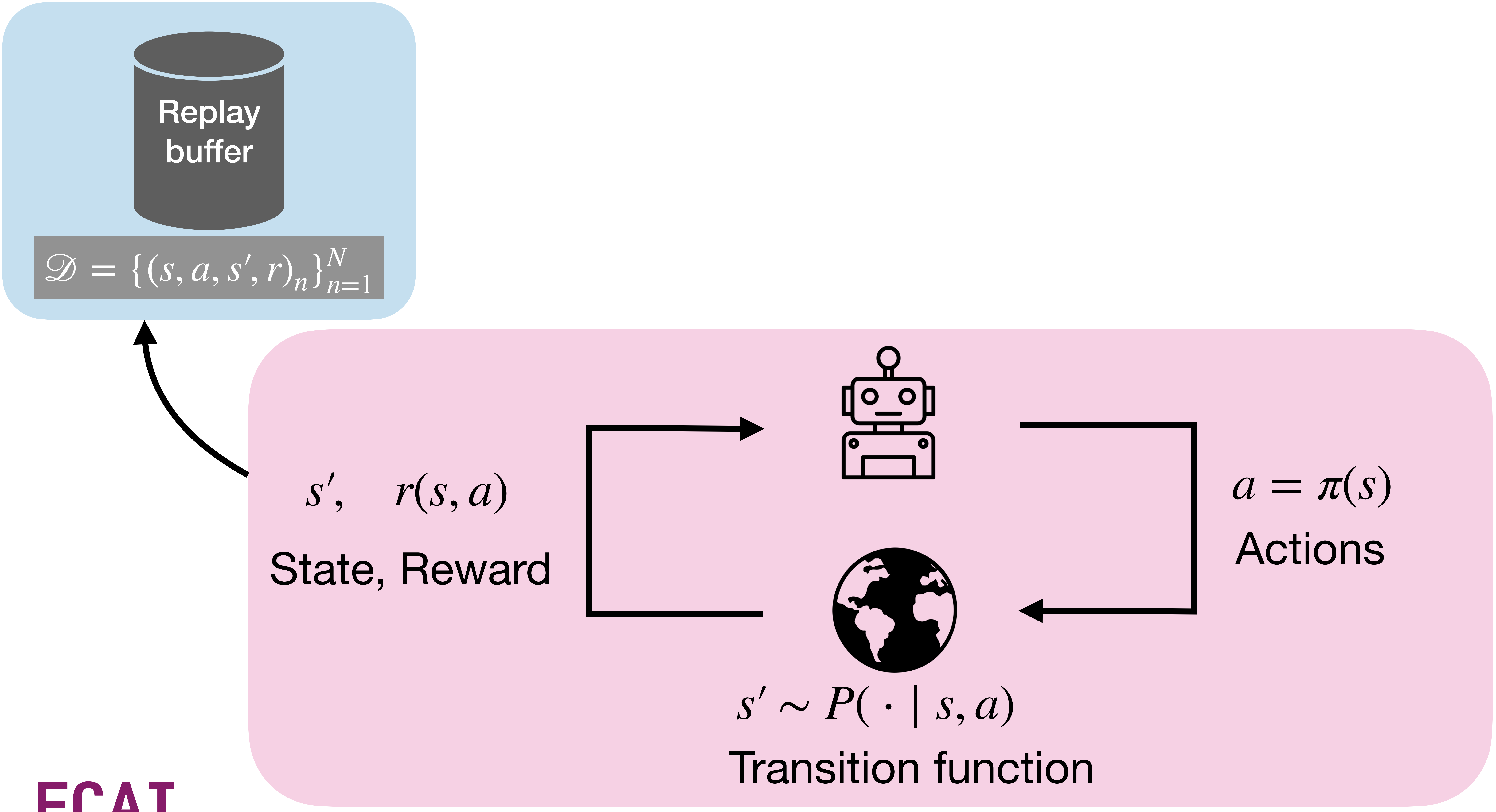


Model-based Reinforcement Learning



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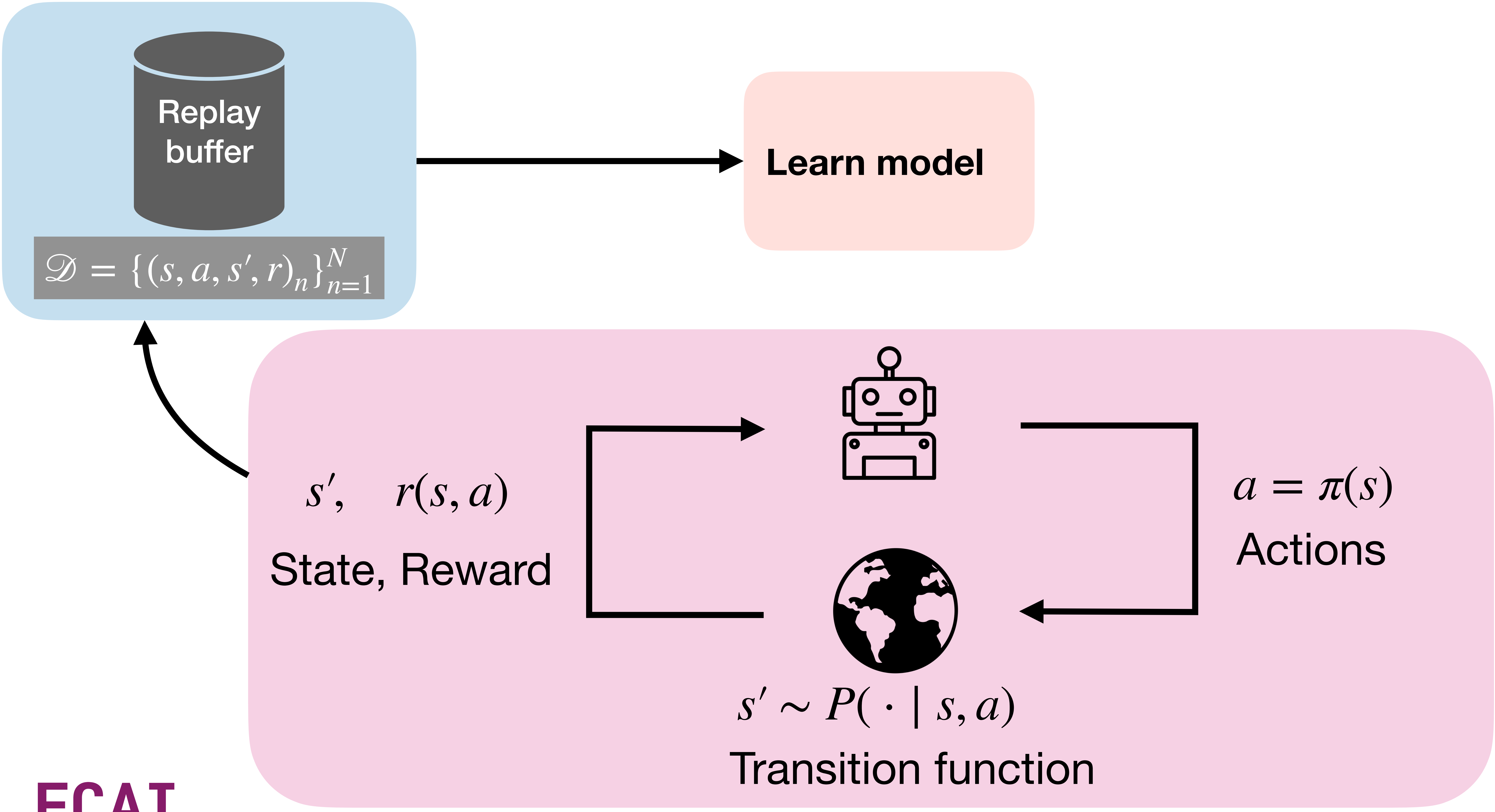

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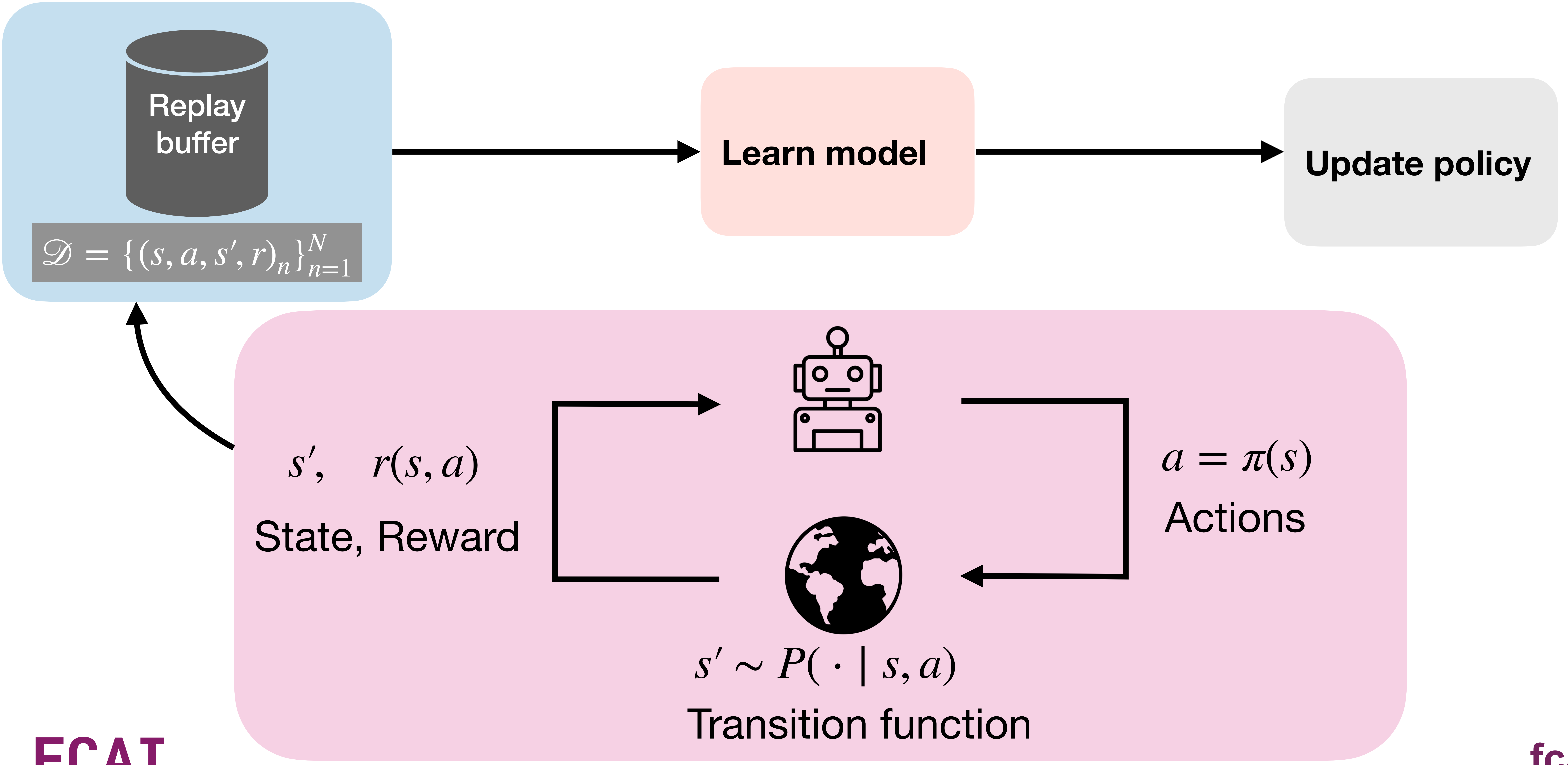

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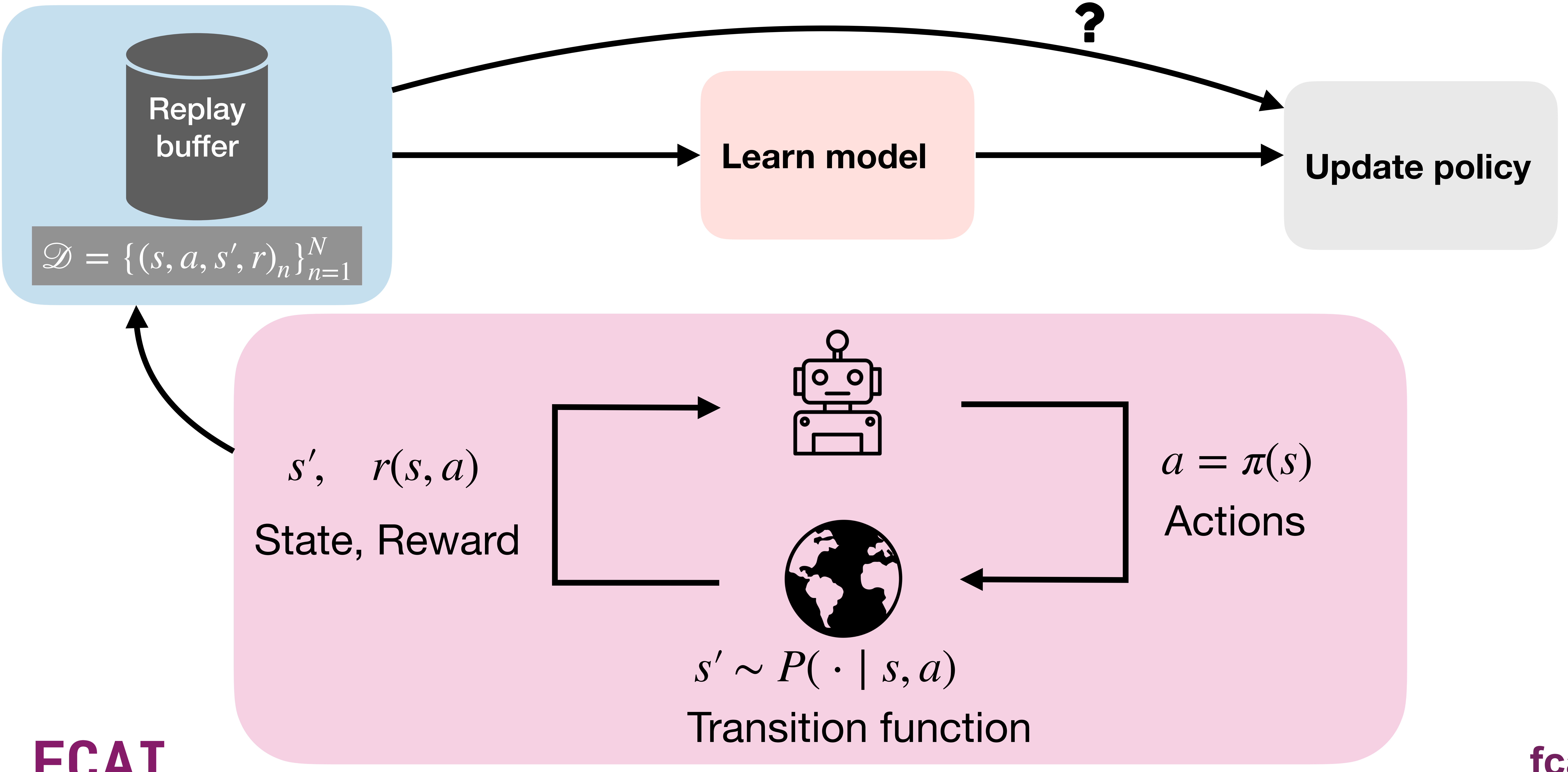

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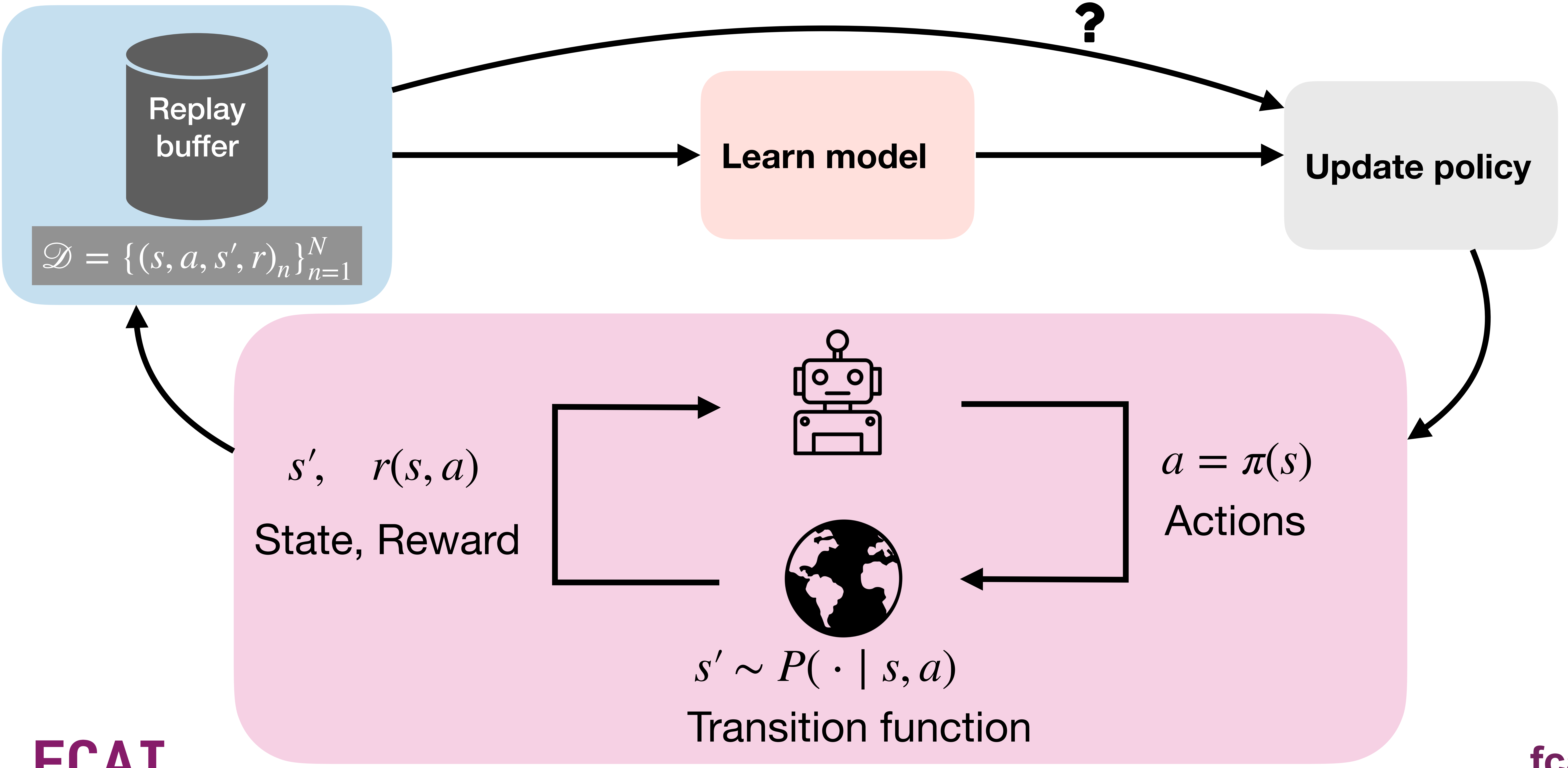

Reinforcement Learning Has Its Drawbacks

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Reinforcement Learning Has Its Drawbacks

RL has a sample efficiency problem!

Model-free vs Model-based RL

Model-free vs Model-based RL

	Model-free	Model-based

Model-free vs Model-based RL

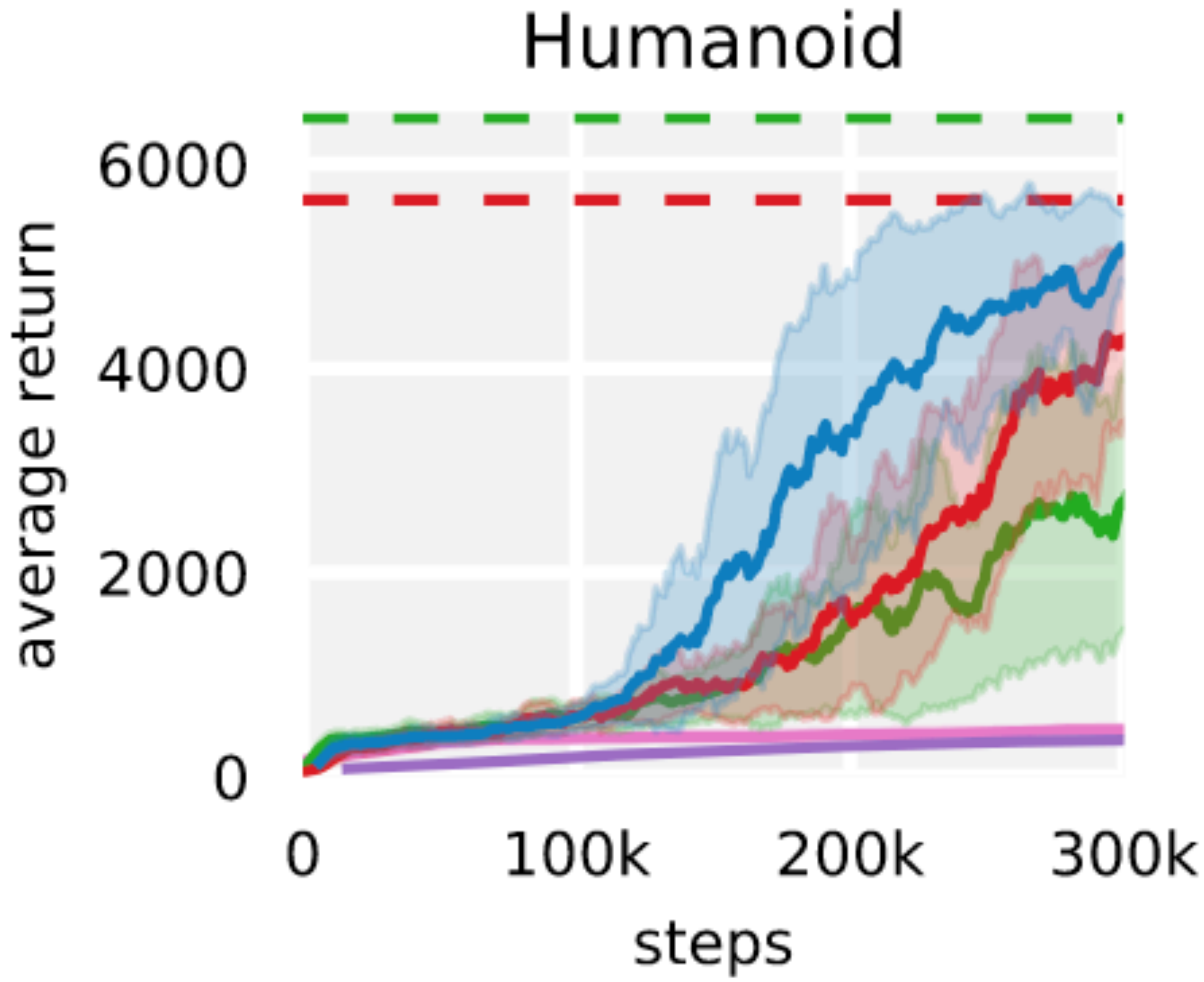
	Model-free	Model-based
Asymptotic performance	✅	Depends

Model-free vs Model-based RL

	Model-free	Model-based
Asymptotic performance	✅	Depends
Sample efficiency	❌	✅

Model-free vs Model-based RL

	Model-
Asymptotic performance	✅
Sample efficiency	❌



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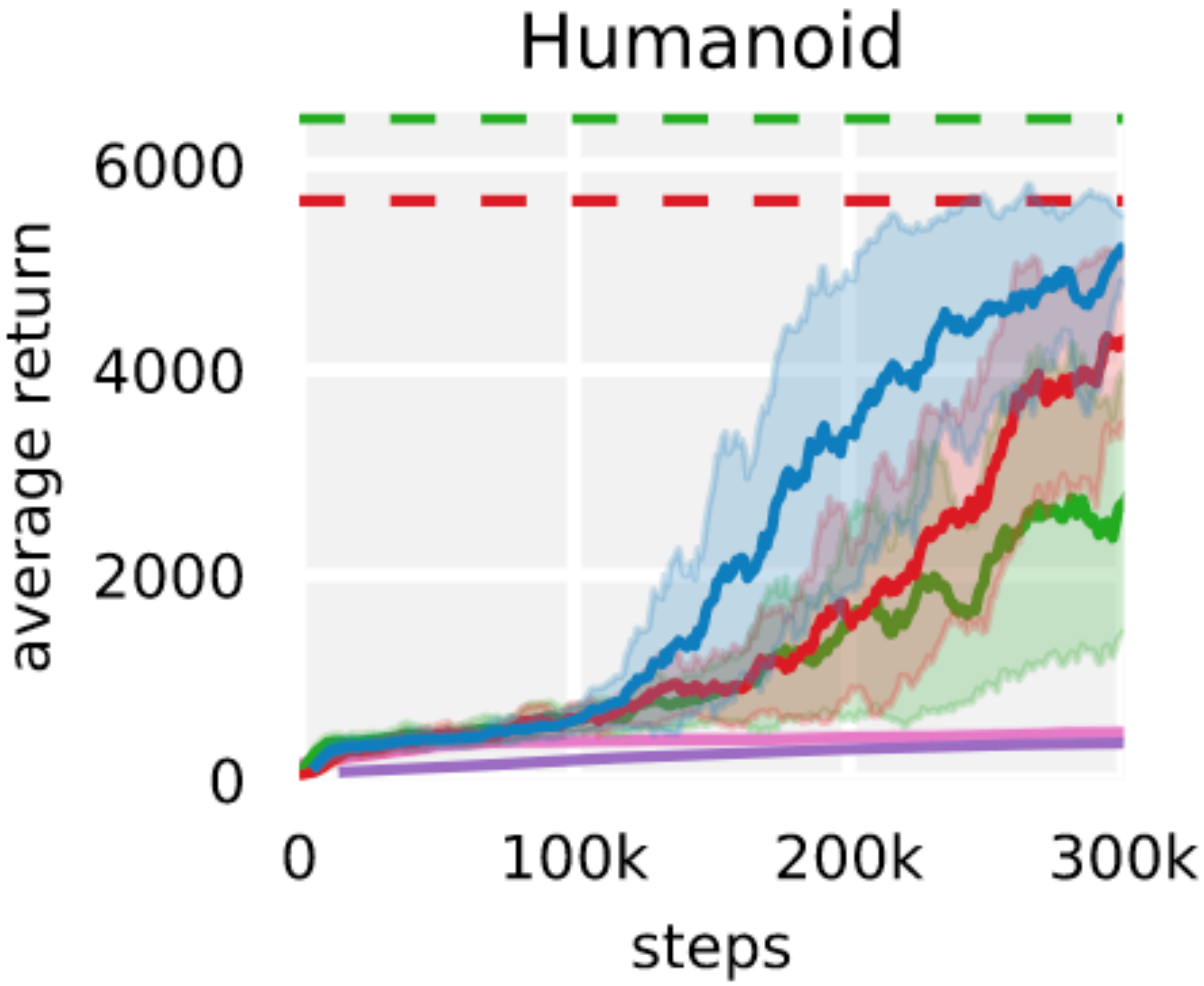

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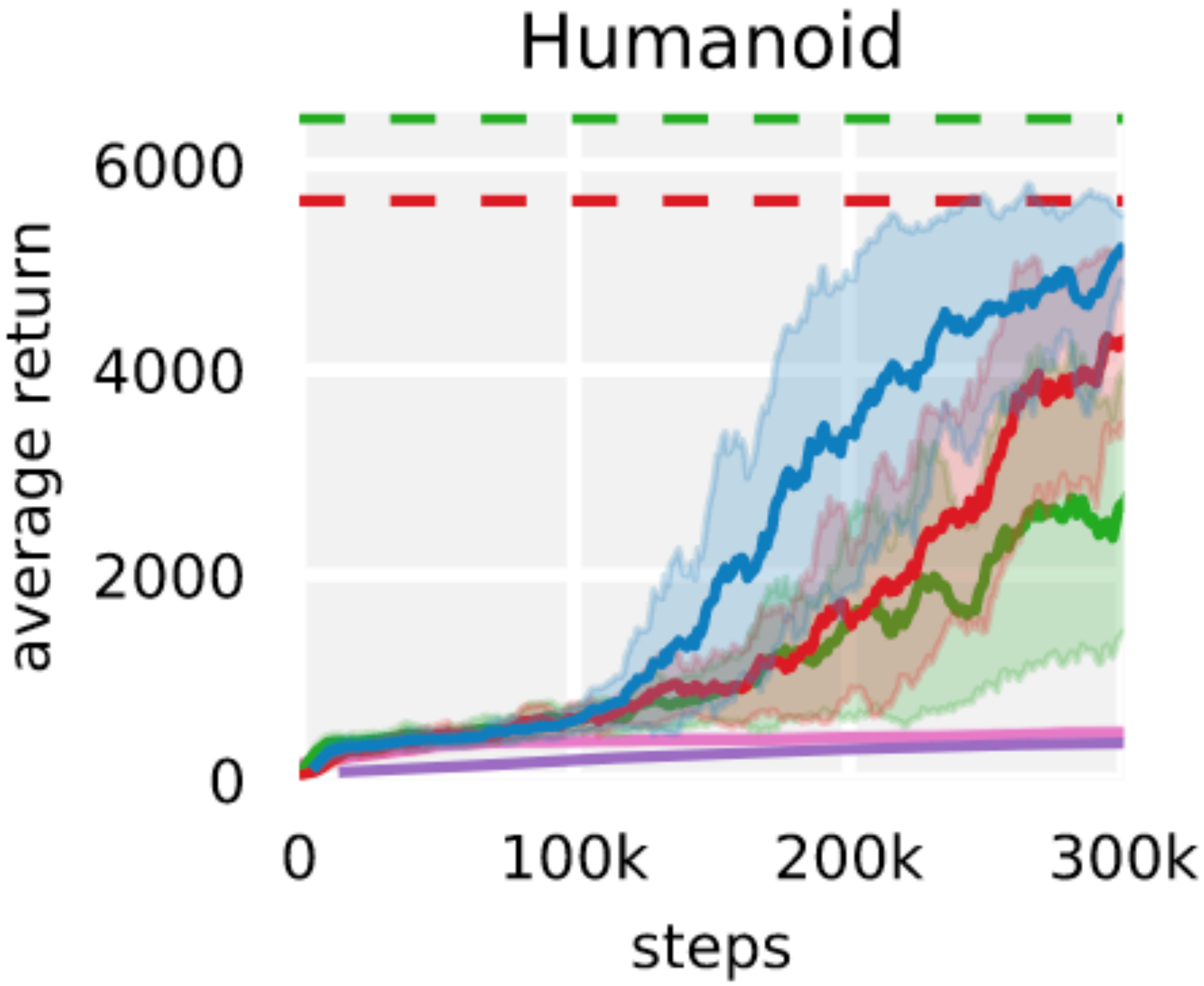

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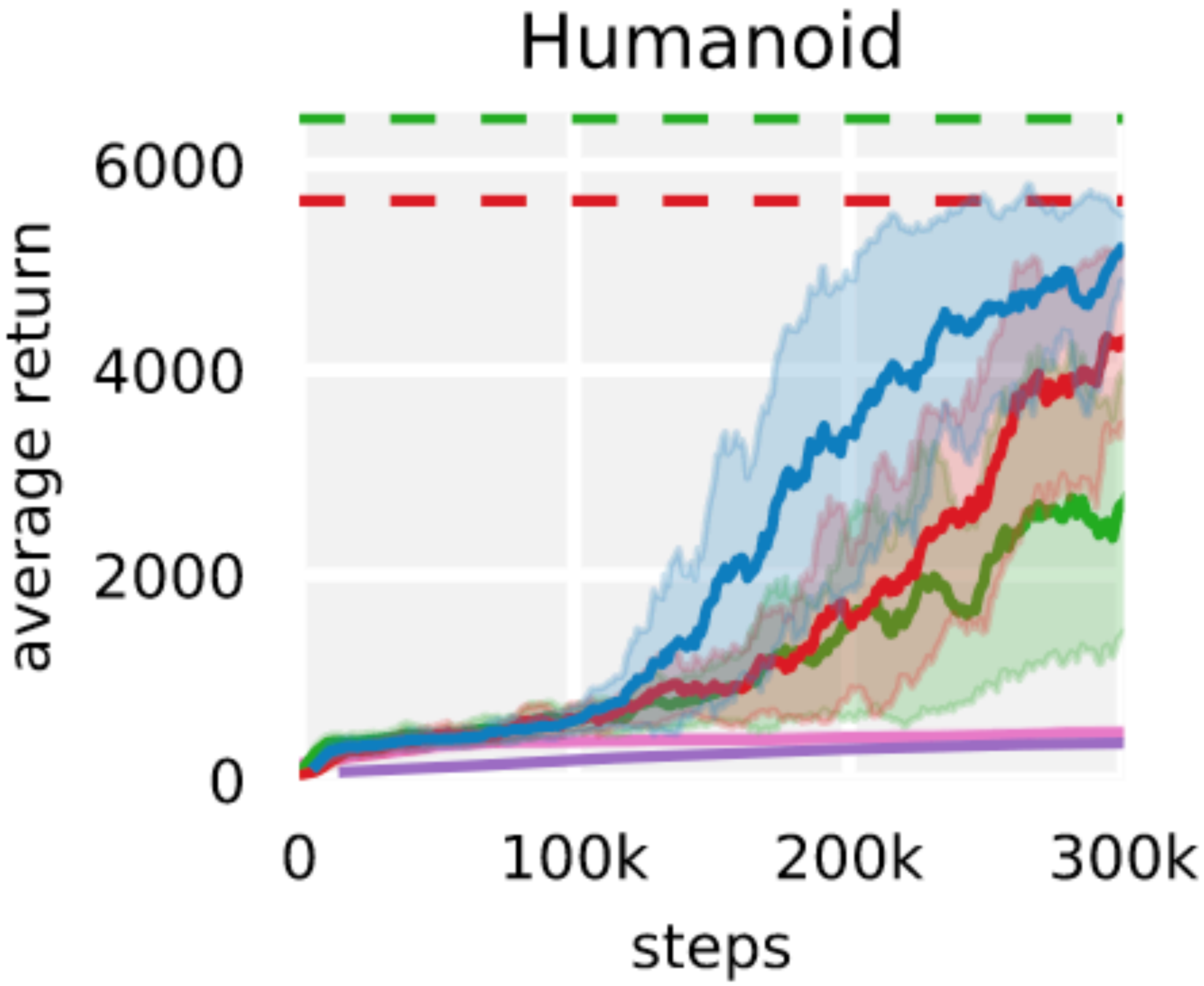
Model-free vs Model-based RL



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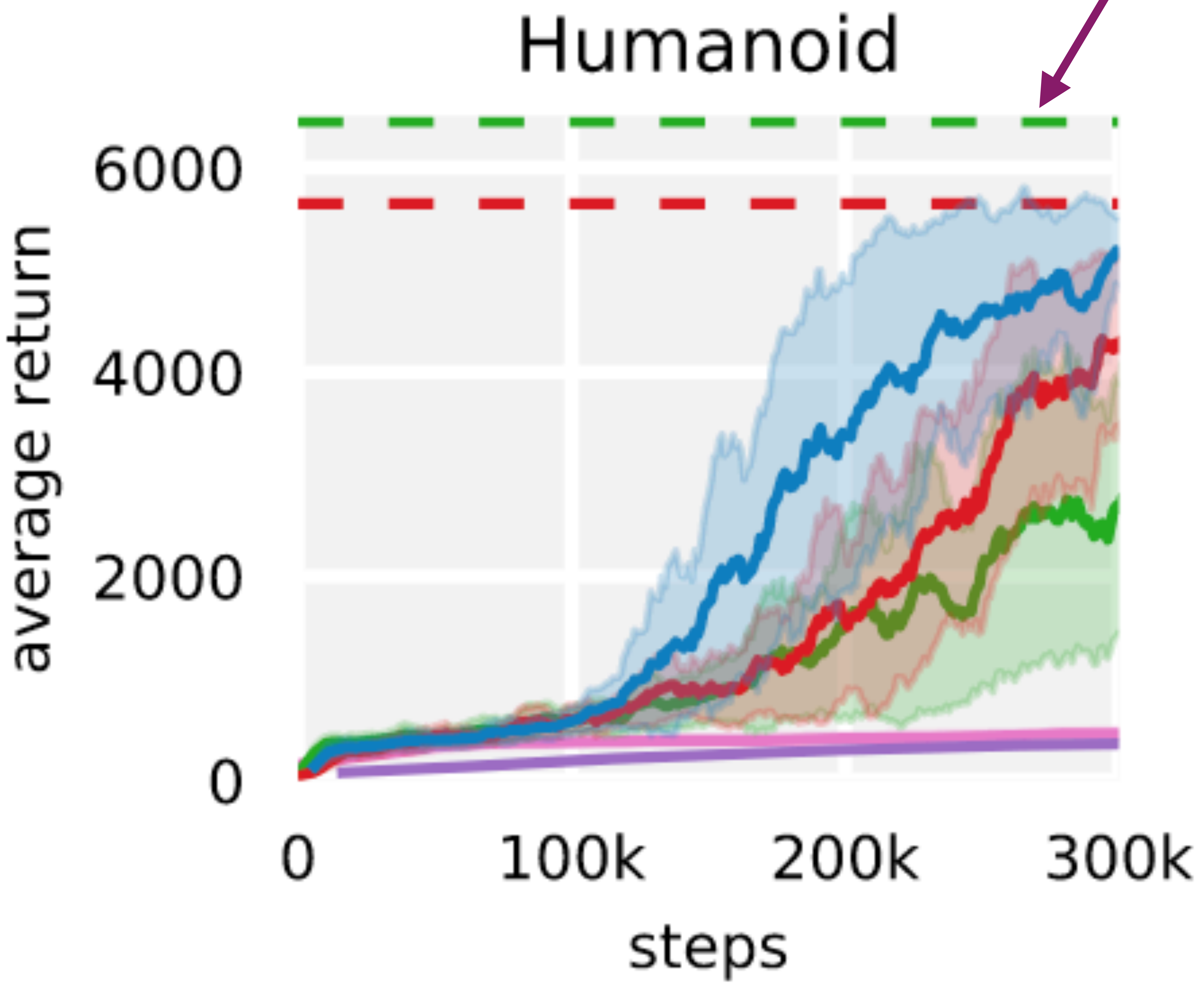
Model-free vs Model-based RL

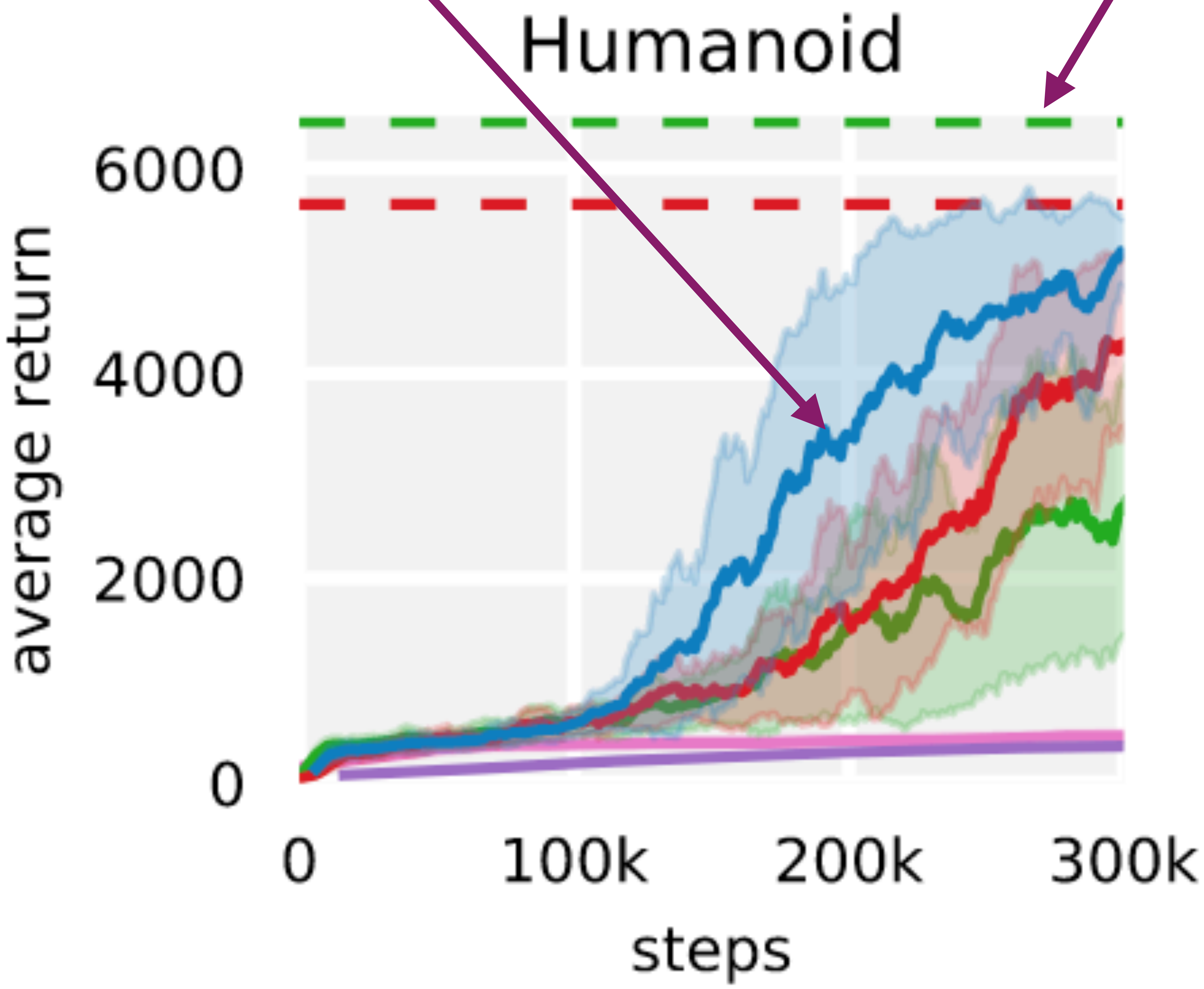
Best asymptotic performance



Model-free vs Model-based RL

Sample efficient

Best asymptotic performance



Model-free vs Model-based RL

	Model-free	Model-based
Asymptotic performance	✅	Depends
Sample efficiency	❌	✅

Model-free vs Model-based RL

	Model-free	Model-based
Asymptotic performance	✅	Depends
Sample efficiency	❌	✅
Computation at deployment	✅	❌/✅

Model-free vs Model-based RL

	Model-free	Model-based
Asymptotic performance	✅	Depends
Sample efficiency	❌	✅
Computation at deployment	✅	❌/✅
Adapting to new tasks	❌	✅

Model-free vs Model-based RL

	Model-free	Model-based
Asymptotic performance	✅	Depends
Sample efficiency	❌	✅
Computation at deployment	✅	❌/✅
Adapting to new tasks	❌	✅
Exploration	❌	✅

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*Definition: a model is a representation that **explicitly** encodes knowledge about the structure of the environment and task.*

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Dynamics (transition) model $s_{t+1} = f(s_t, a_t)$

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Definition: a model is a representation that ***explicitly*** *encodes knowledge about the structure of the environment and task.*

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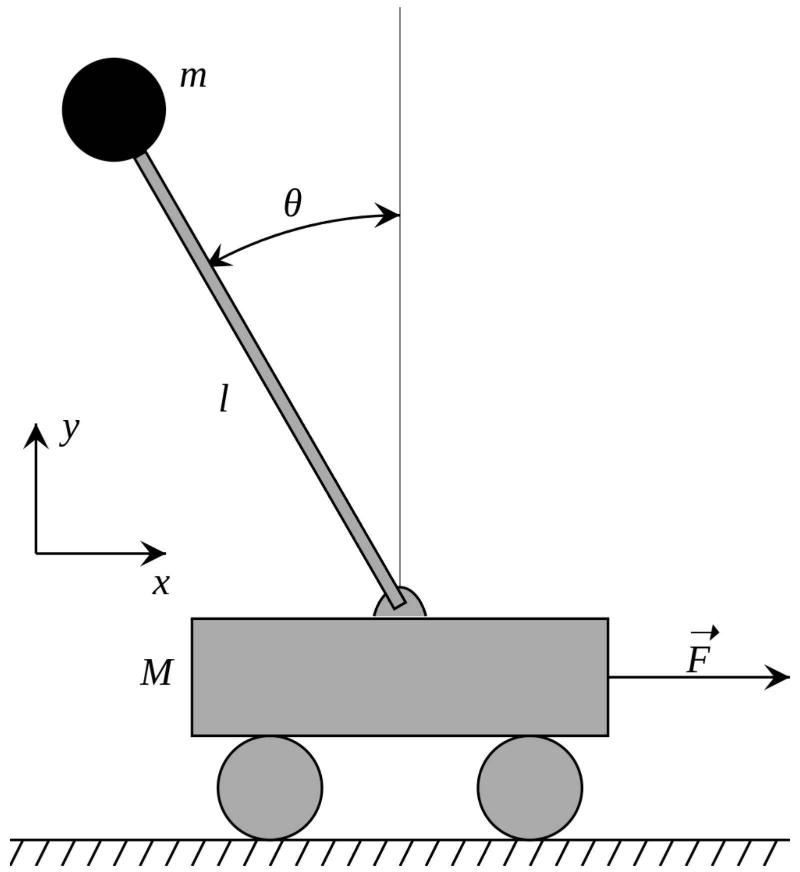
Typically this is what’s meant in model-based RL

What is a “Model”?

What is a “Model”?



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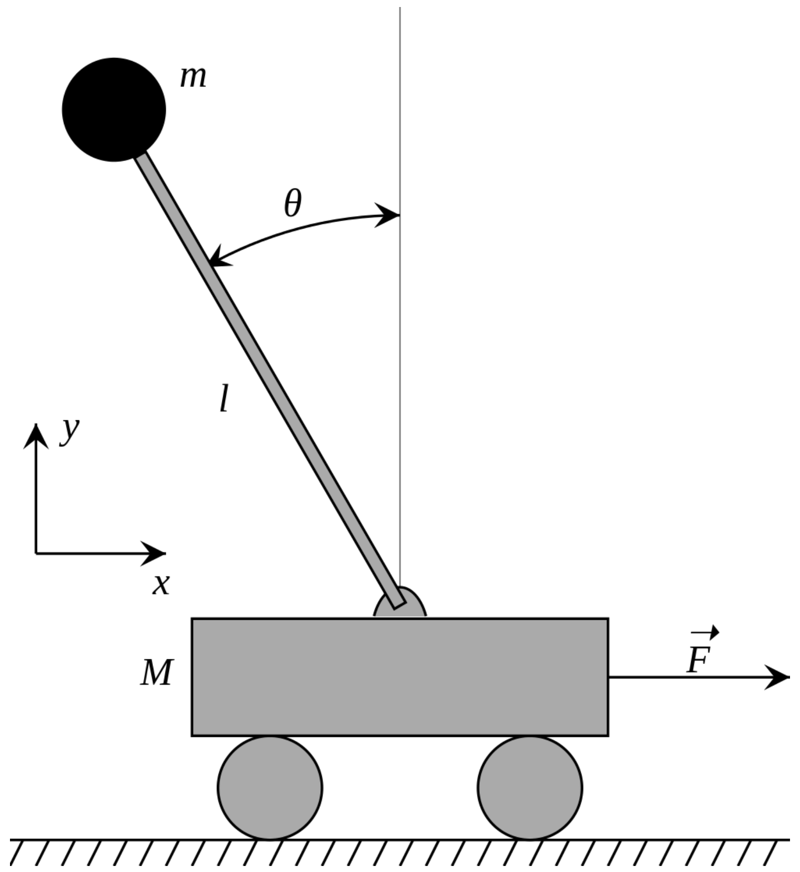

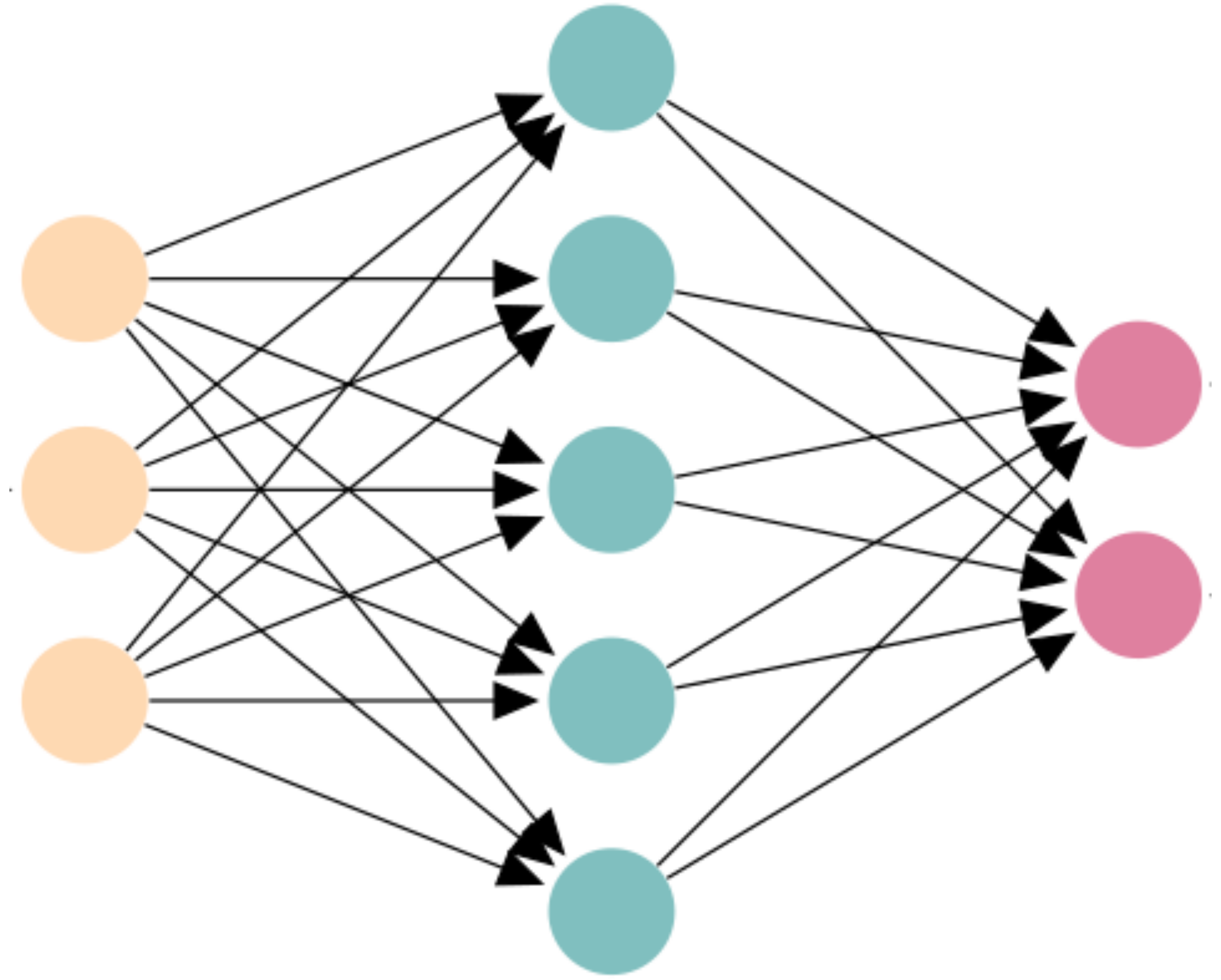

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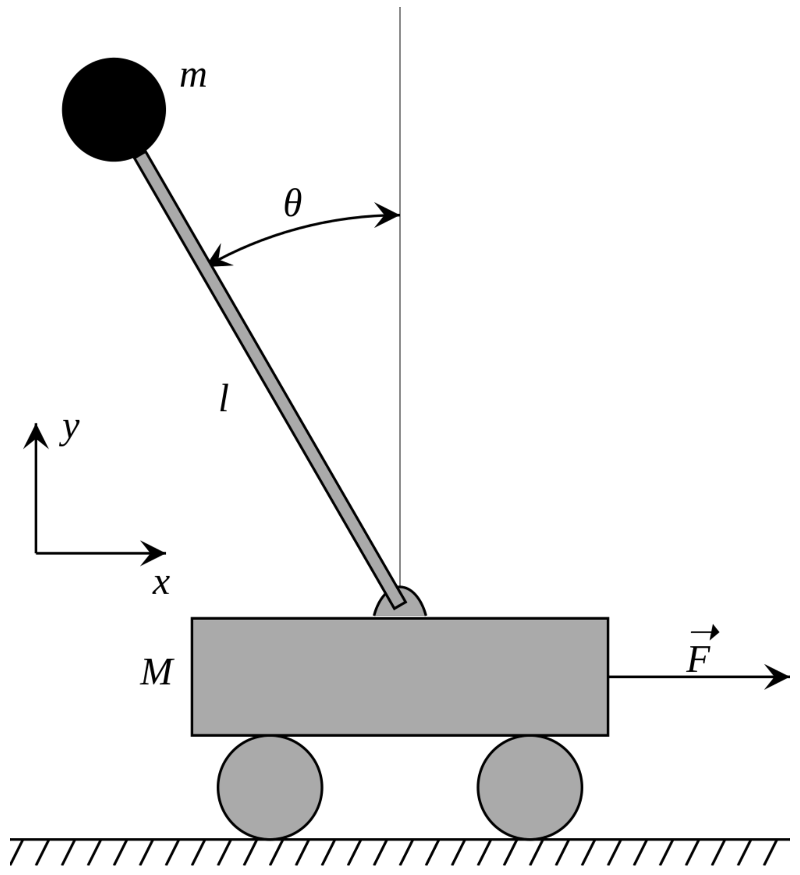

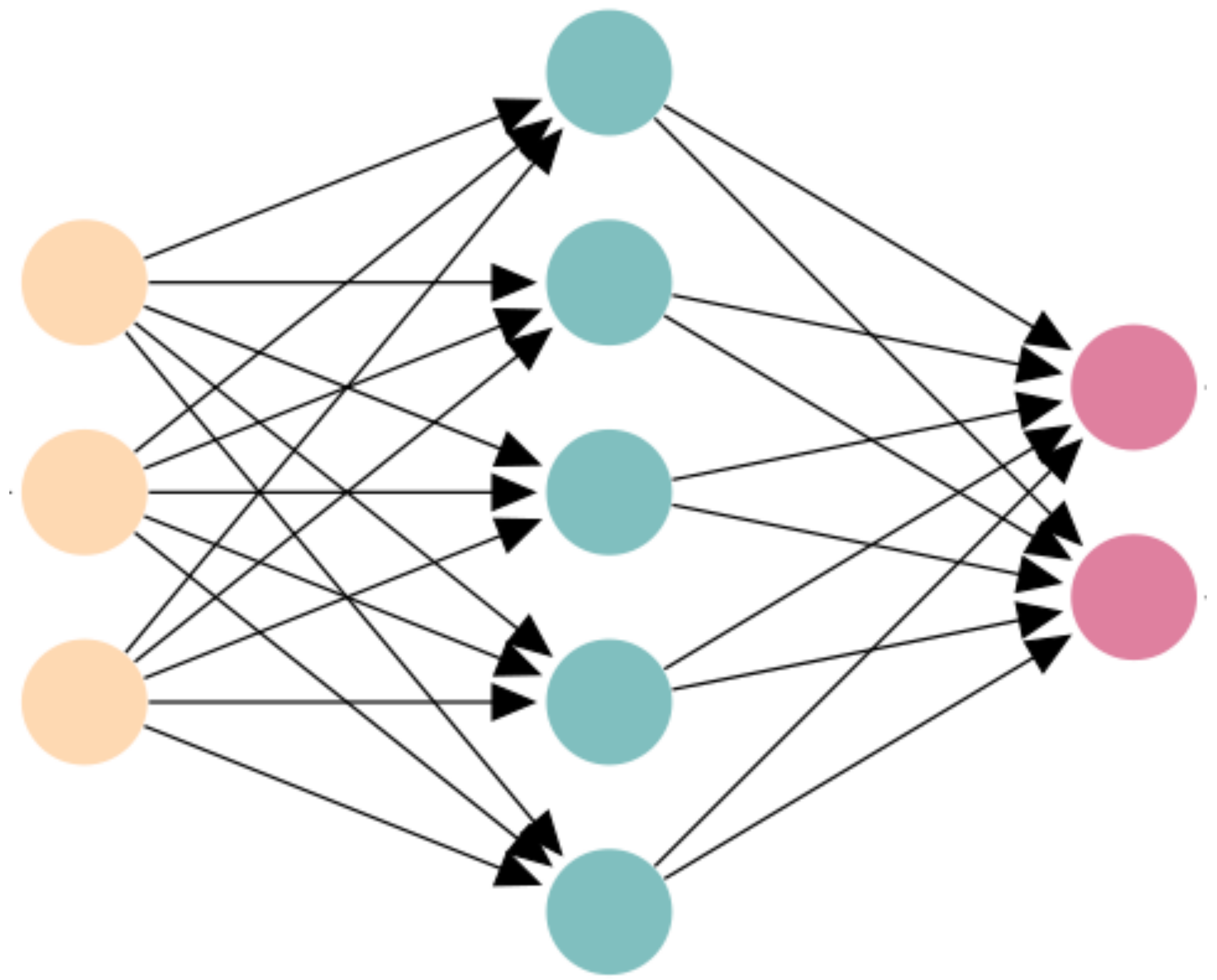

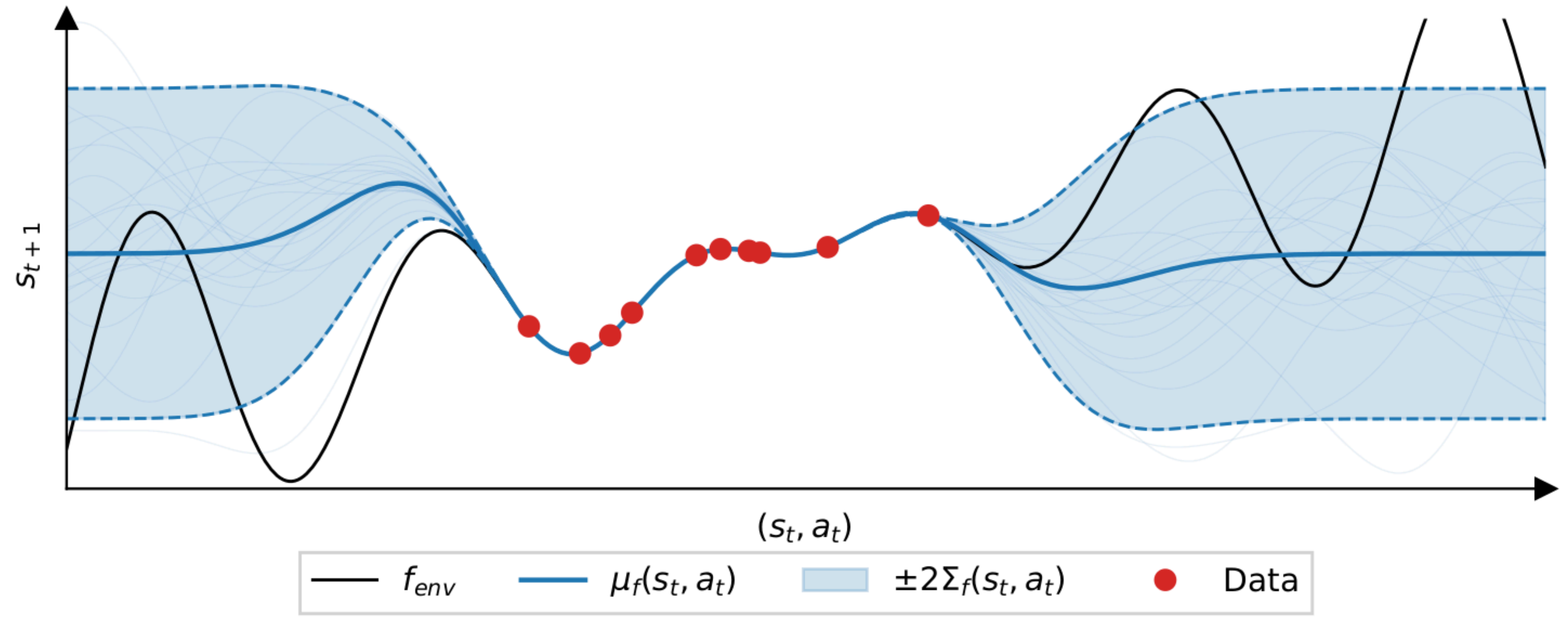

Learning Objectives

Understand

1. ~~What a "model" is in model-based RL~~
2. How a "model" can aid decision making
3. Differences between background and decision-time planning
4. Decision-time planning strategies for continuous actions
5. Sources of uncertainty in model-based RL
6. Rationale and insights for decision-making under uncertainty

Planning

How Do We Use The "Model”?

Background vs Decision-time Planning

Time of
Planning

How Do We Use The "Model"?

Background vs Decision-time Planning

Time of Planning

Decision-time Planning

How Do We Use The "Model”?

Background vs Decision-time Planning

Time of Planning

Decision-time Planning

- Find best action for current situation

How Do We Use The "Model”?

Background vs Decision-time Planning

Time of Planning

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- Find best action for current situation

Background Planning

How Do We Use The "Model”?

Background vs Decision-time Planning

Time of Planning

Decision-time Planning

- Find best action for current situation

Background Planning

- Learn (from past data) how to act in any situation

How Do We Use The "Model”?

Background vs Decision-time Planning

Time of Planning

Decision-time Planning

- Find best action for current situation

Continuous actions

Background Planning

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How Do We Use The "Model”?

Background vs Decision-time Planning

Time of Planning

Decision-time Planning

- Find best action for current situation

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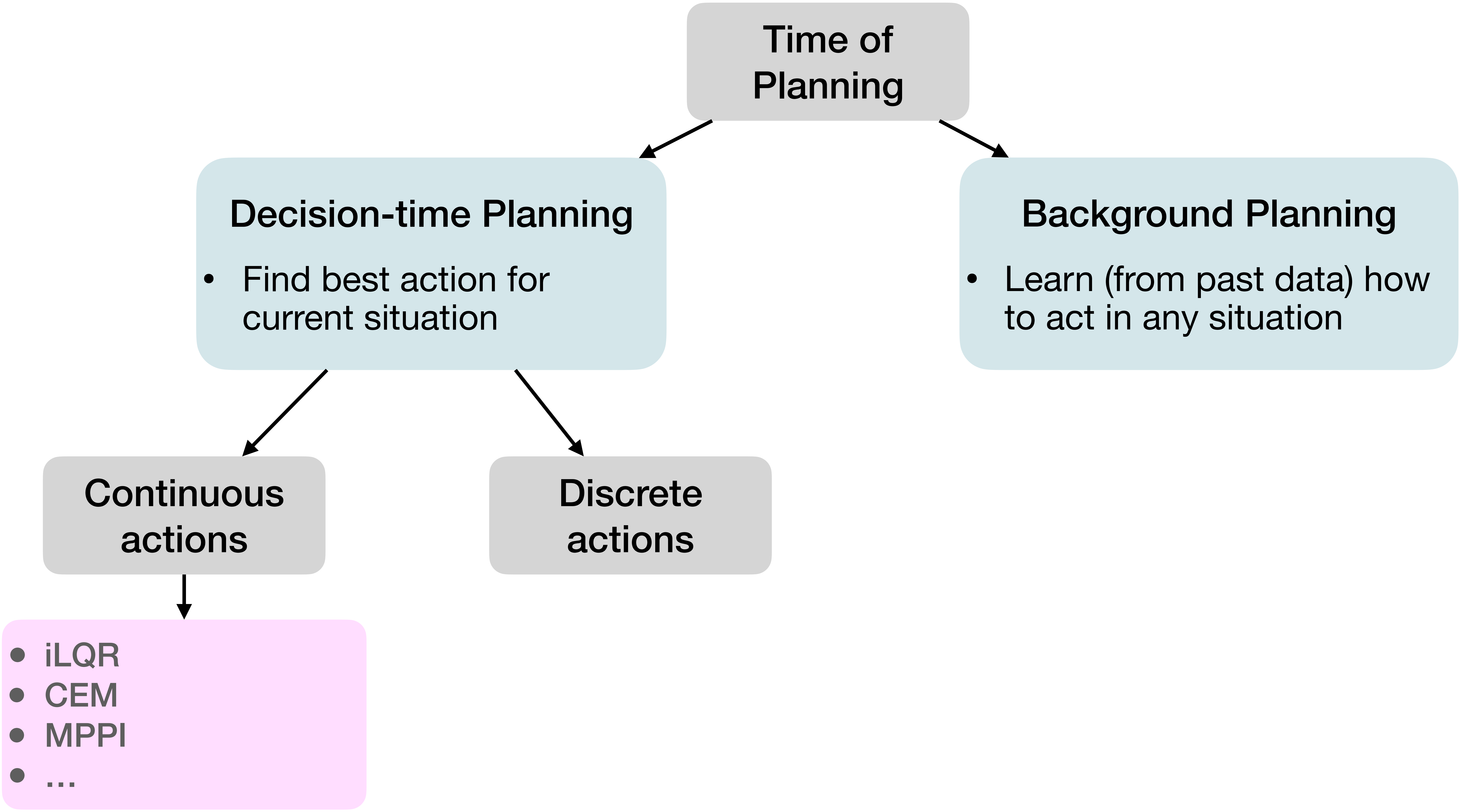
Continuous actions

- iLQR
- CEM
- MPPI
- ...

How Do We Use The "Model”?

Background vs Decision-time Planning



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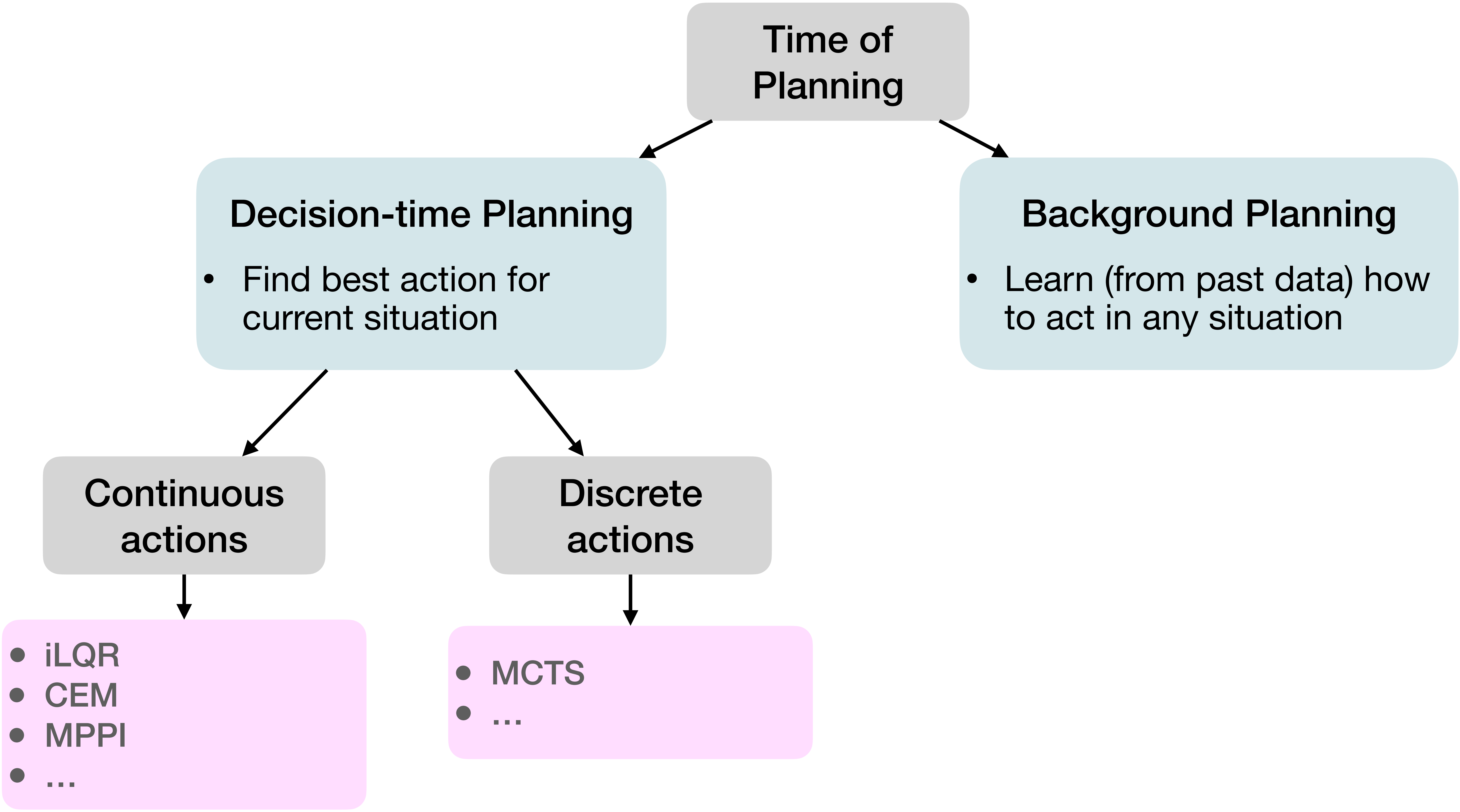

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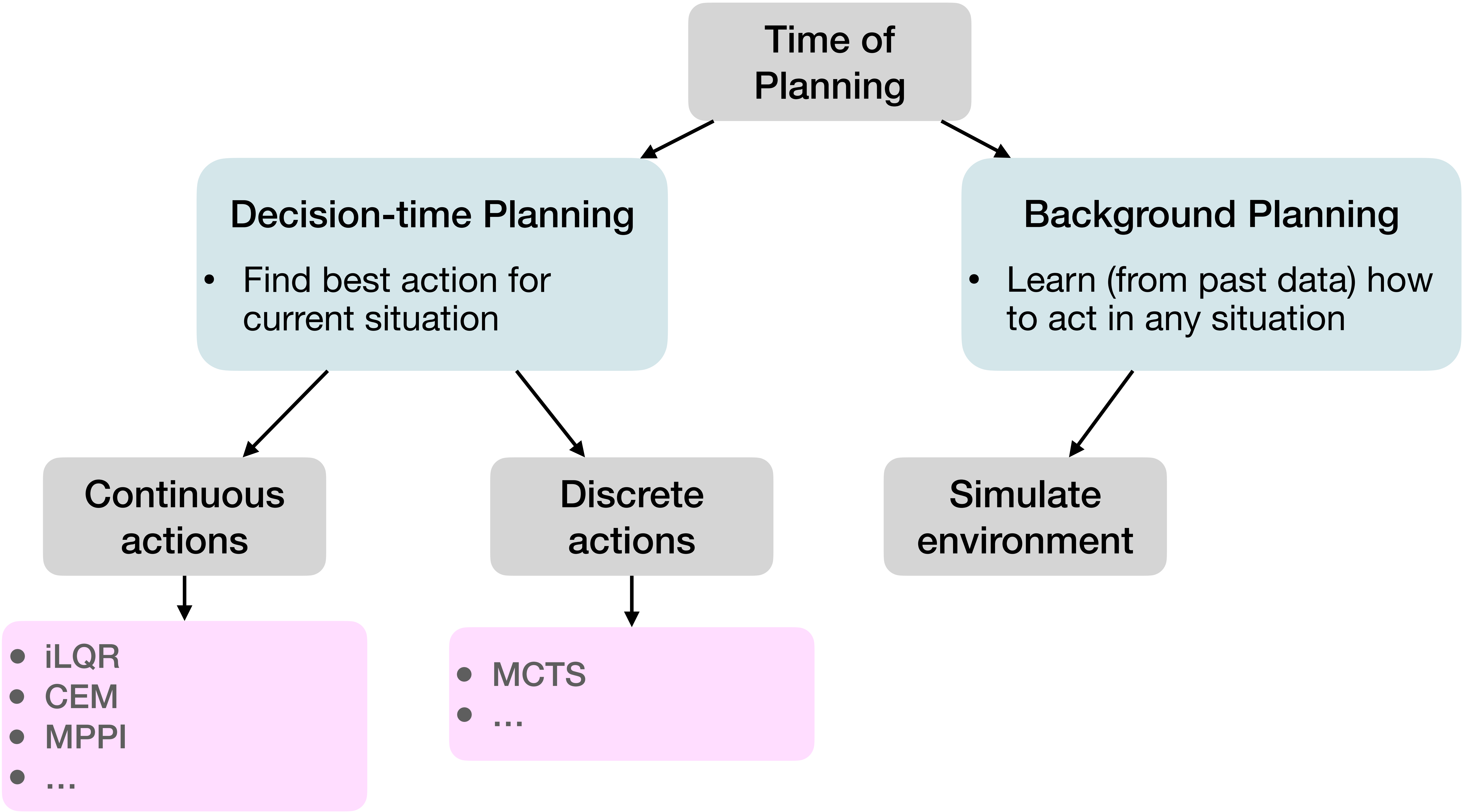

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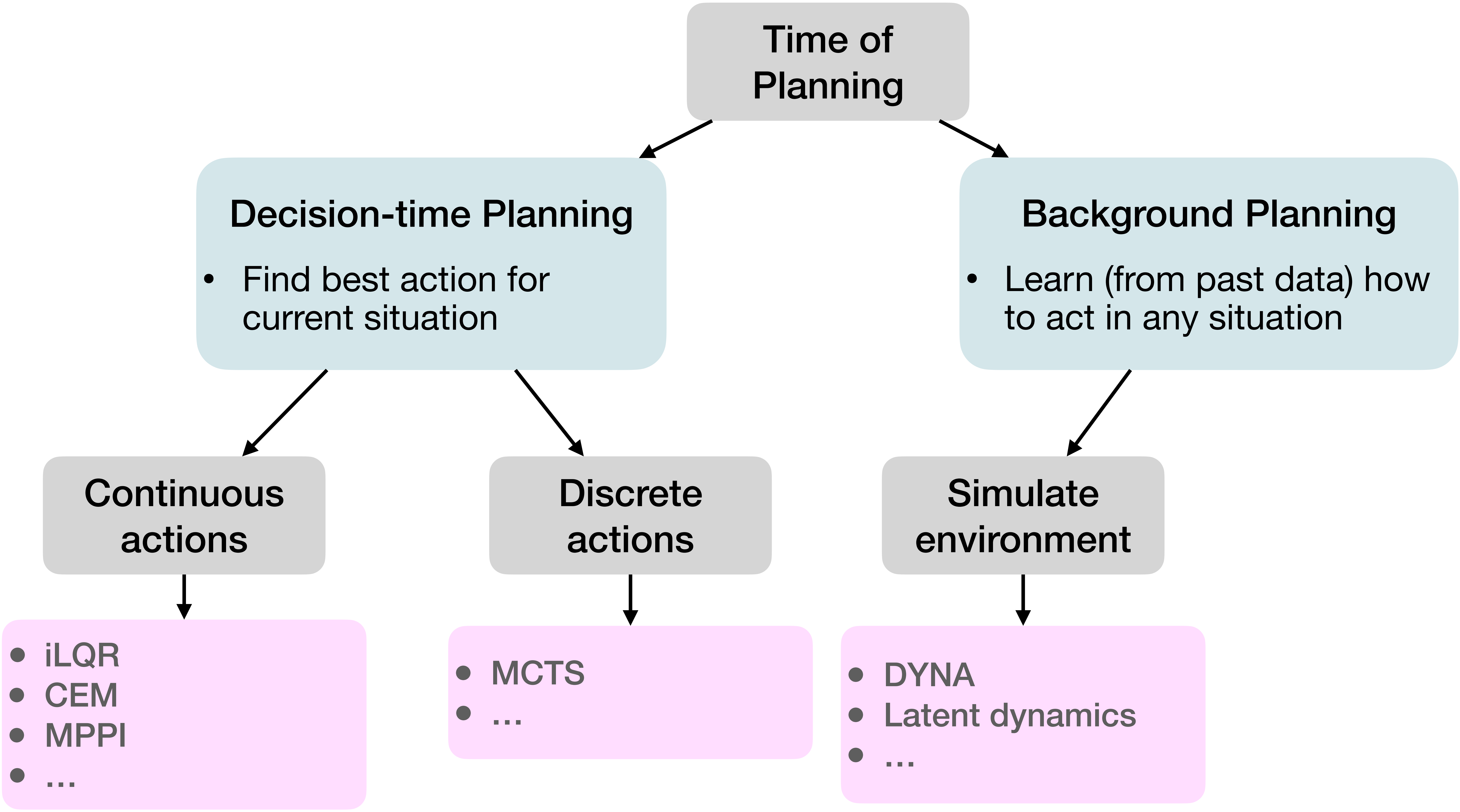

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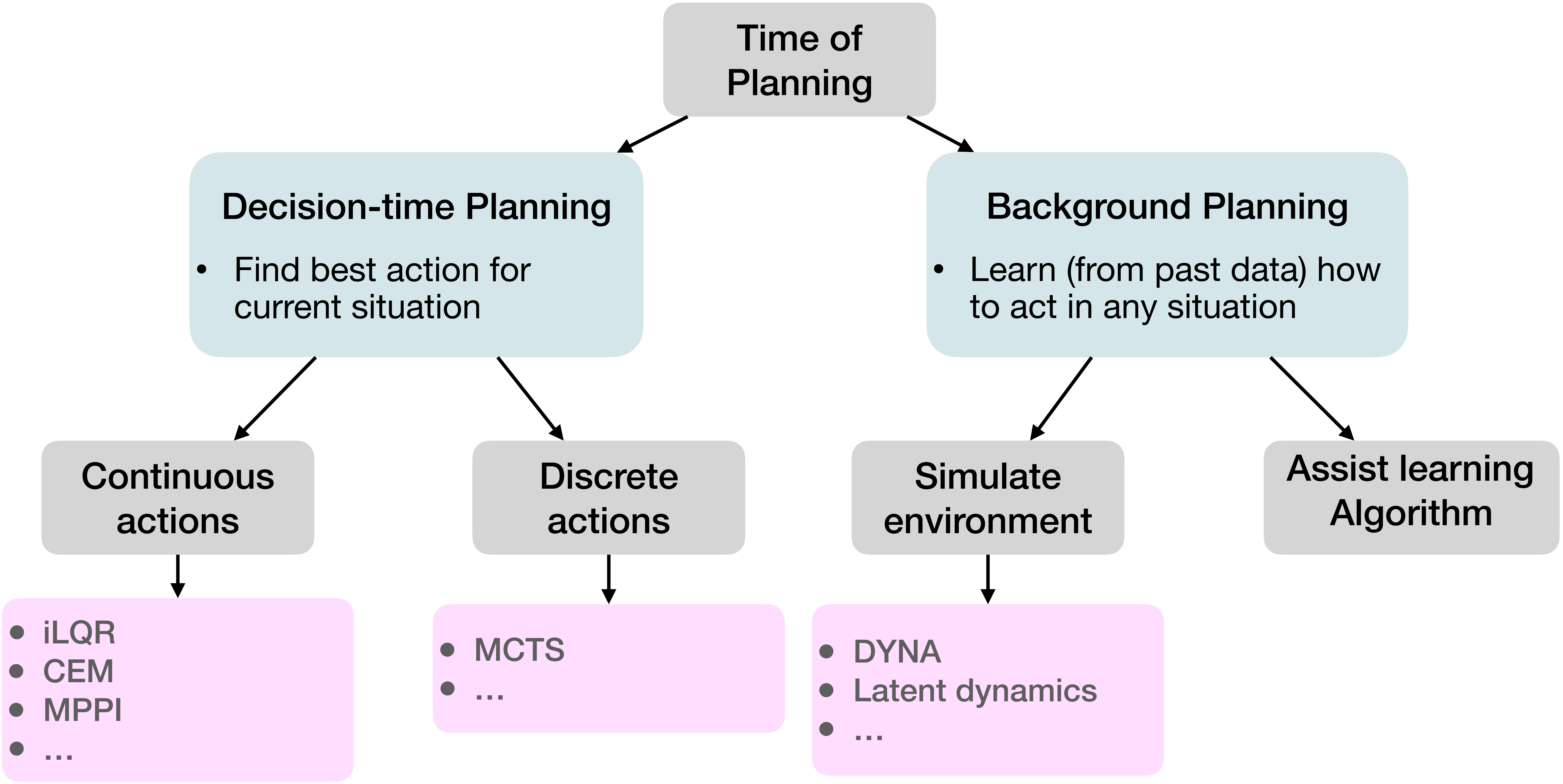

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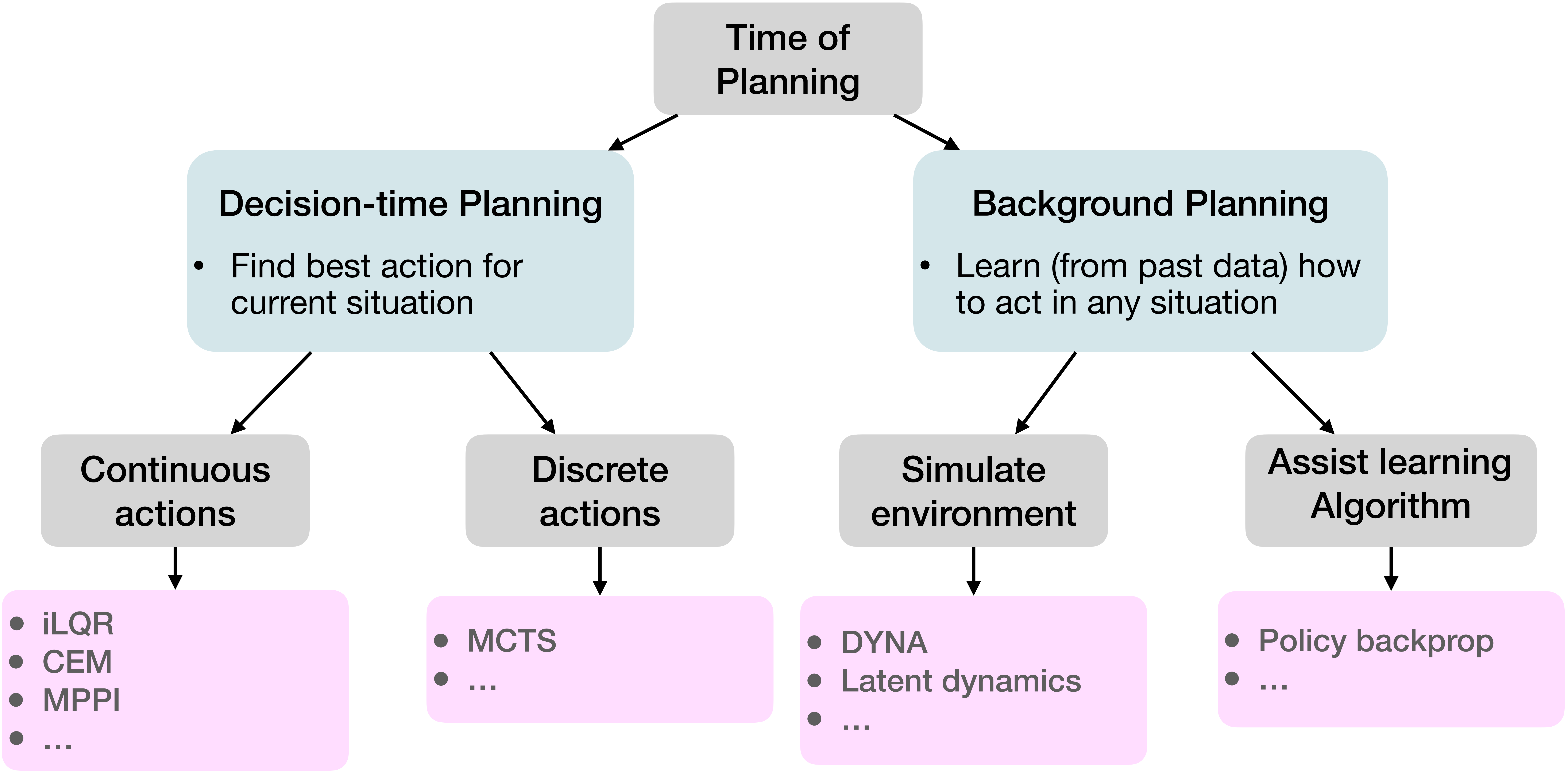

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Background vs Decision-time Planning

Background planning

Decision-time planning

Background vs Decision-time Planning

Background planning

Learn how to act in any situation

Decision-time planning

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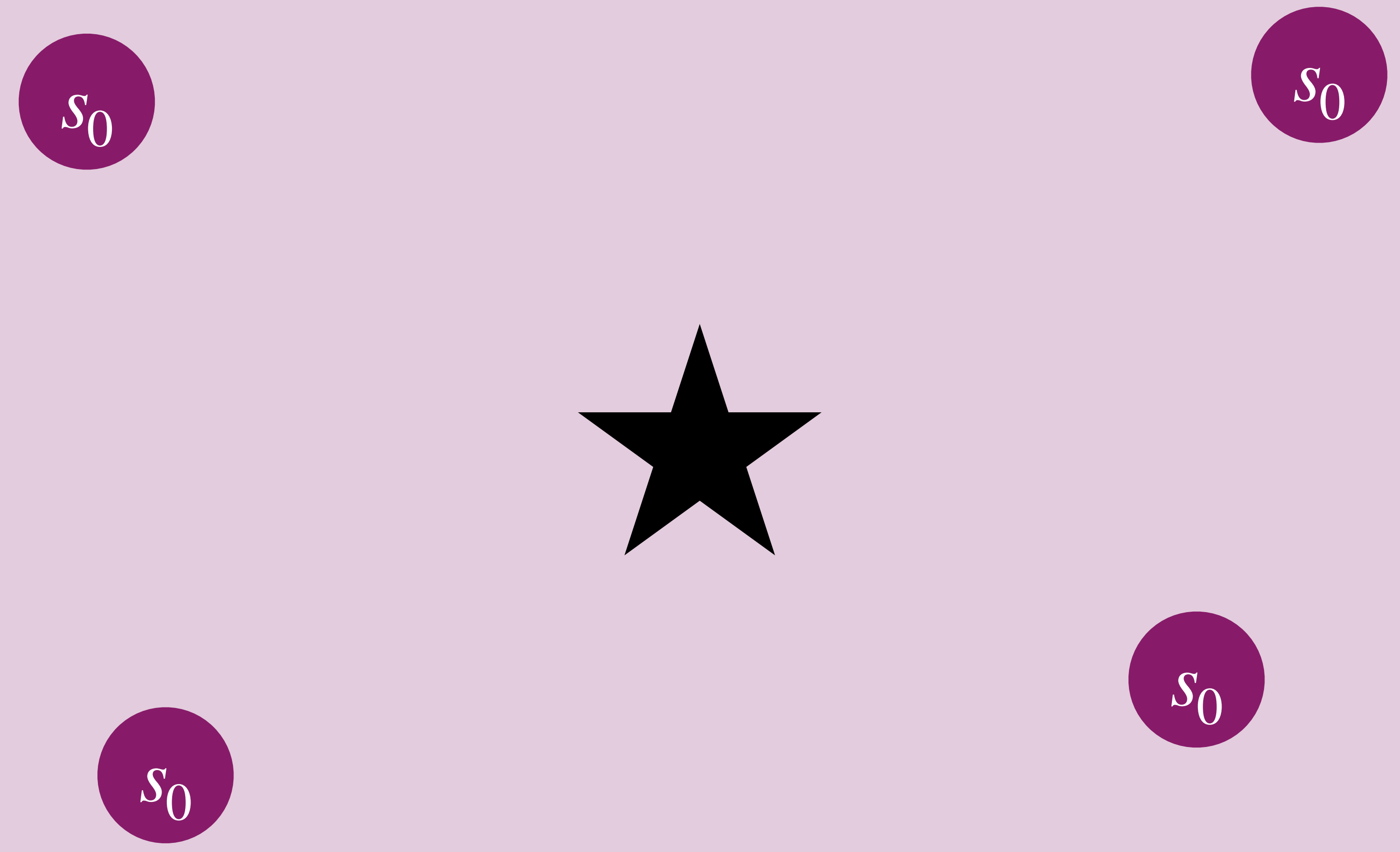

Decision-time planning

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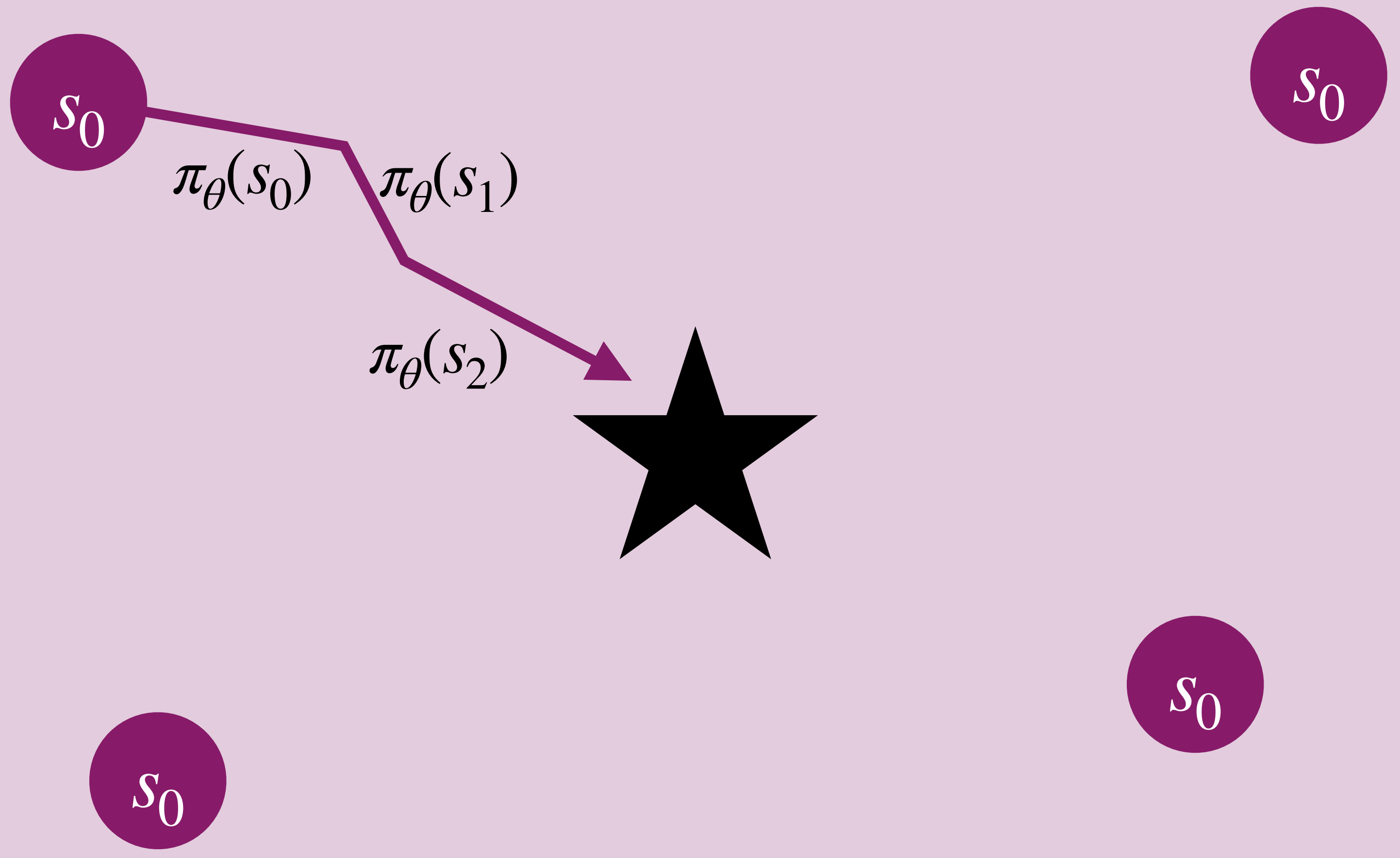

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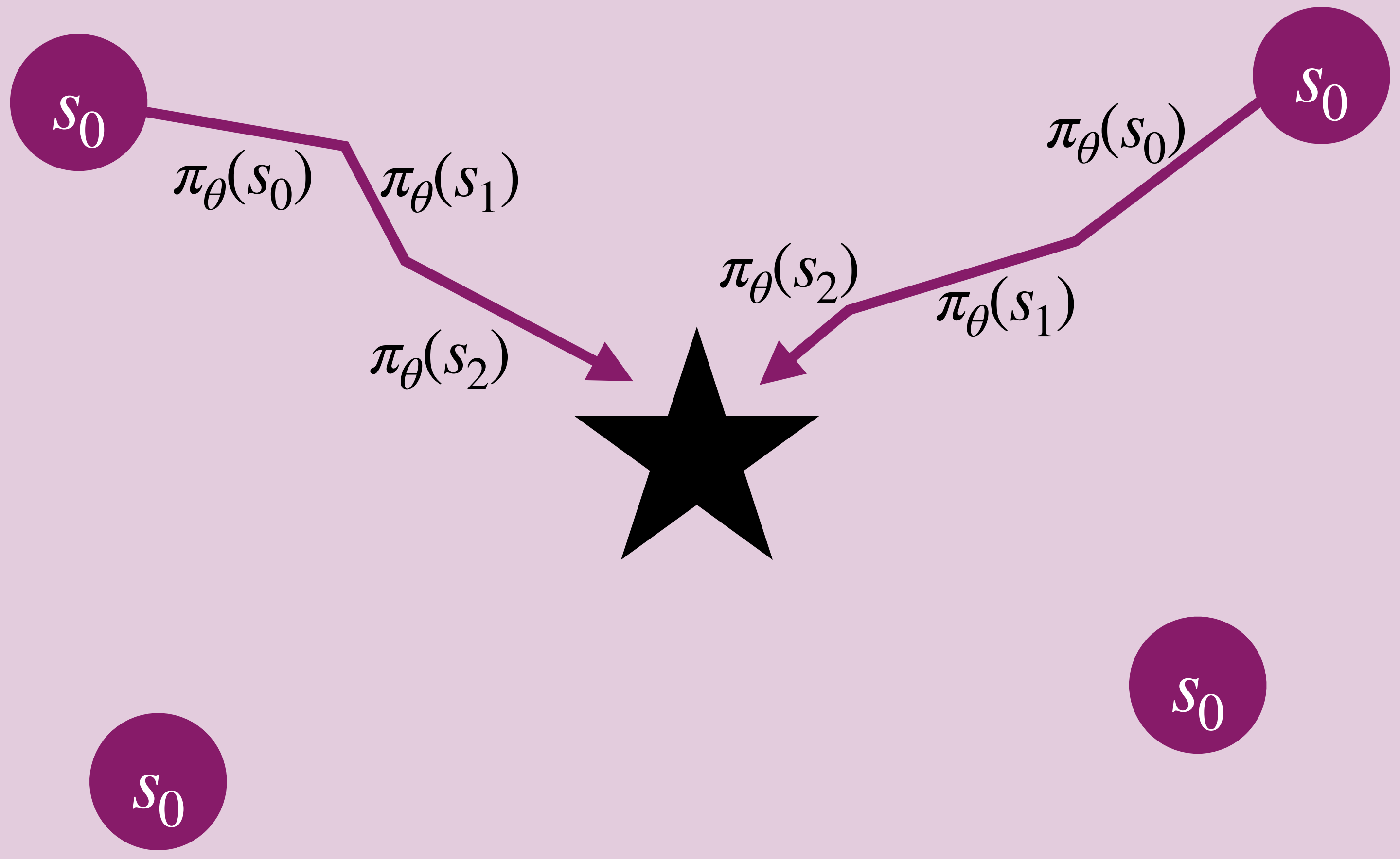

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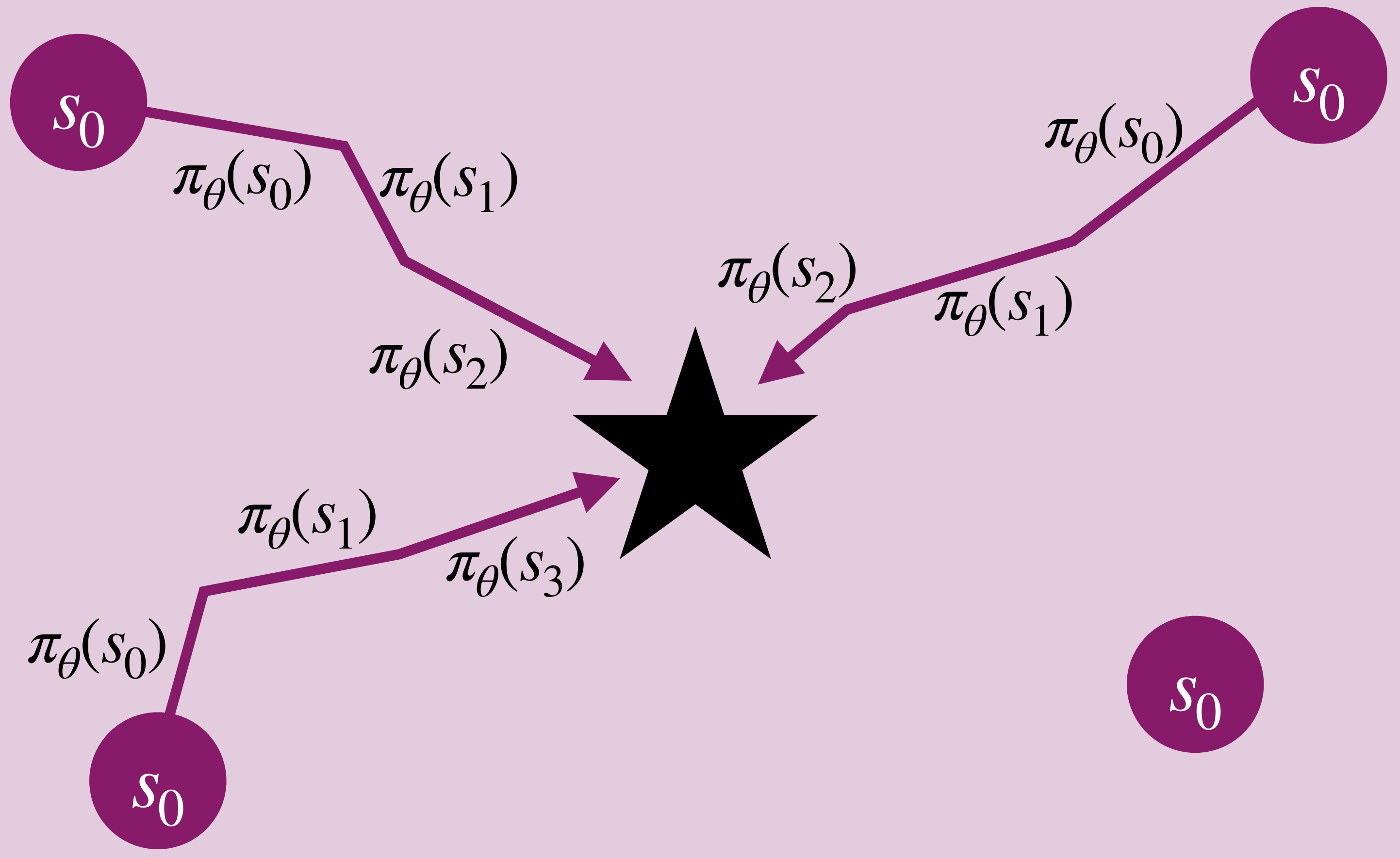

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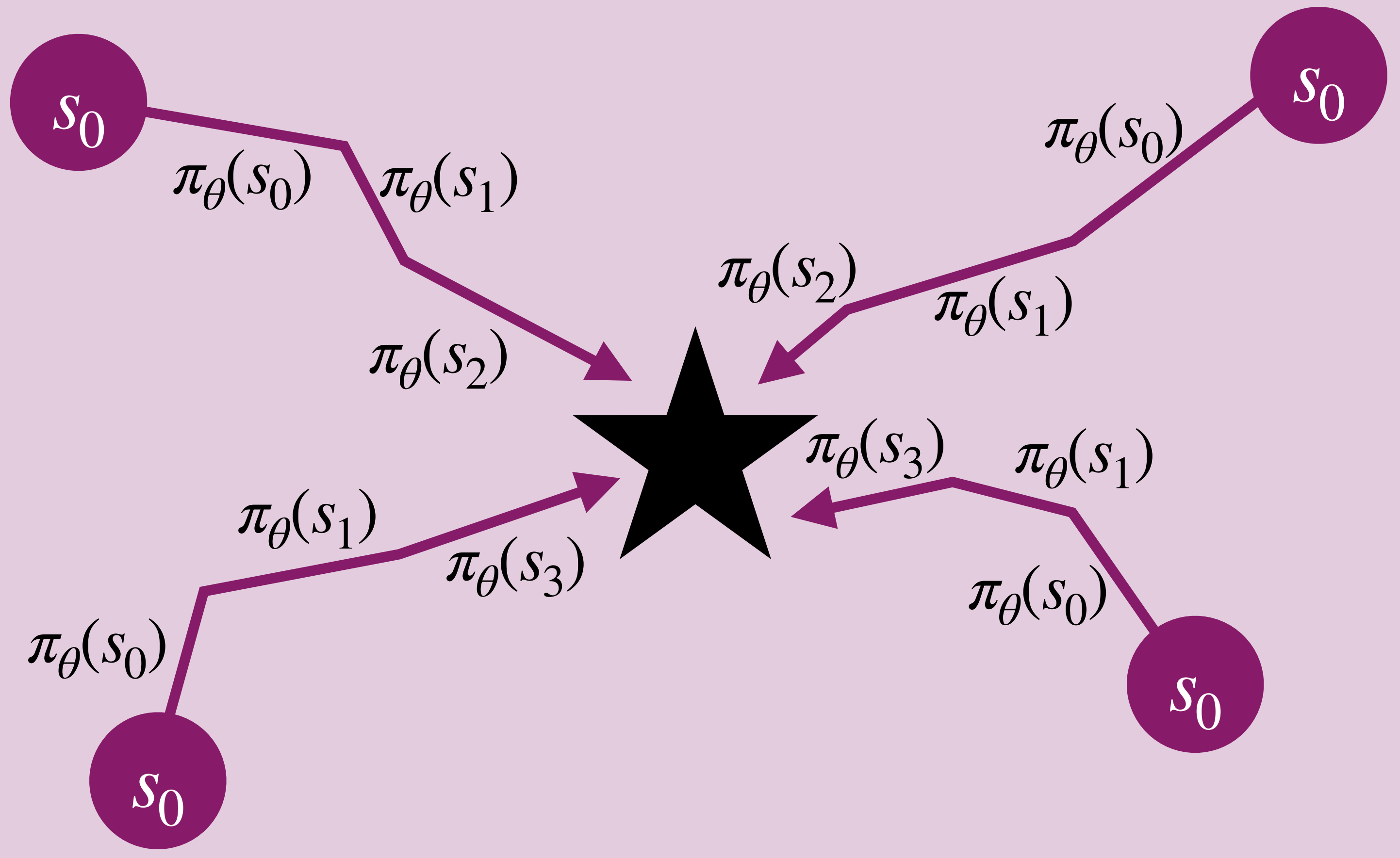

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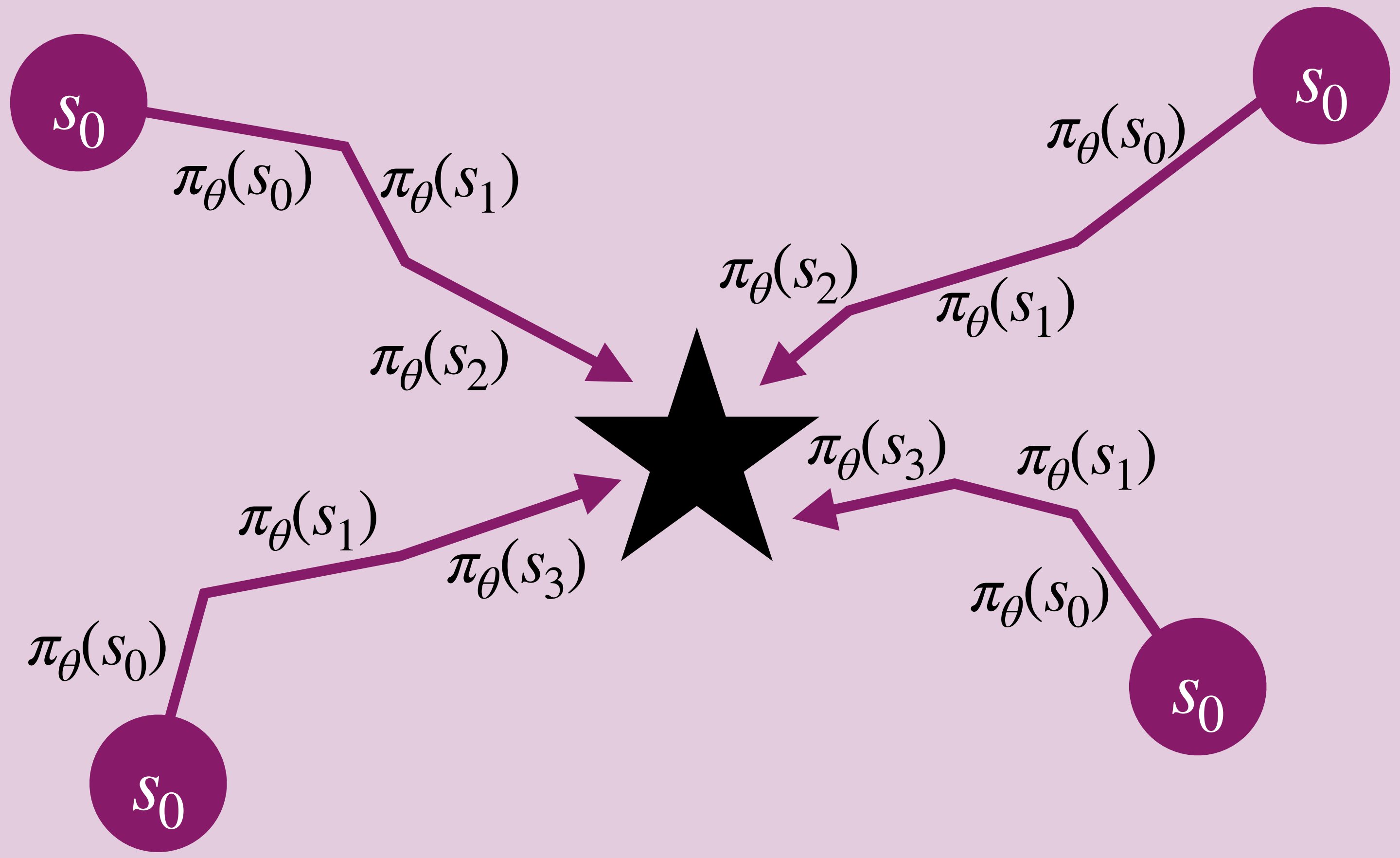

Optimisation variables: θ

Decision-time planning

Background vs Decision-time Planning

Background planning

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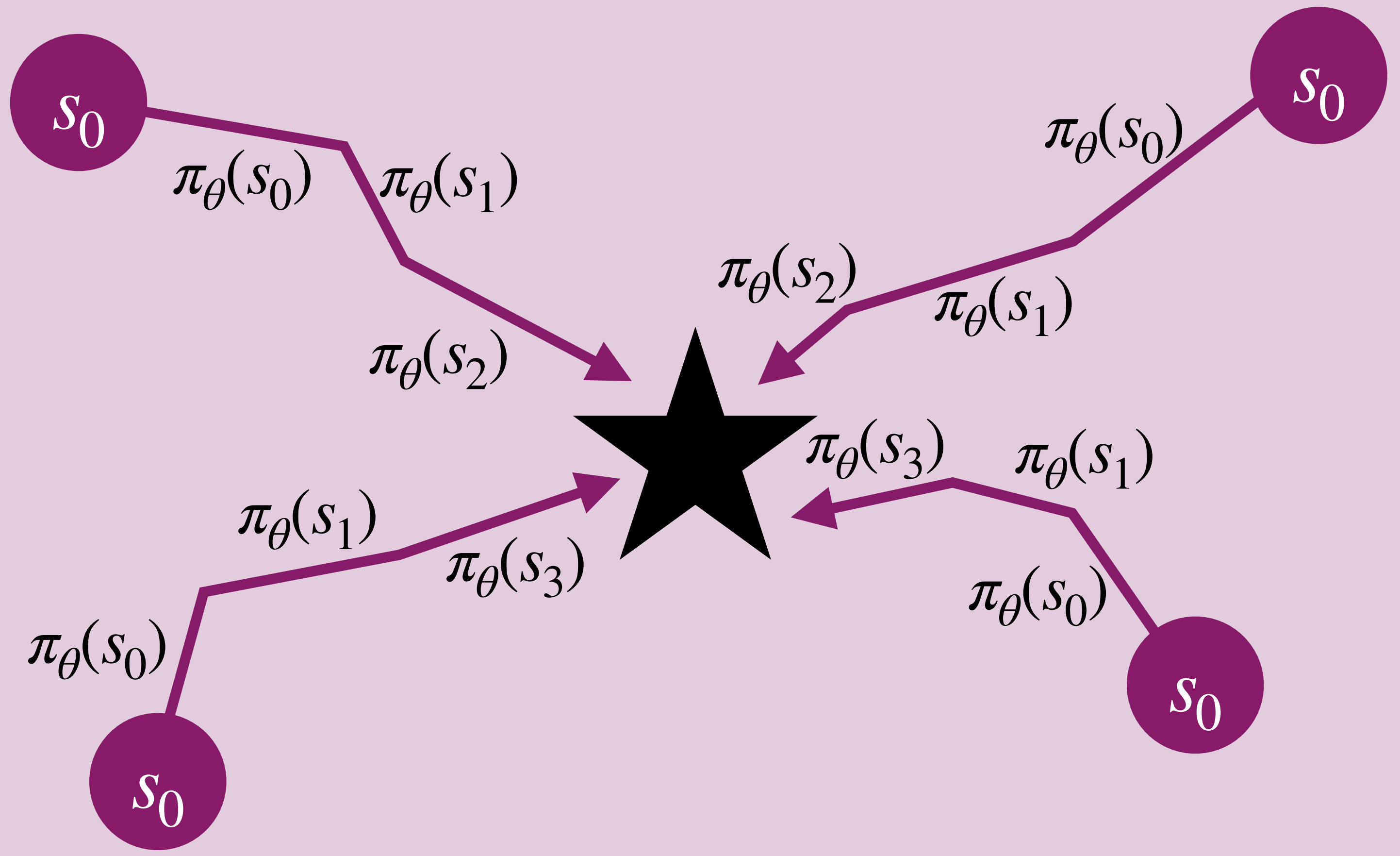

Optimisation variables: θ

Parameters of policy $\pi_\theta(s)$, value $Q_\theta(s, a)$, etc

Decision-time planning

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Background vs Decision-time Planning

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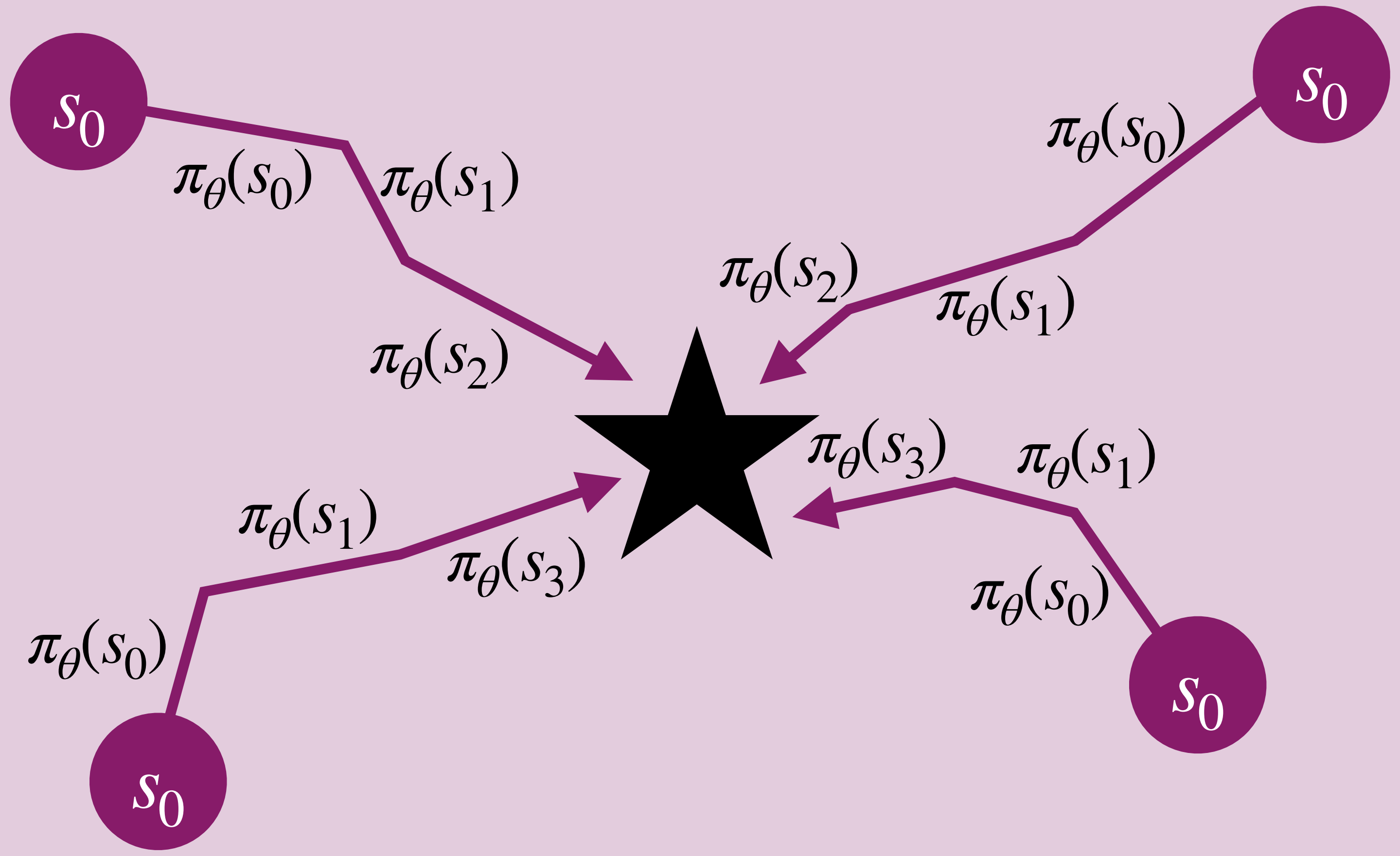

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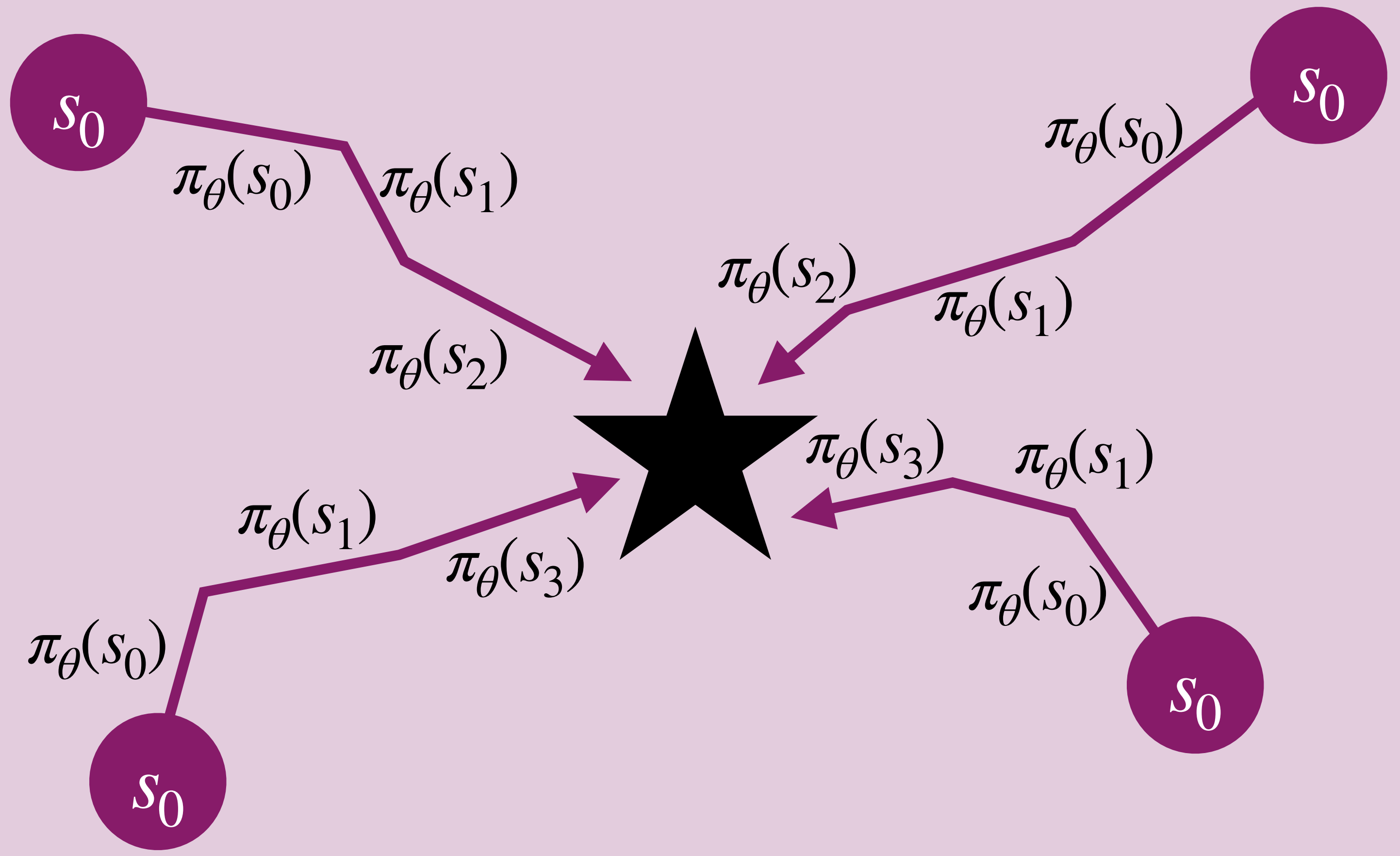

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Decision-time planning

Find best action for current situation

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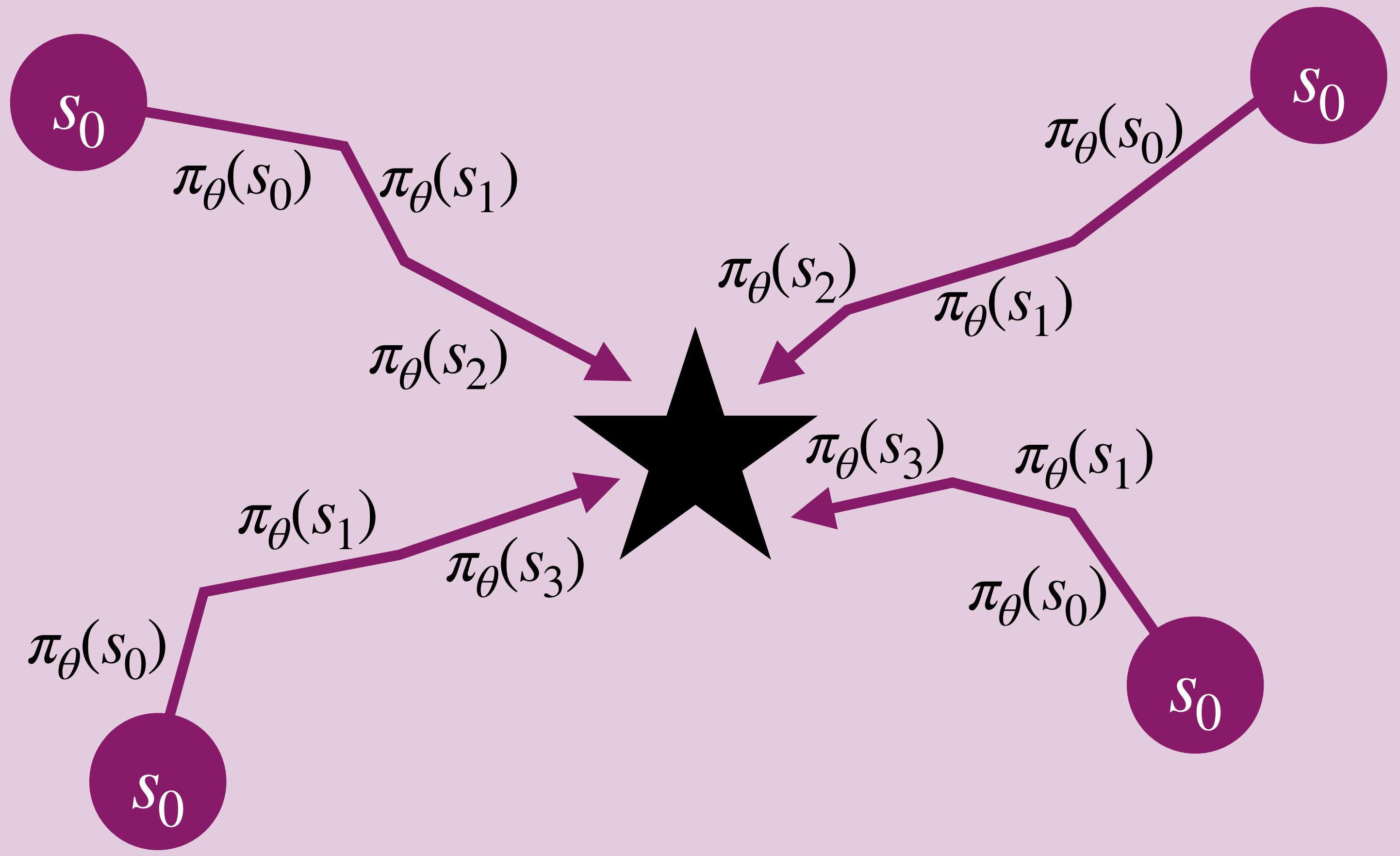

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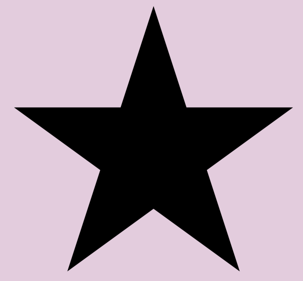

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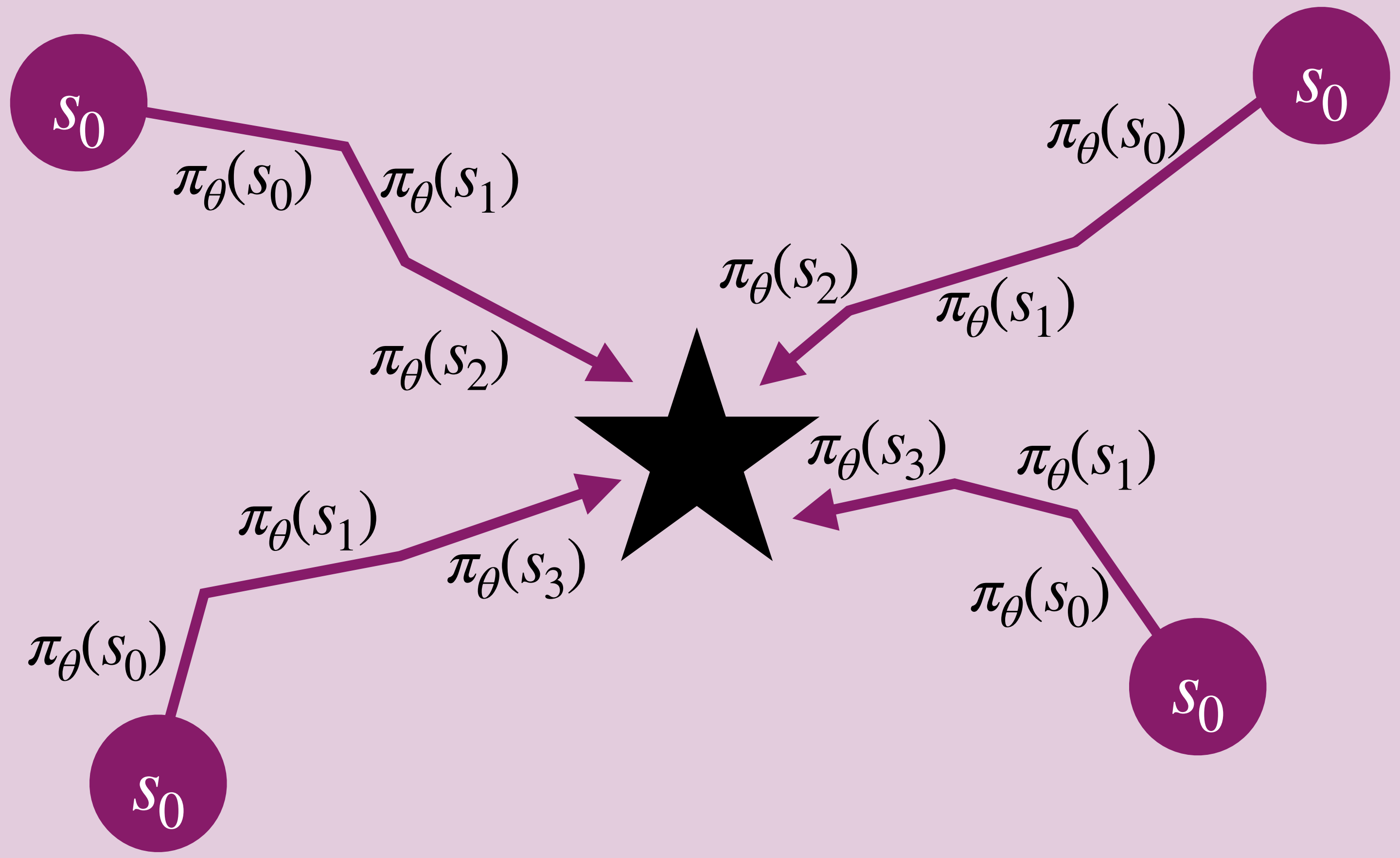

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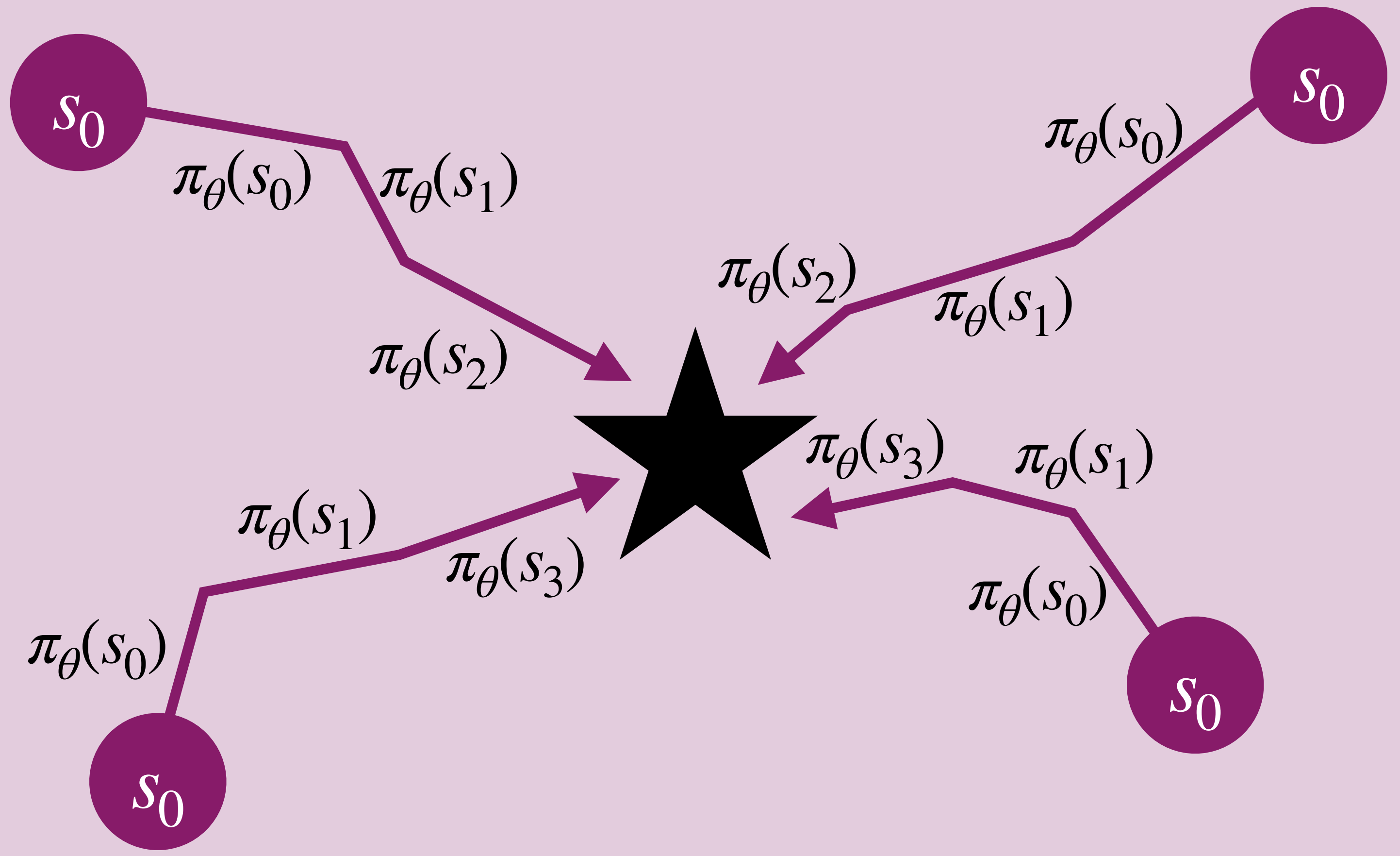

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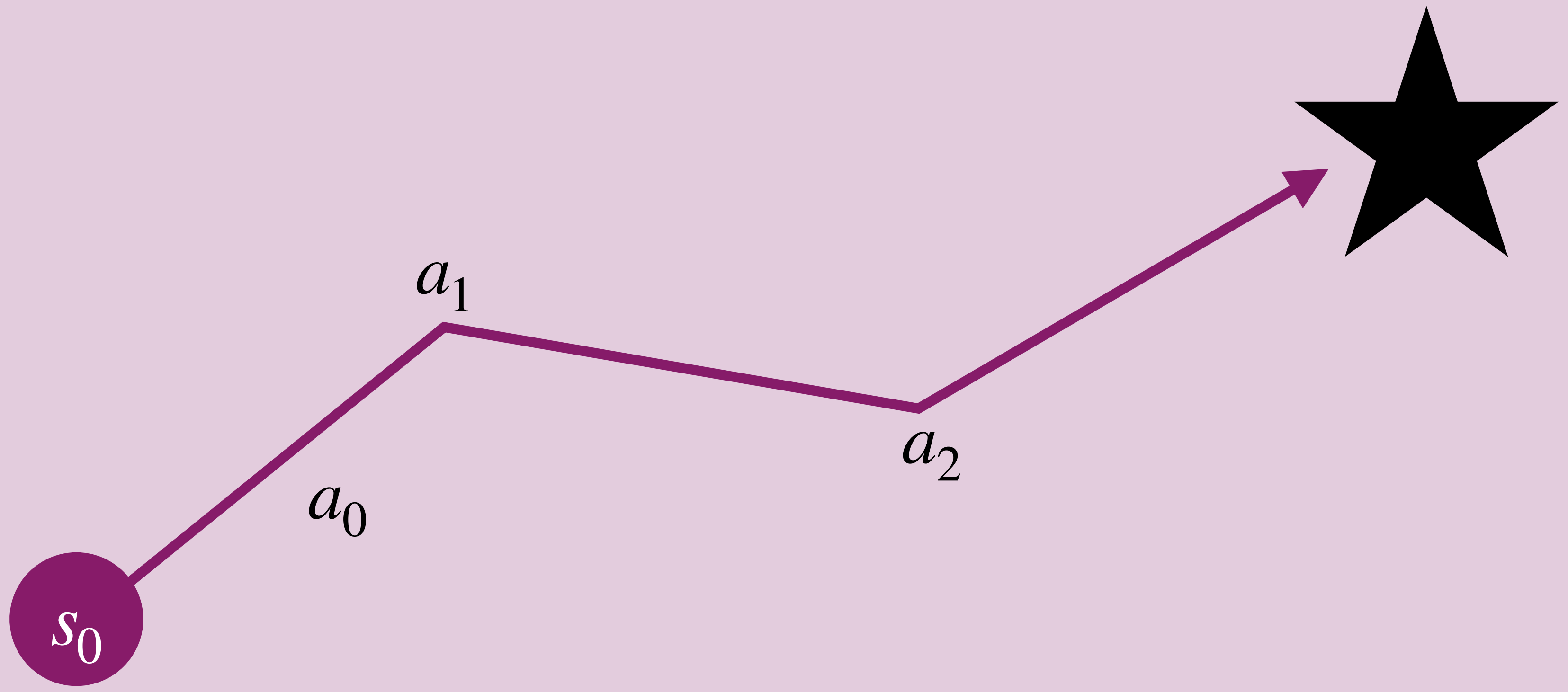

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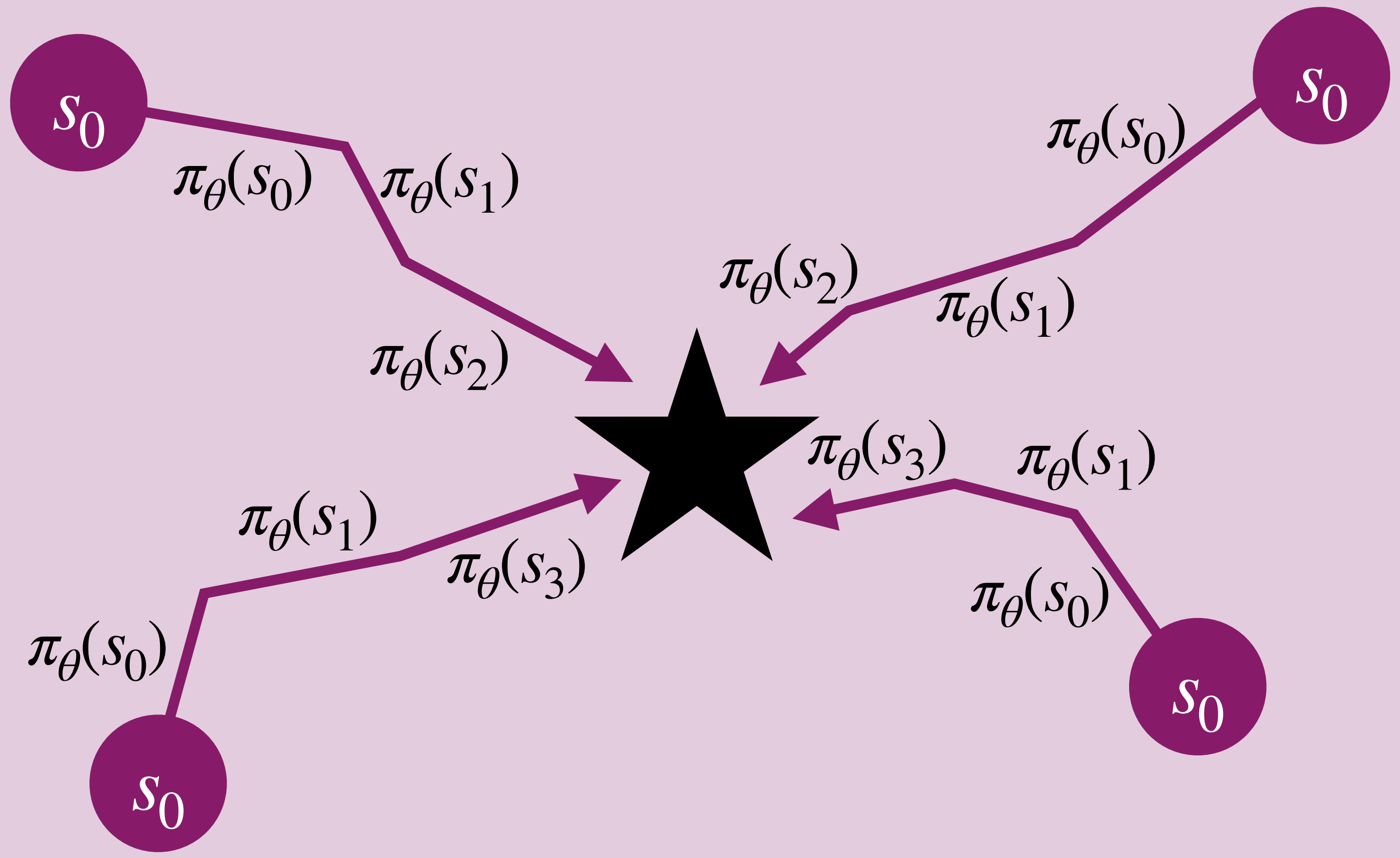

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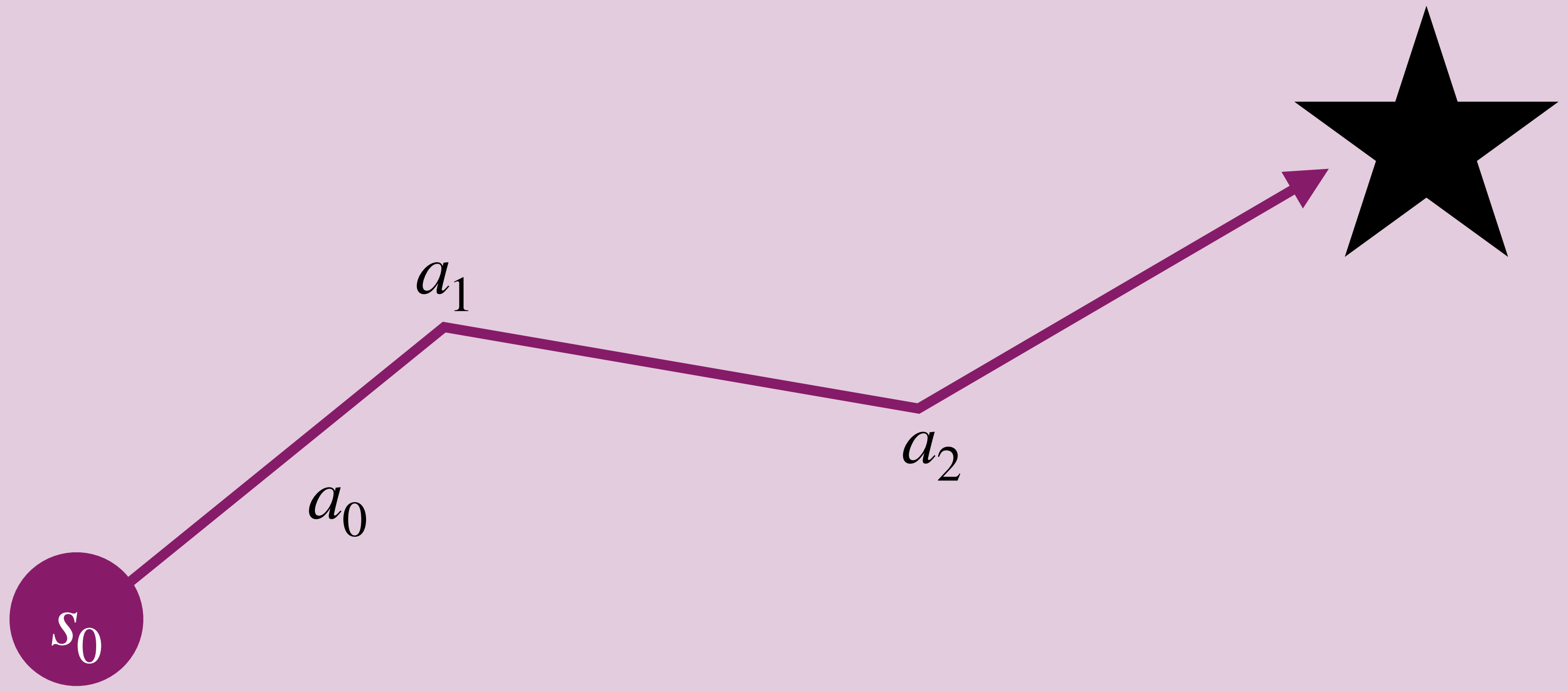

Optimisation variables: $a_0, \ldots, a_H$

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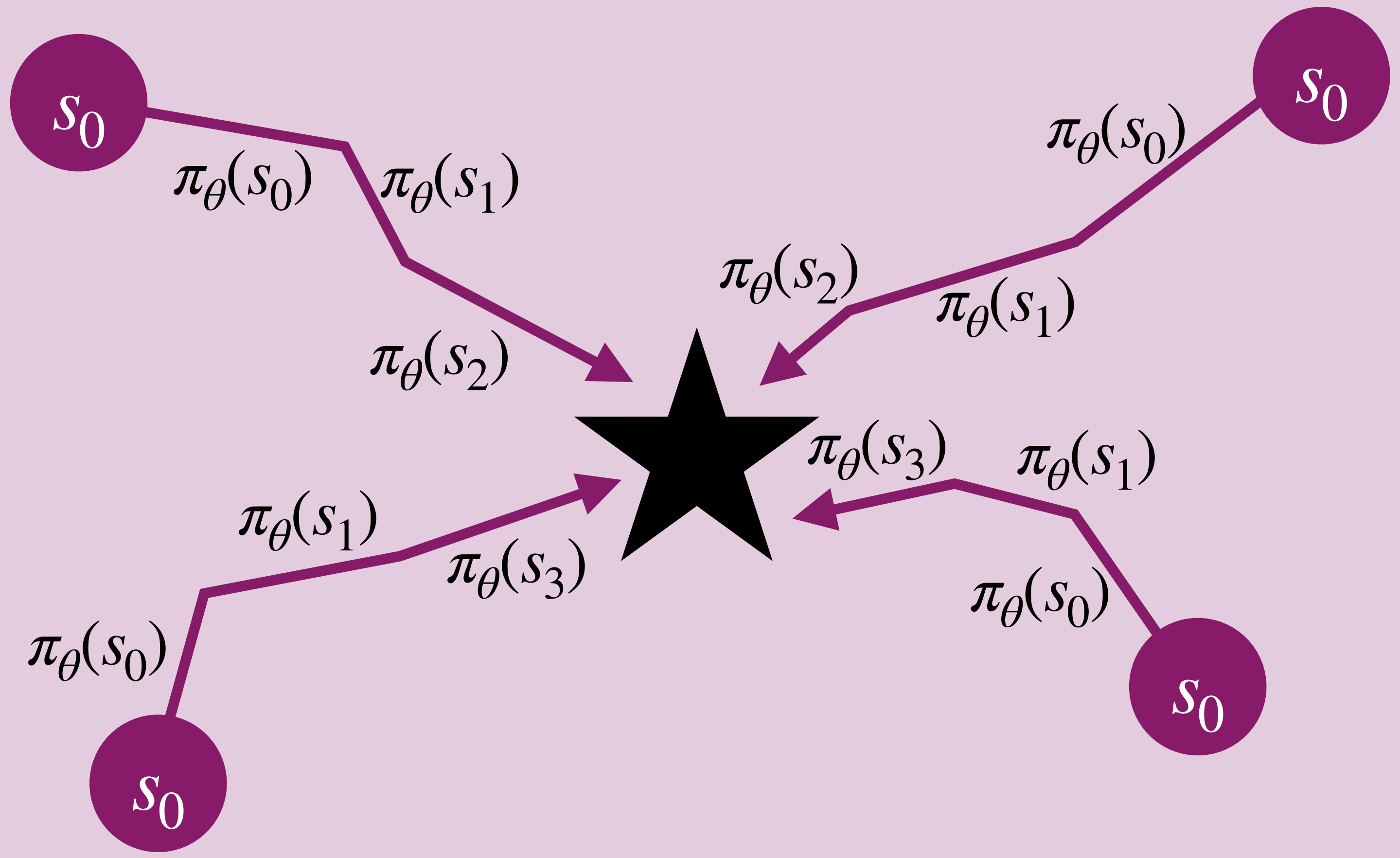

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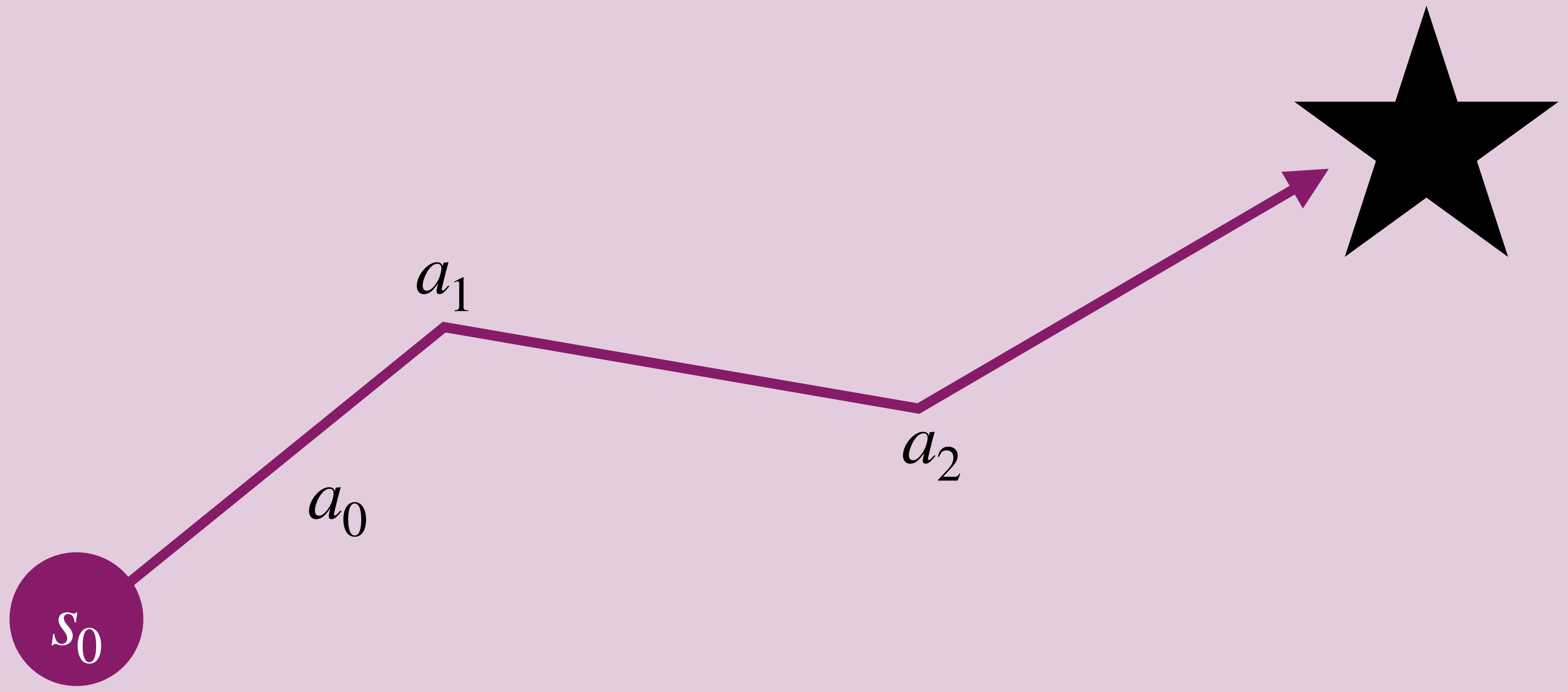

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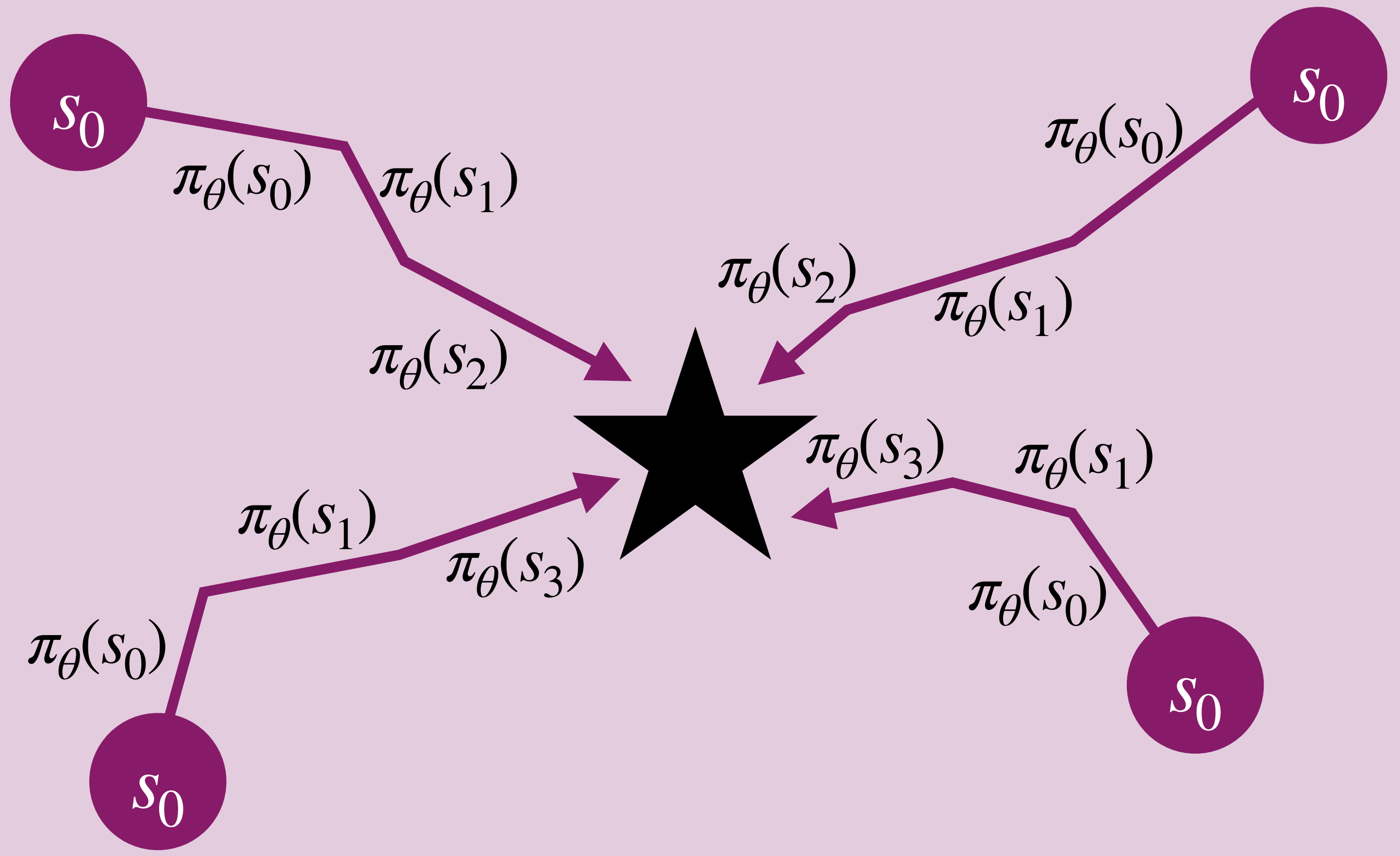
Sequence of actions (and maybe also states)

Background vs Decision-time Planning

Background planning

Learn how to act in any situation



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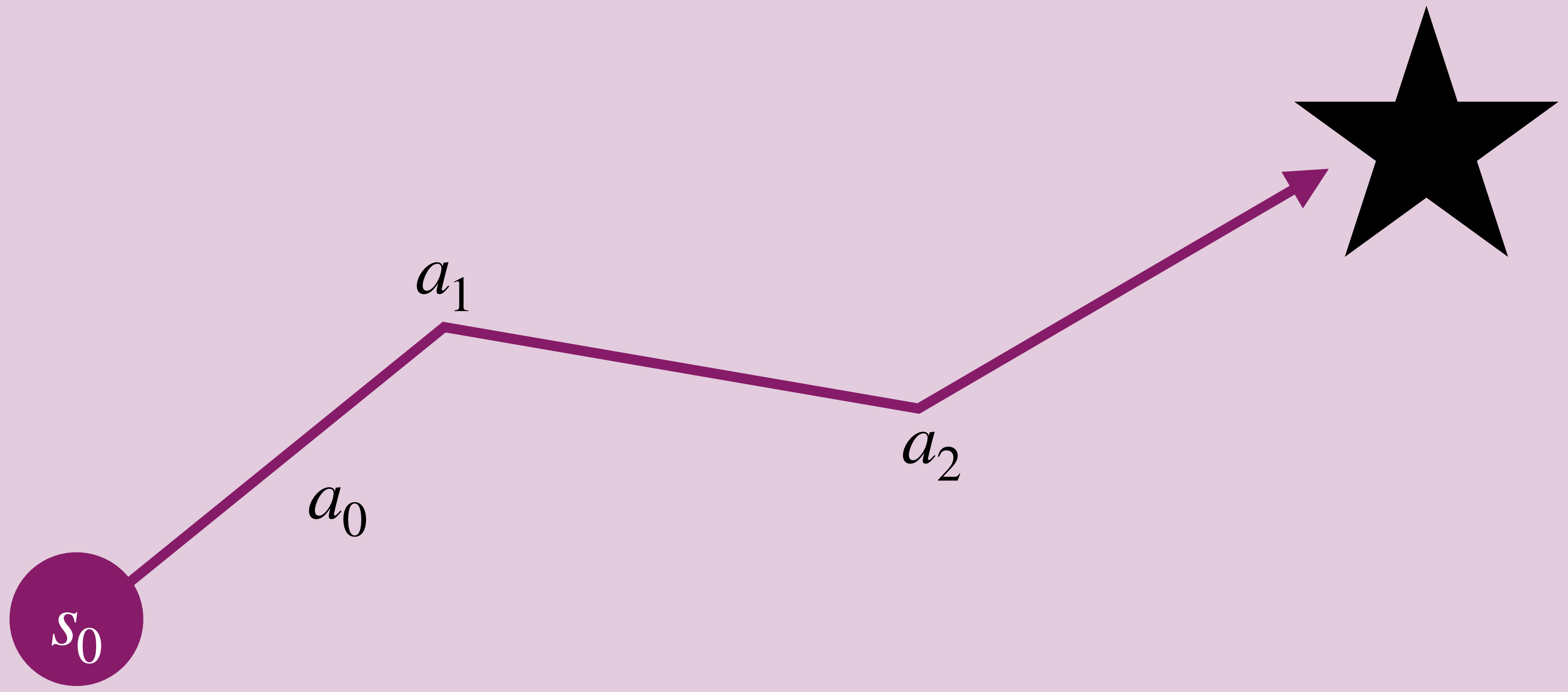
Decision-time planning

Find best action for current situation



Optimisation variables: $a_0, \ldots, a_H$

Sequence of actions (and maybe also states)

$$J(a_0, \ldots, a_H) = \sum_{t=0}^{H} r(s_t, a_t)$$

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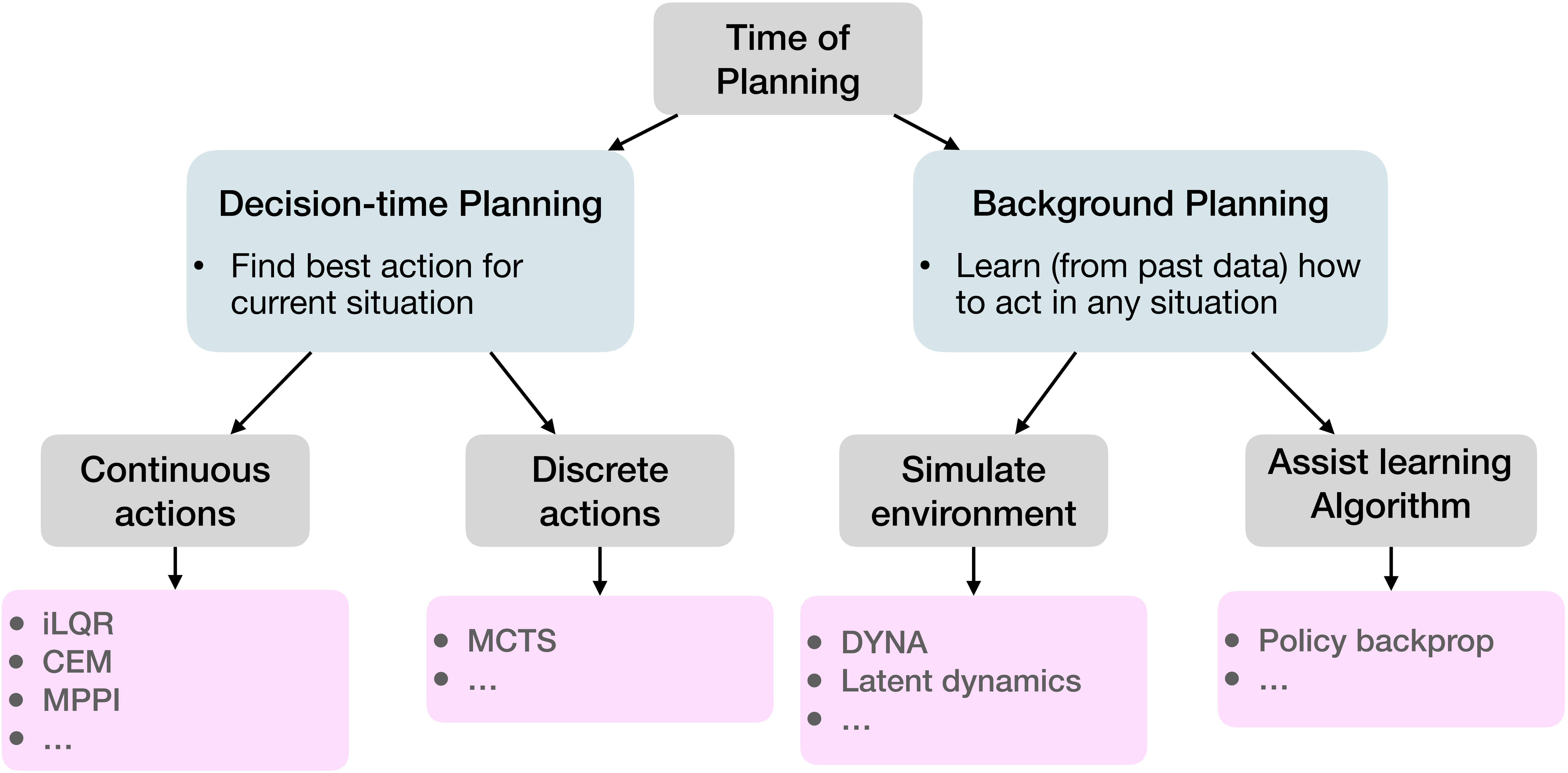
How Do We Use The "Model”?

Background vs Decision-time Planning



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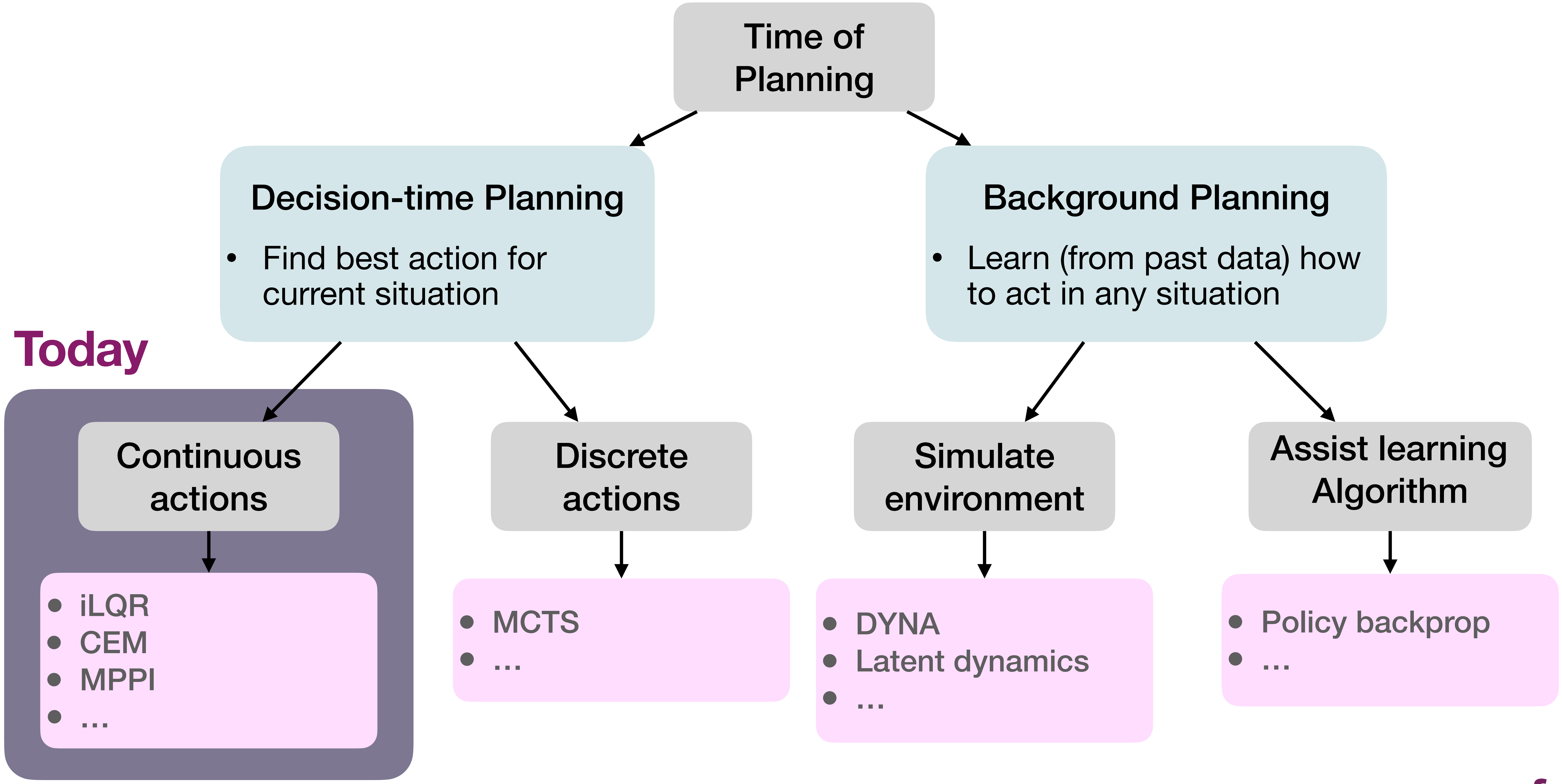

Decision-time Planning (Continuous Actions)

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We’ll start by assuming known, deterministic dynamics

Decision-time Planning (Continuous Actions)

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$$s_{t+1} = f(s_t, a_t)$$

Decision-time Planning

Trajectory optimisation

Decision-time Planning

Trajectory optimisation

Observe state s

Decision-time Planning

Trajectory optimisation

Observe state s

Plan $a_0, \ldots, a_H$ to maximise return $\sum_{t=0}^{H} \gamma^t r(s_t, a_t) \quad$ s.t. $s_0 = s$

Decision-time Planning

Trajectory optimisation

Observe state s

Plan $a_0, \ldots, a_H$ to maximise return $\sum_{t=0}^{H} \gamma^t r(s_t, a_t) \quad$ s.t. $s_0 = s$

Execute each action

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Trajectory Optimisation

Shooting methods

Collocation methods

Trajectory Optimisation

Shooting methods

Optimisation variables: $a_0, \ldots, a_H$

Actions

$$J(a_{0:H}) = \sum_{t=0}^{H} r(s_t, a_t)$$

Collocation methods

Trajectory Optimisation

Shooting methods

Optimisation variables: $a_0, \ldots, a_H$

Actions

$$J(a_{0:H}) = \sum_{t=0}^{H} r(s_t, a_t)$$

Collocation methods

Optimisation variables: $a_0, s_0, \ldots, a_H, s_H$

Actions and states

$$J(a_{0:H}, s_{0:H}) = \sum_{t=0}^{H} r(s_t, a_t)$$

$$\text{s.t. } \|s_{t+1} - f(s_t, a_t)\| = 0$$

Shooting Methods

Illustration

Shooting Methods

Illustration

$$J(a_{0:H}) = \gamma^0 r(s_0, a_0) + \gamma^1 r(f(s_0, a_0), a_1) + \ldots + \gamma^H r(f(f(\ldots), a_{H-1}), a_H)$$

Shooting Methods

Illustration

$$\boxed{J(a_{0:H})} = \gamma^0 r(s_0, a_0) + \gamma^1 r(f(s_0, a_0), a_1) + \ldots + \gamma^H r(f(f(\ldots), a_{H-1}), a_H)$$

Optimising actions

Shooting Methods

Illustration

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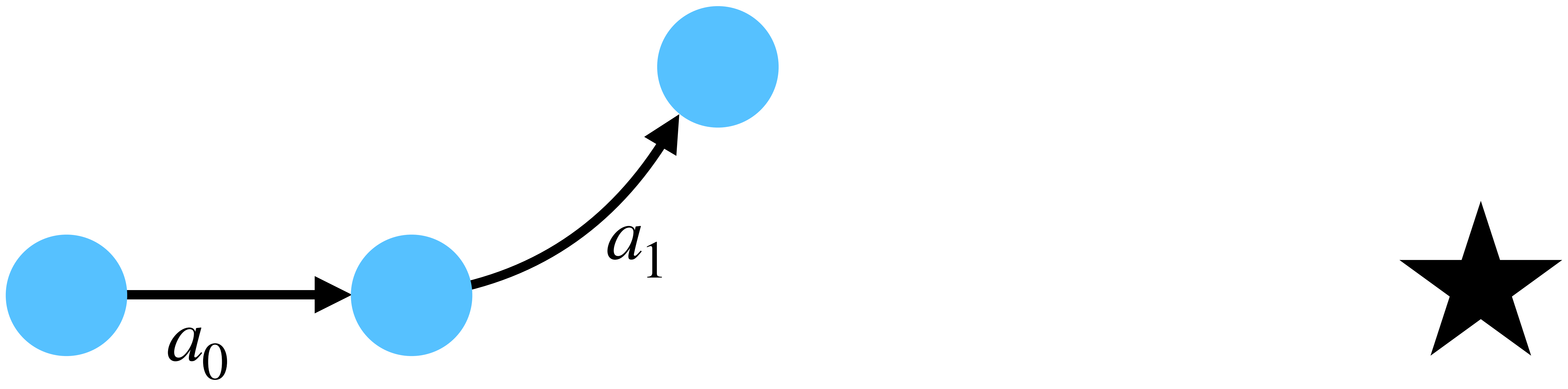
Optimising actions



Shooting Methods

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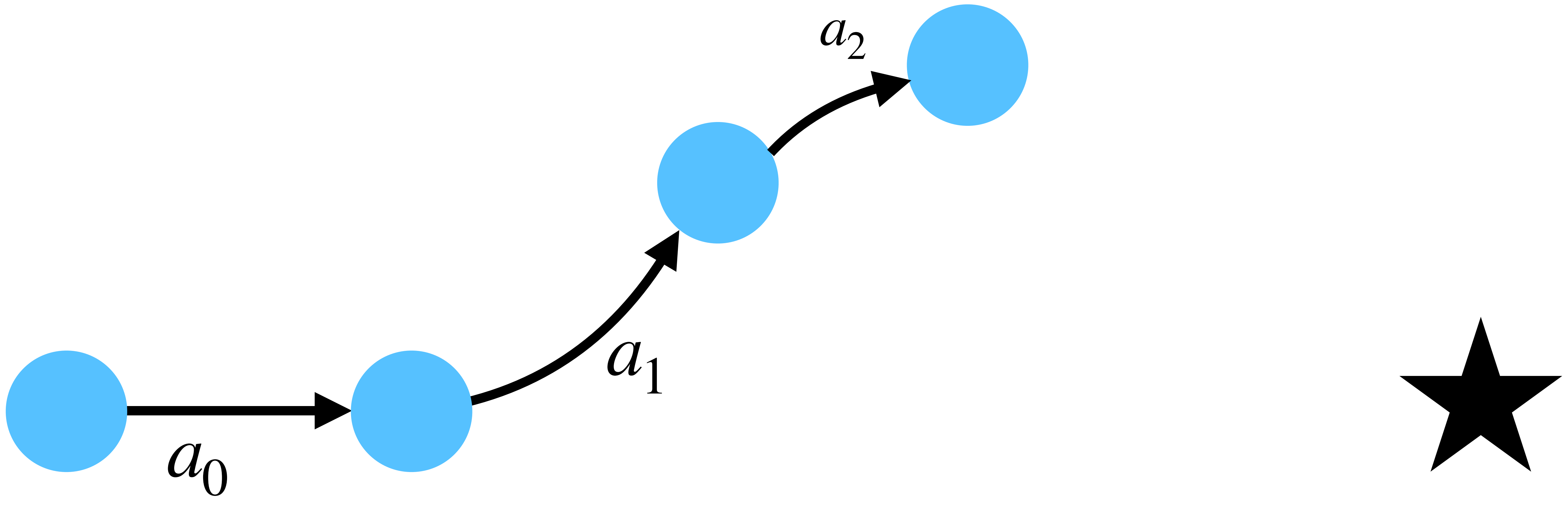

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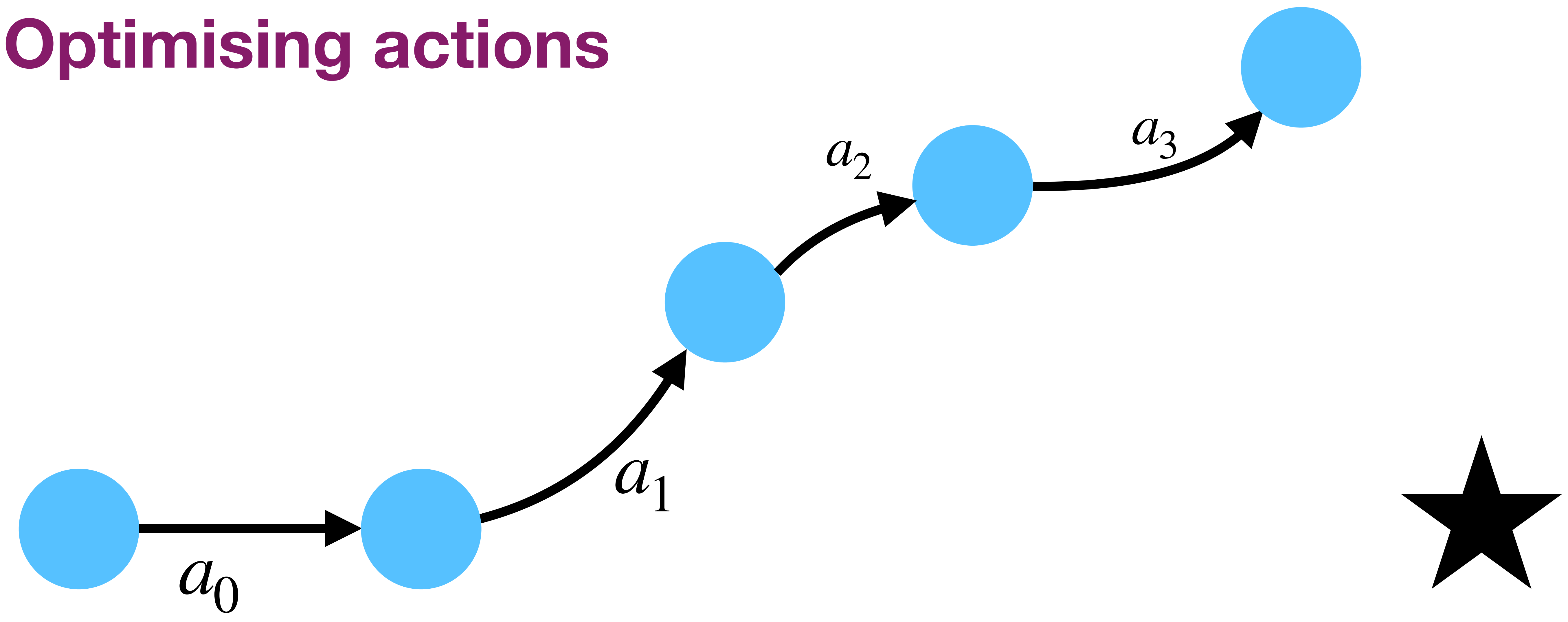

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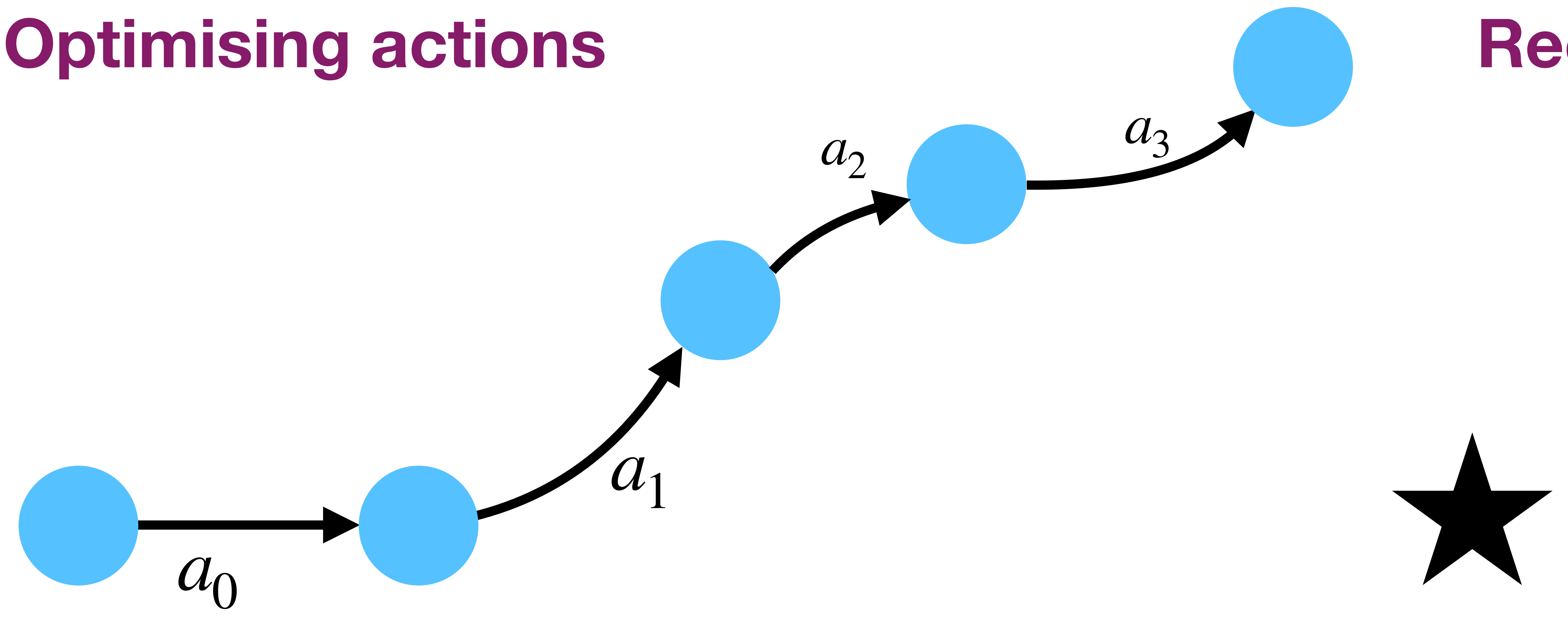
Optimising actions

Recursively evaluate dynamics



Shooting Methods

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Optimising actions

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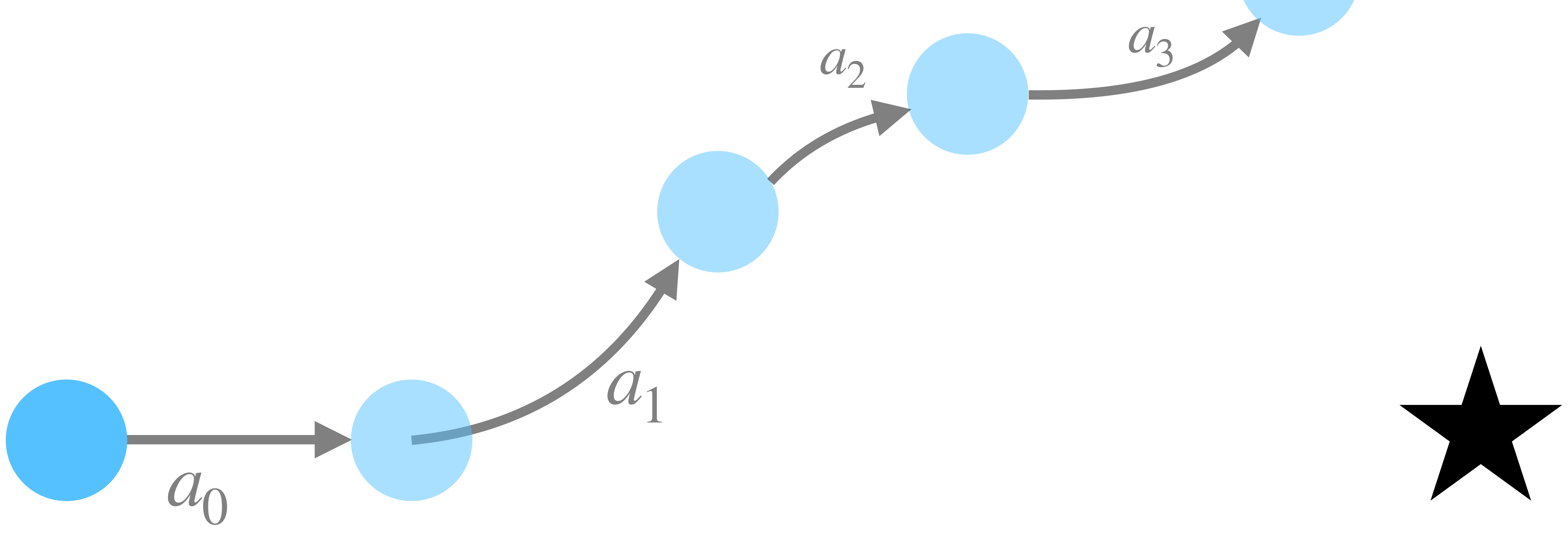

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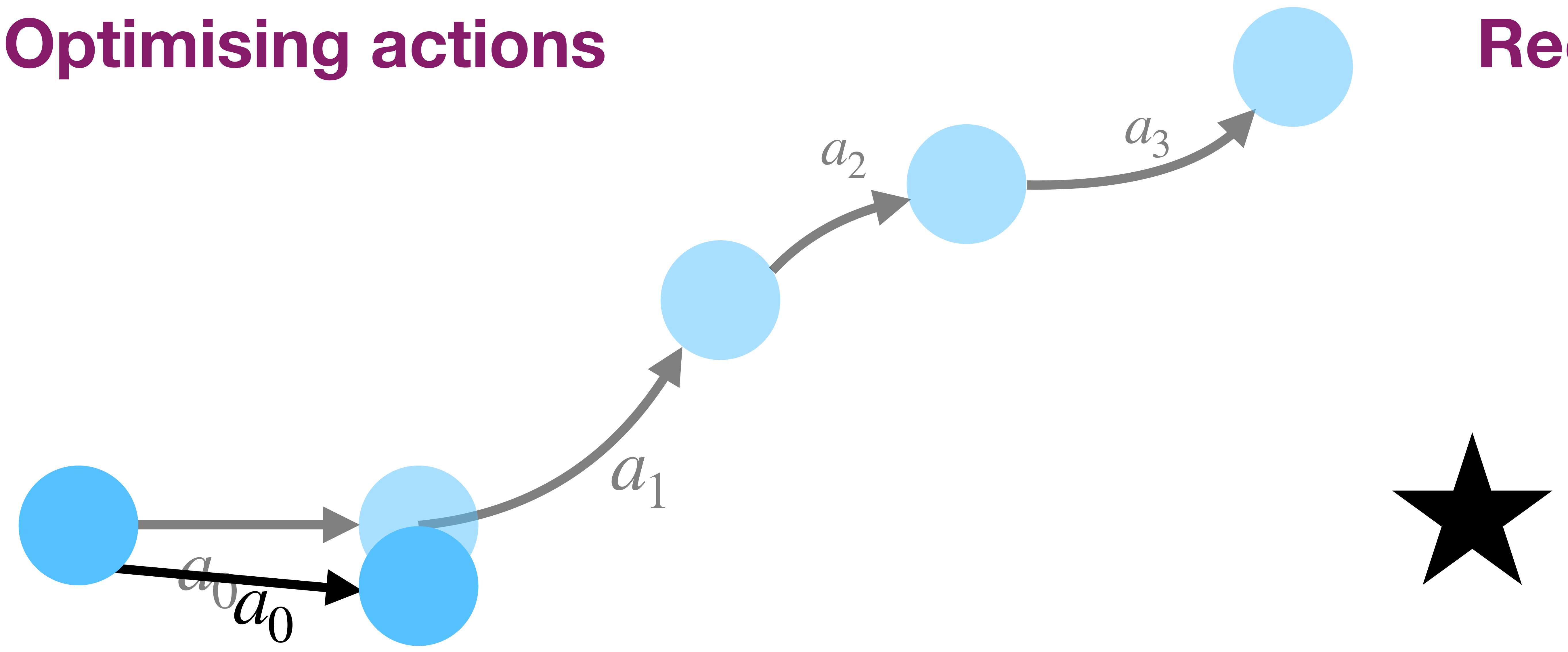
Optimising actions

Recursively evaluate dynamics



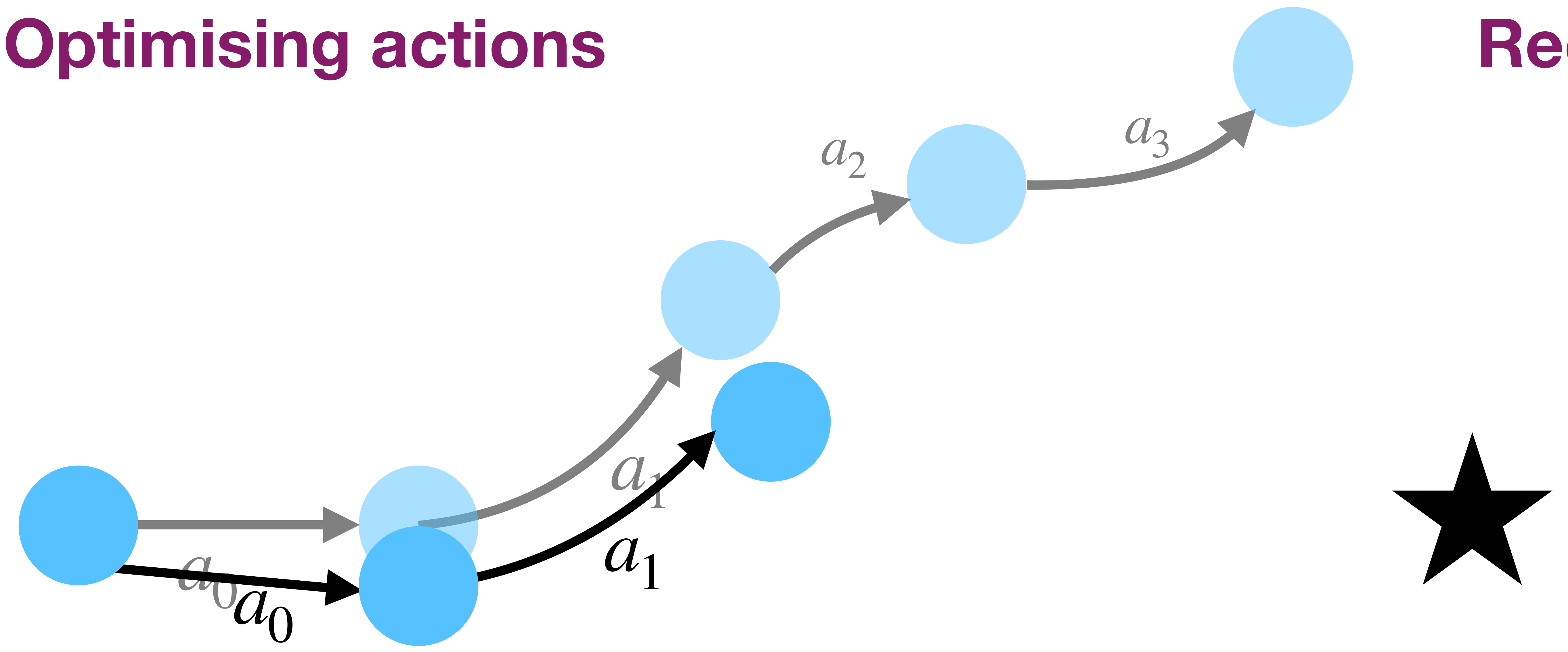
Shooting Methods

Illustration

$$\boxed{J(a_{0:H})} = \gamma^0 r(s_0, a_0) + \gamma^1 r(f(s_0, a_0), a_1) + \ldots + \gamma^H r\boxed{(f(f(\ldots), a_{H-1}), a_H)}$$

Optimising actions

Recursively evaluate dynamics



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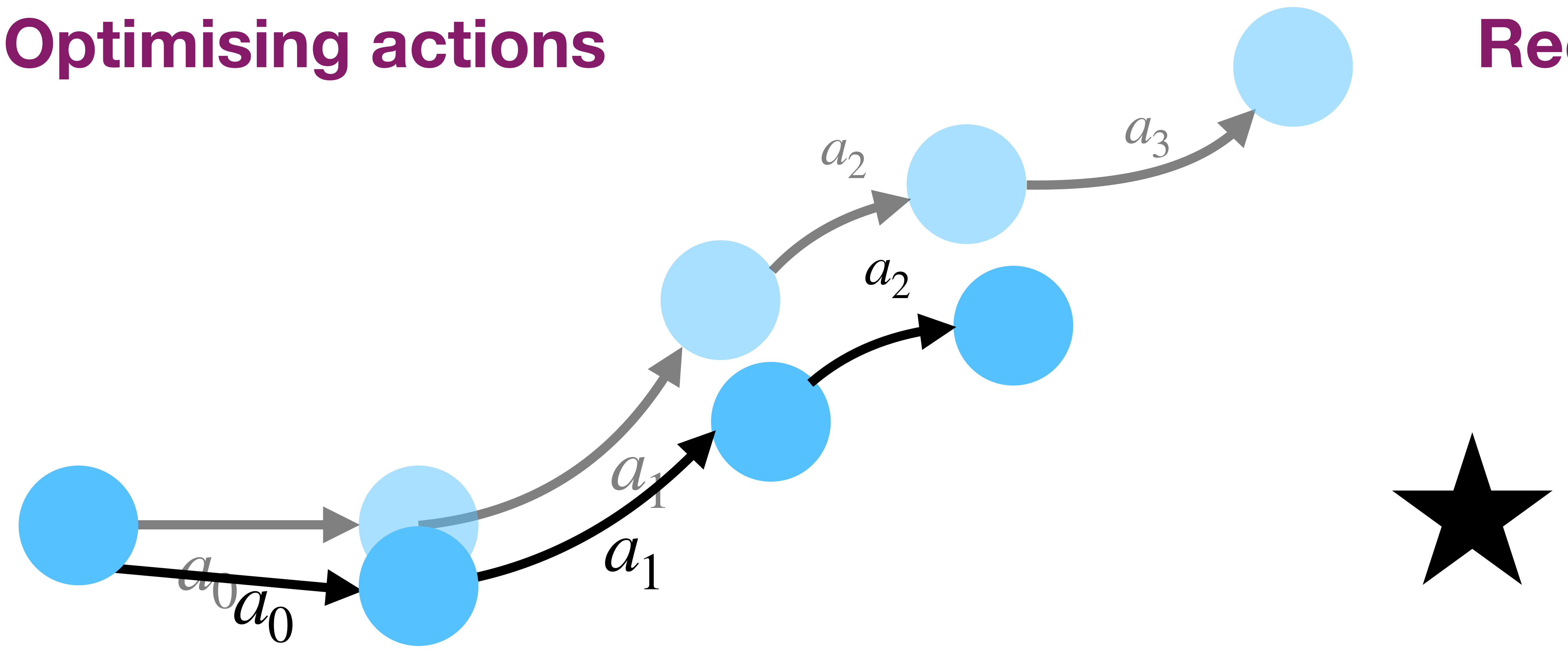
Optimising actions

Recursively evaluate dynamics



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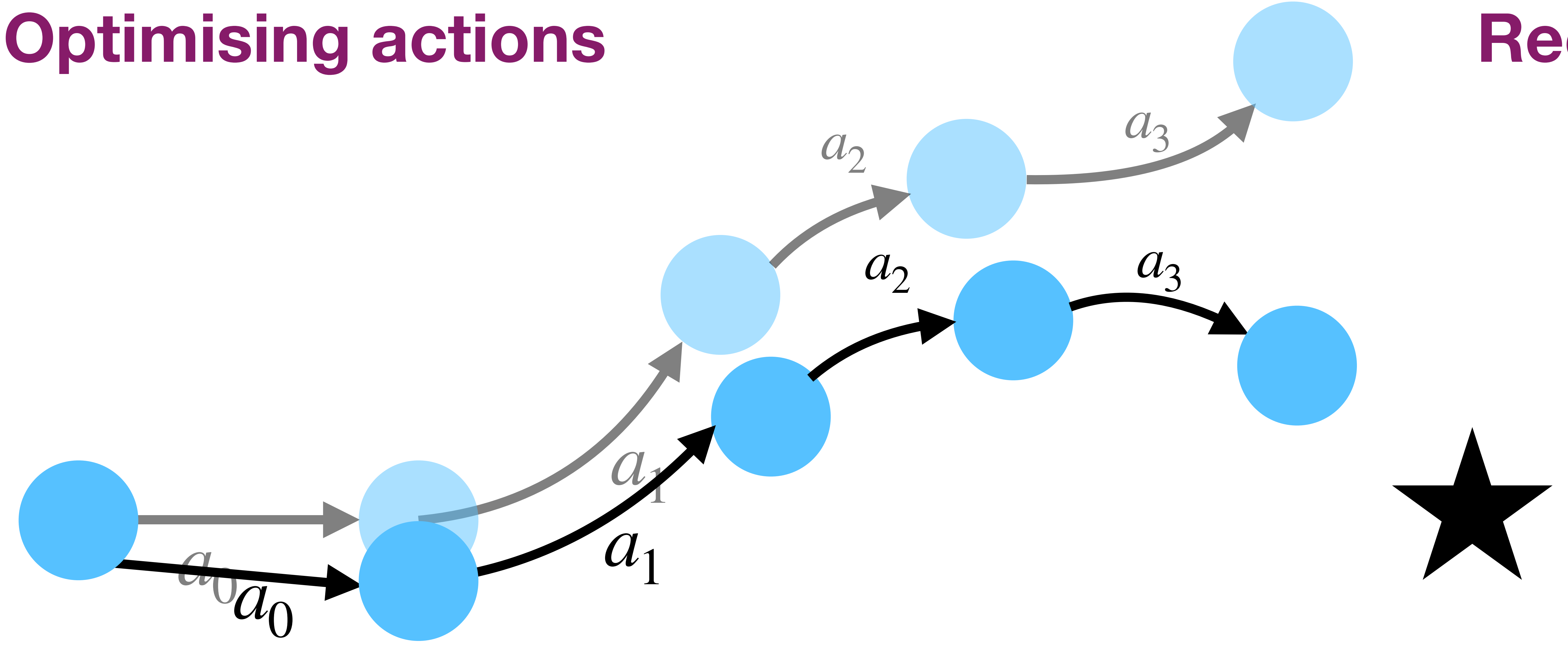
Optimising actions **Recursively evaluate dynamics**



Shooting Methods

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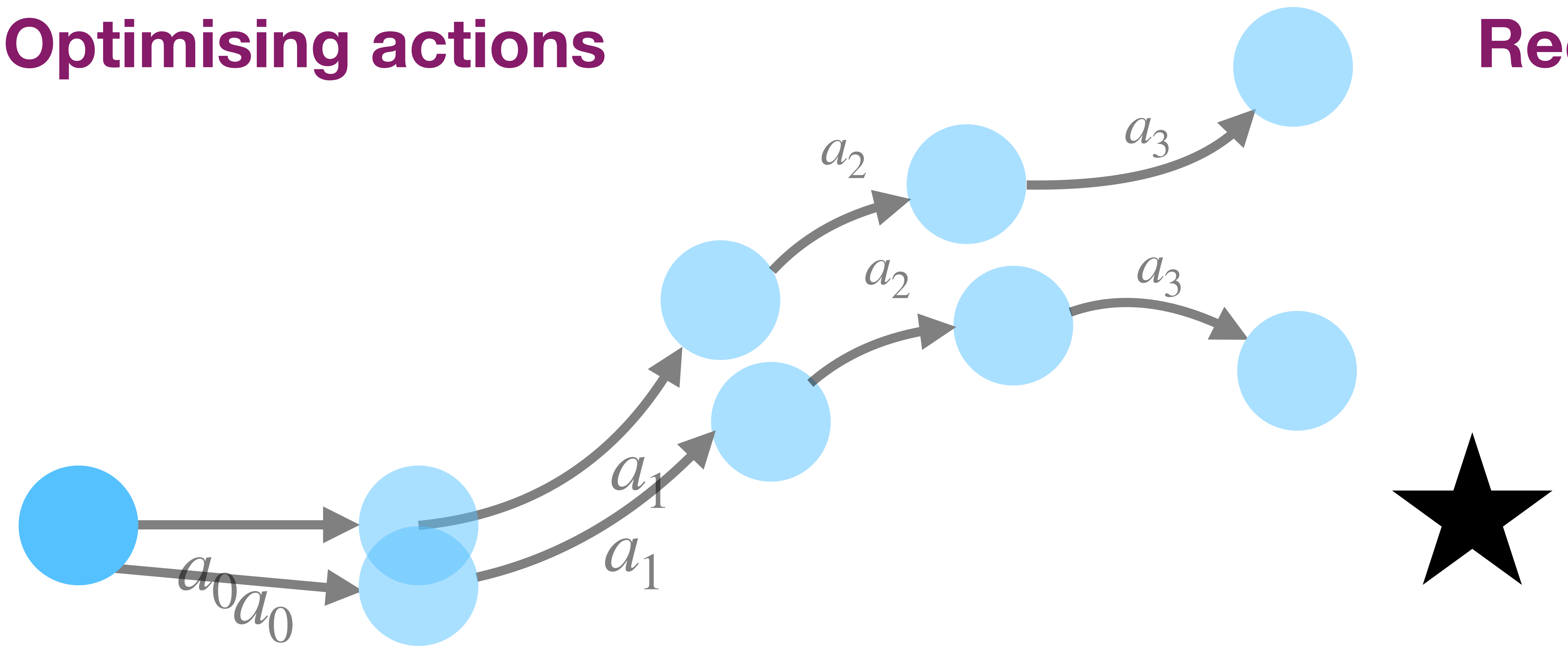
Optimising actions

Recursively evaluate dynamics



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Optimising actions

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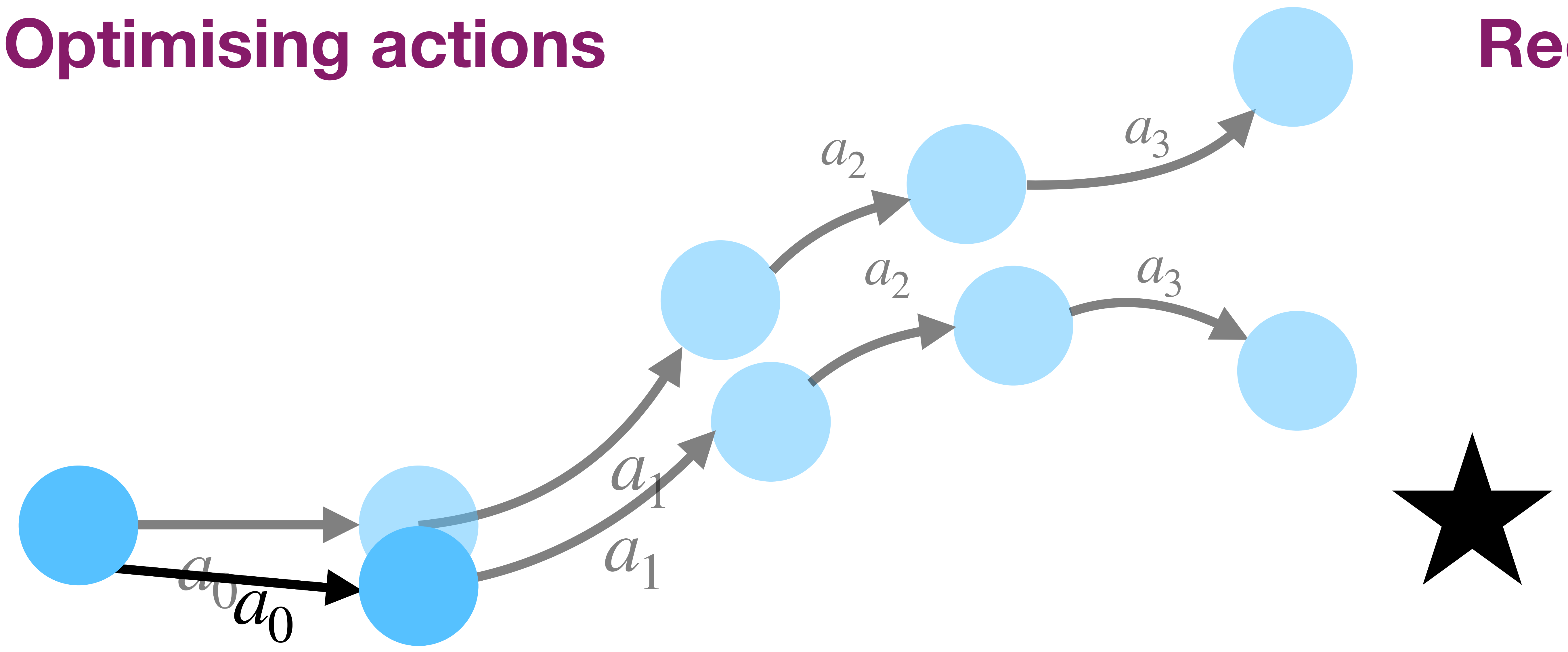

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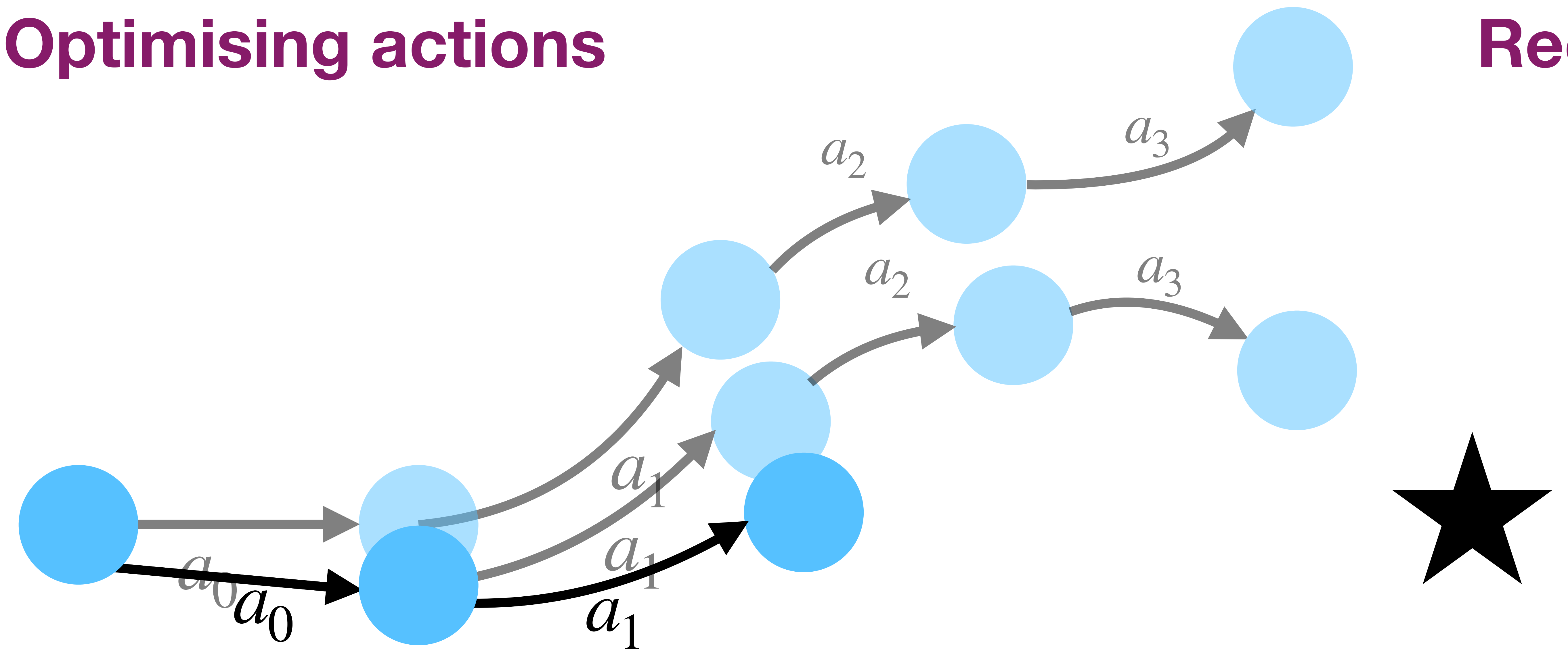
Optimising actions

Recursively evaluate dynamics



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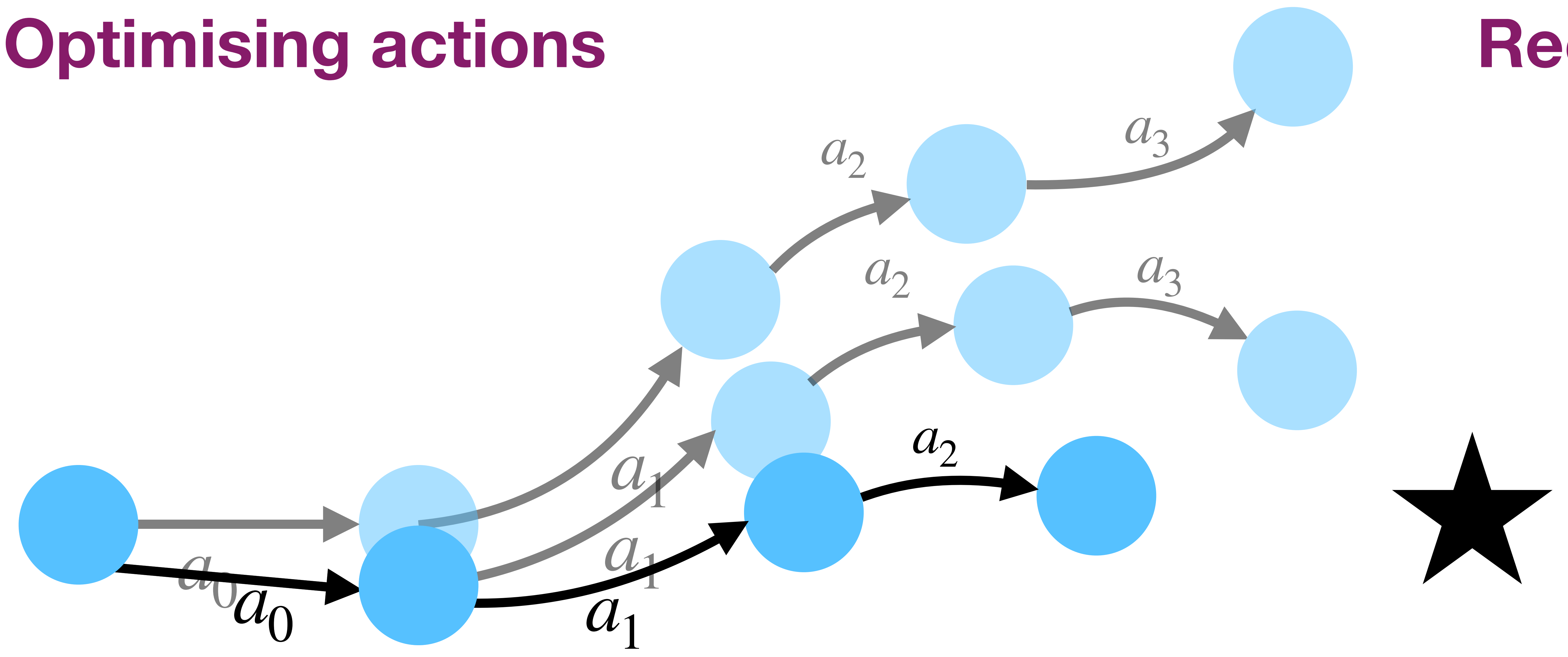
Optimising actions

Recursively evaluate dynamics



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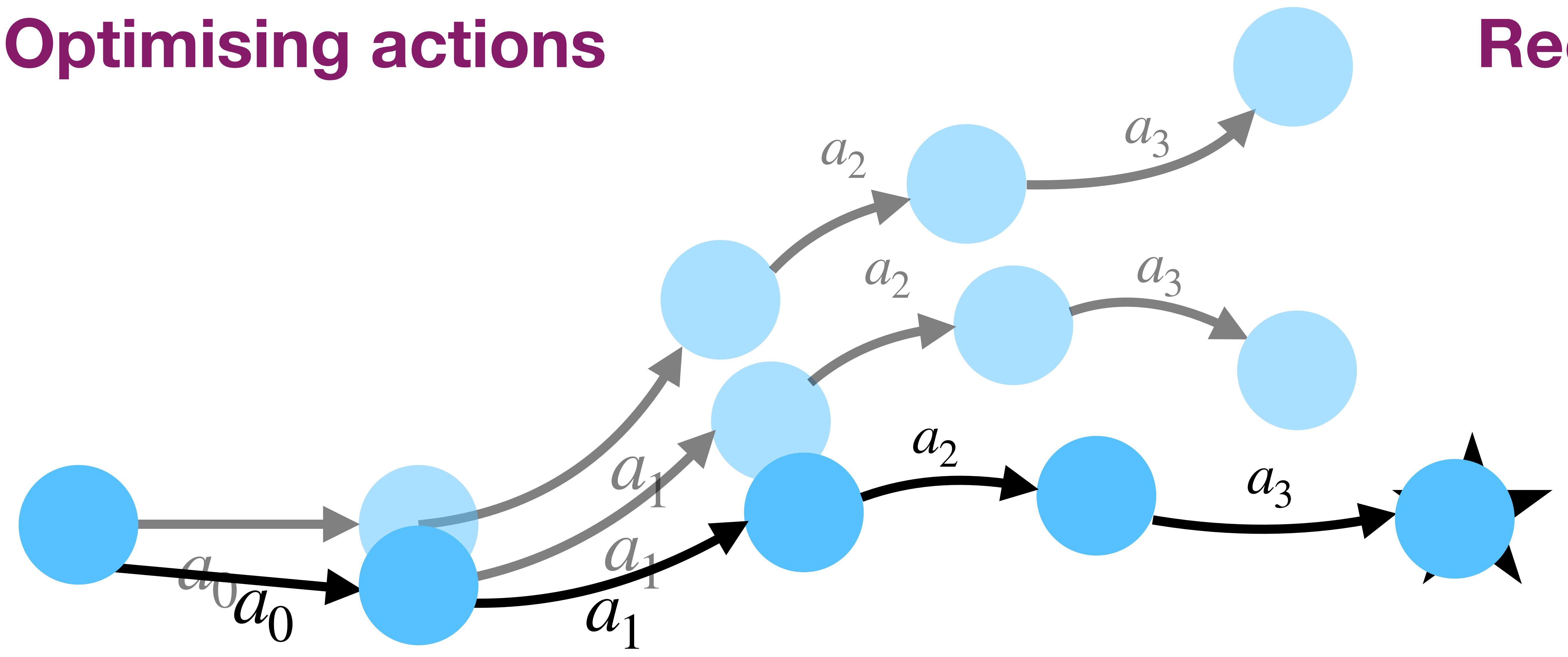
Optimising actions

Recursively evaluate dynamics



Shooting Methods

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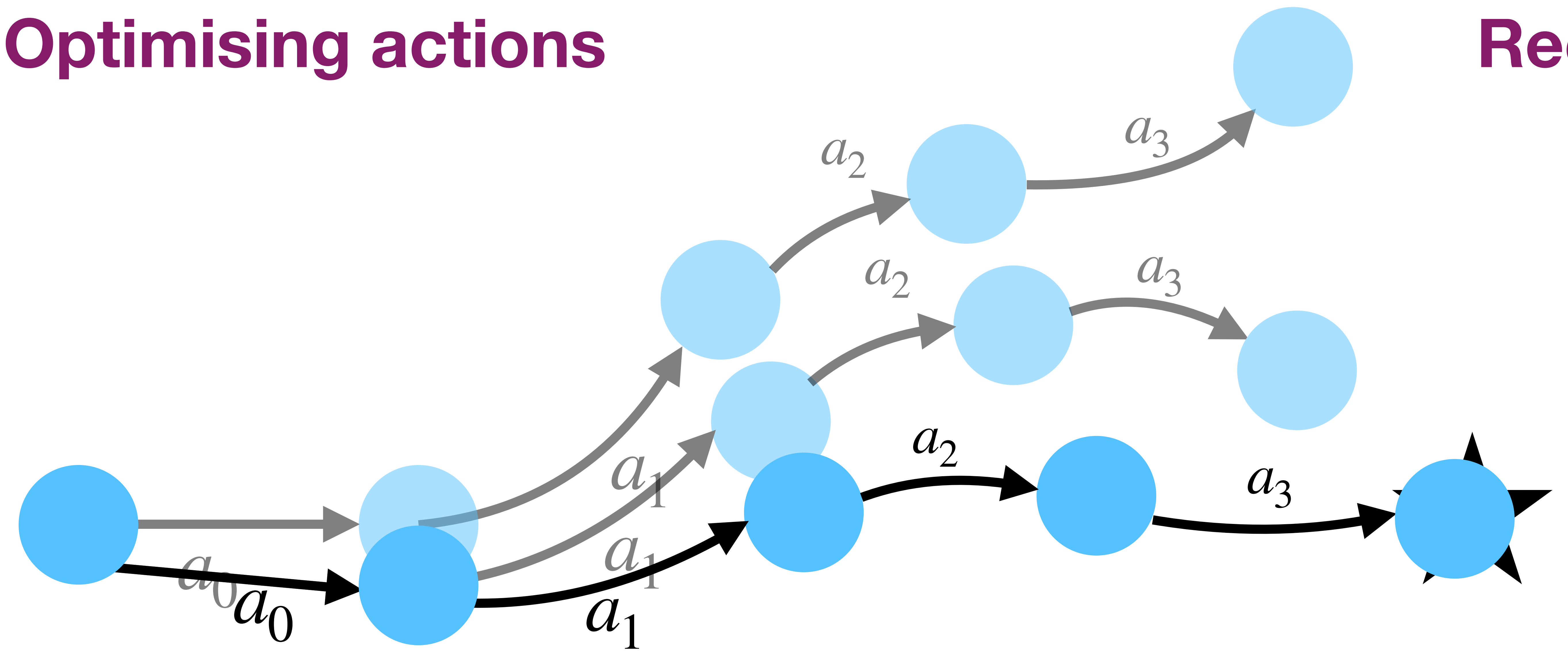
Optimising actions

Recursively evaluate dynamics



Shooting Methods
Illustration

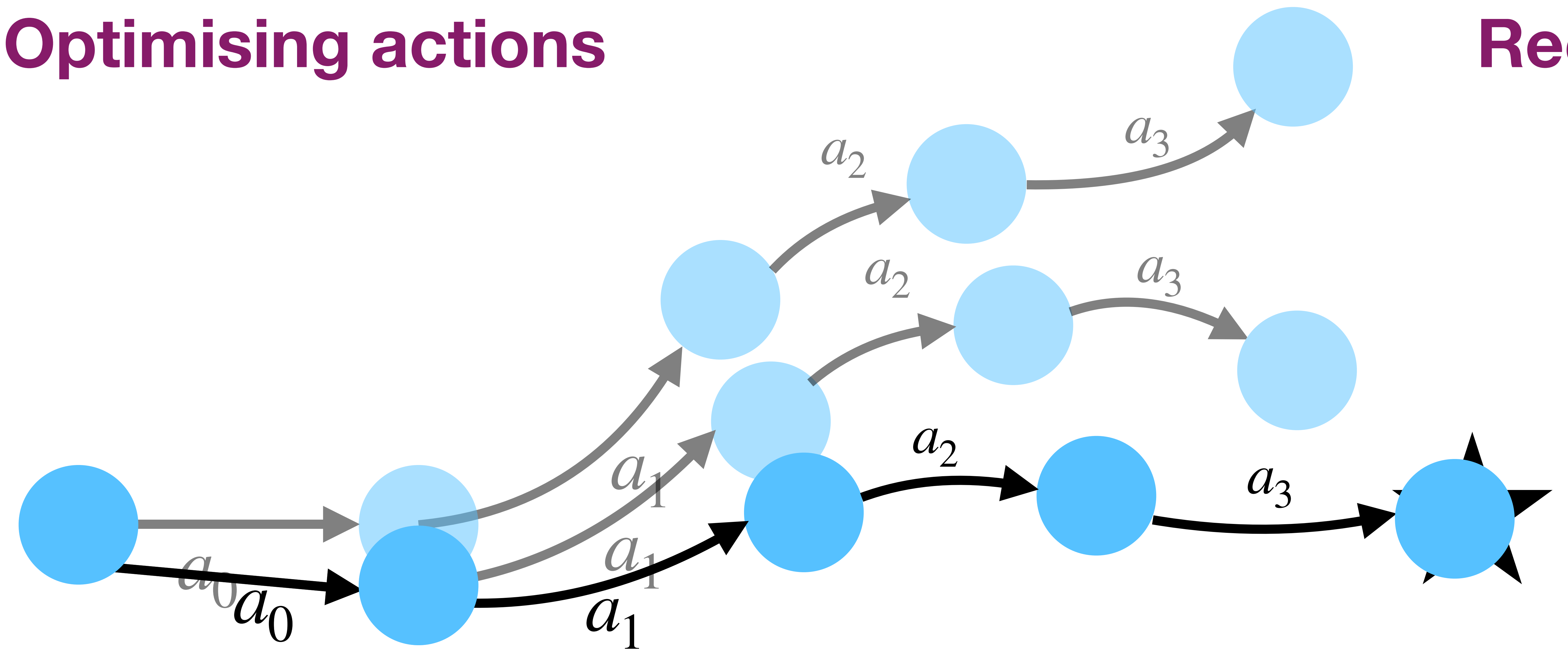
Gradient based approaches are fast

But local minima

$$\boxed{J(a_{0:H})} = \gamma^0 r(s_0, a_0) + \gamma^1 r(f(s_0, a_0), a_1) + \ldots + \gamma^H r(\boxed{f(f(\ldots), a_{H-1}), a_H)}$$

Optimising actions

Recursively evaluate dynamics



Shooting Methods

Illustration

Gradient based approaches are fast

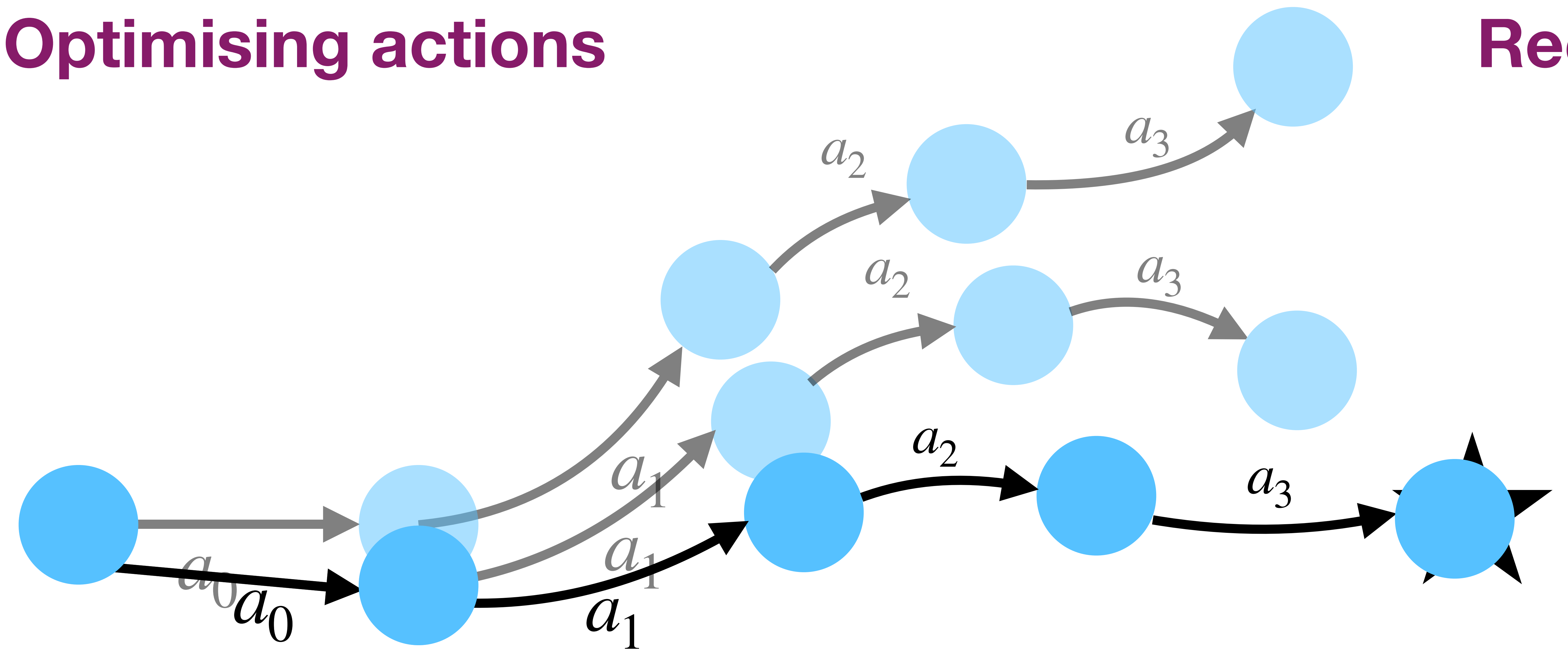
But local minima

And vanishing/exploding gradients

$$\boxed{J(a_{0:H})} = \gamma^0 r(s_0, a_0) + \gamma^1 r(f(s_0, a_0), a_1) + \ldots + \gamma^H r(\boxed{f(f(\ldots), a_{H-1}), a_H)}$$

Optimising actions

Recursively evaluate dynamics

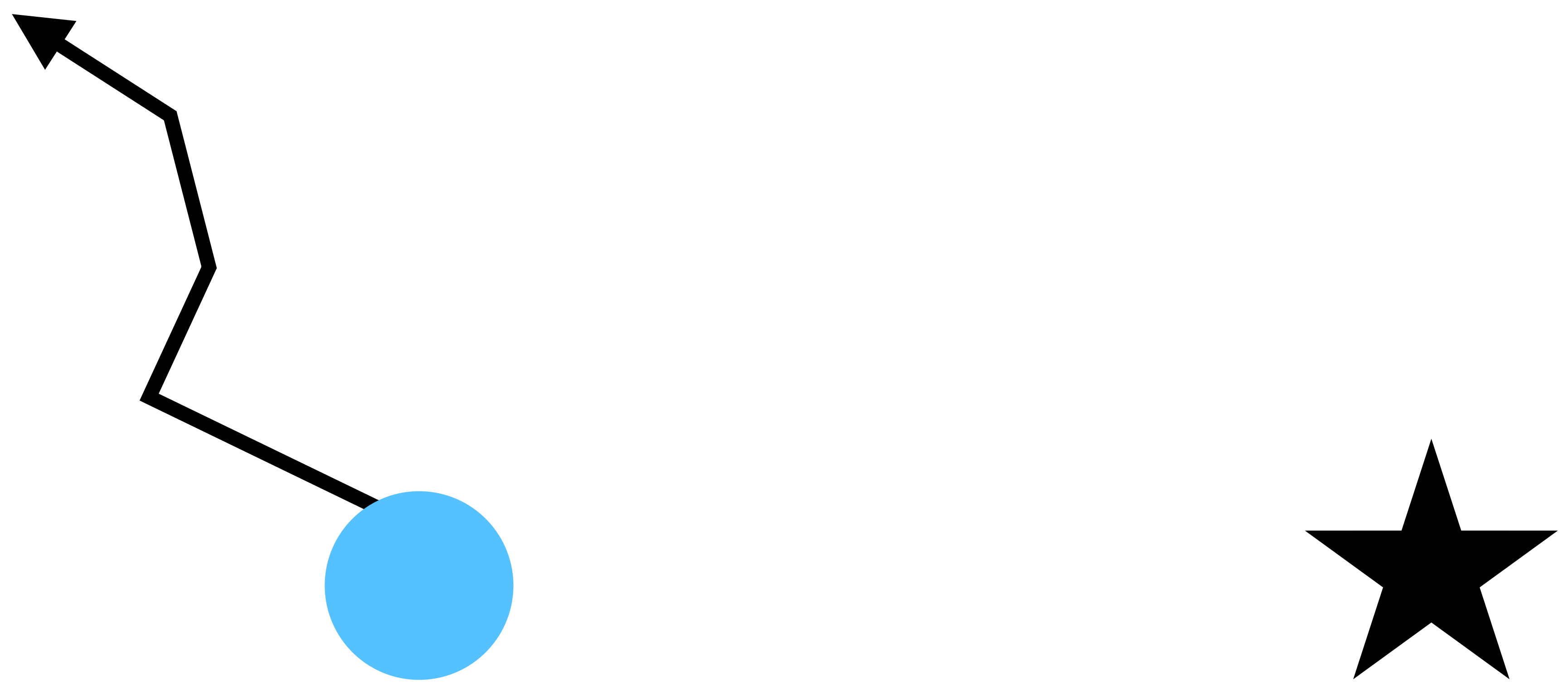


Shooting Methods

Random shooting

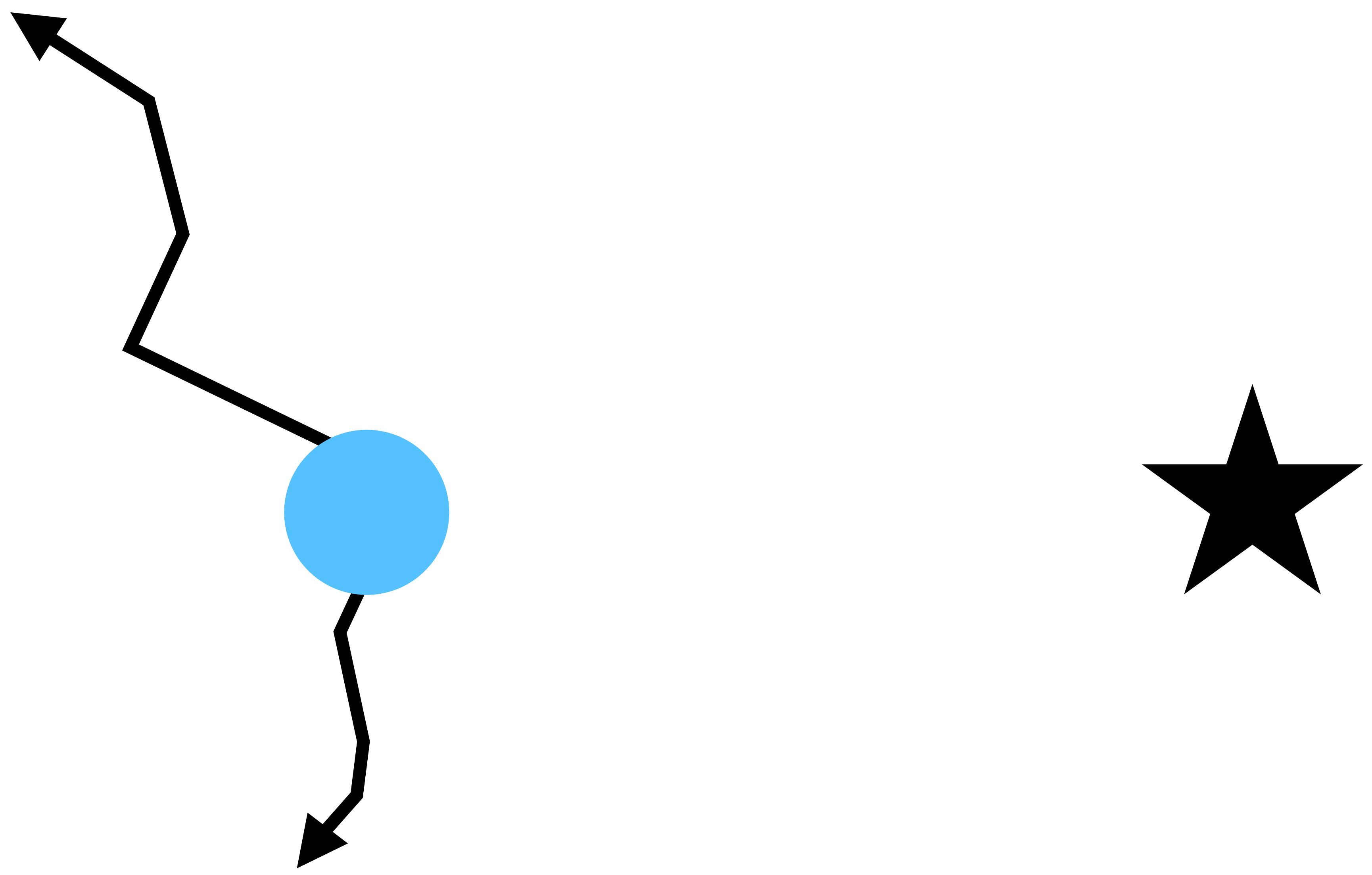
Shooting Methods

Random shooting



Shooting Methods

Random shooting



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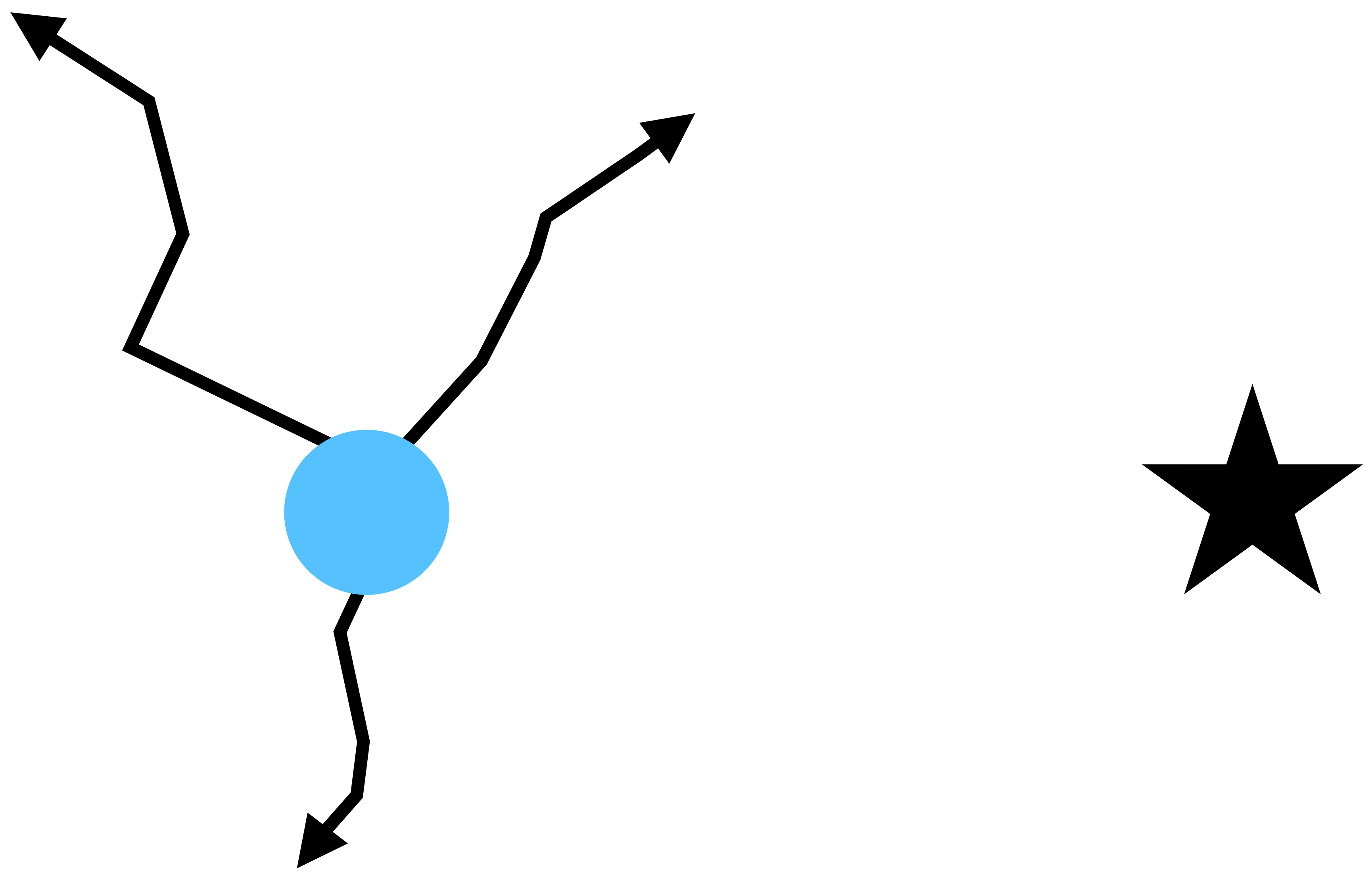
Shooting Methods

Random shooting



Shooting Methods

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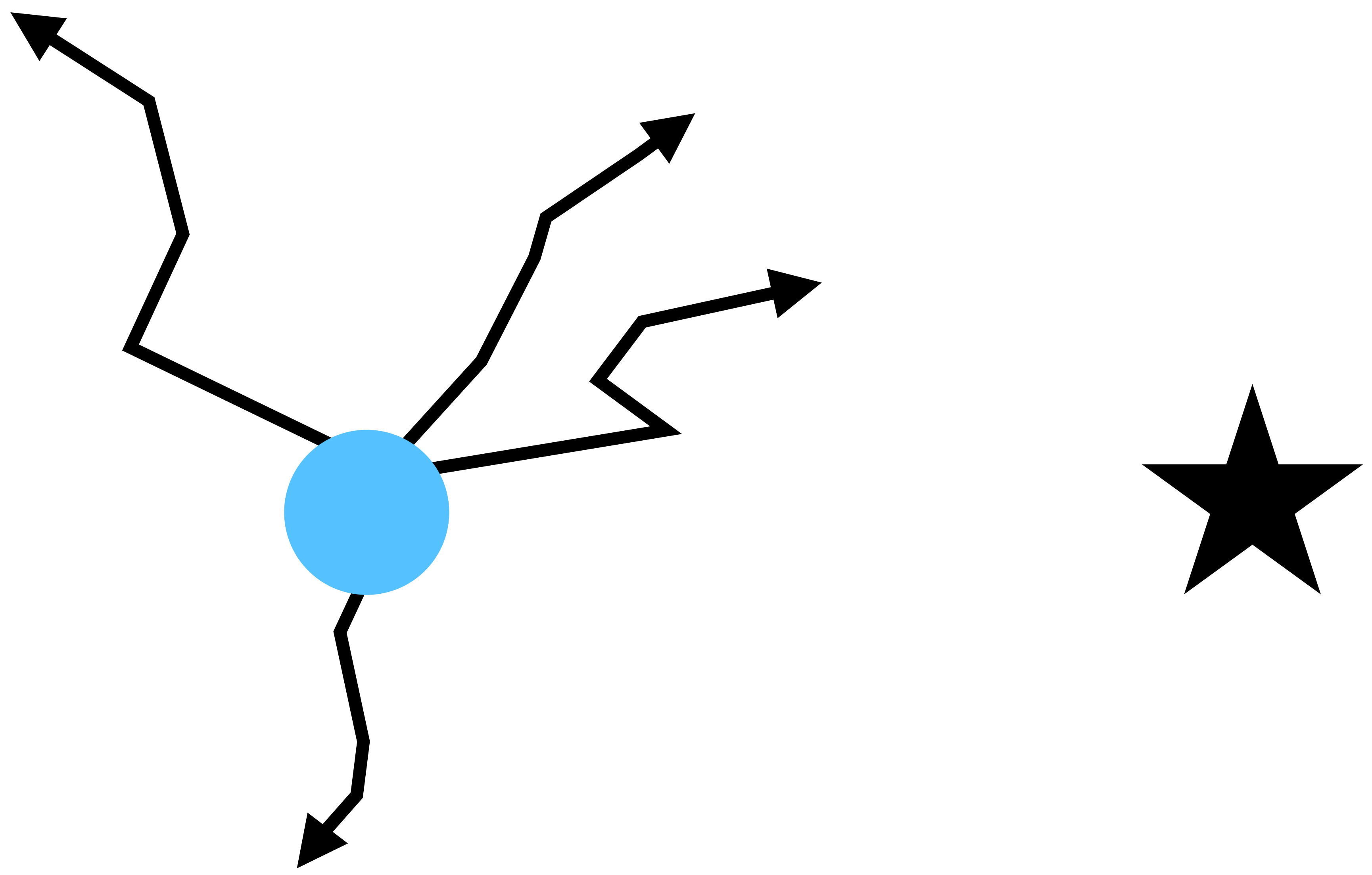

Shooting Methods

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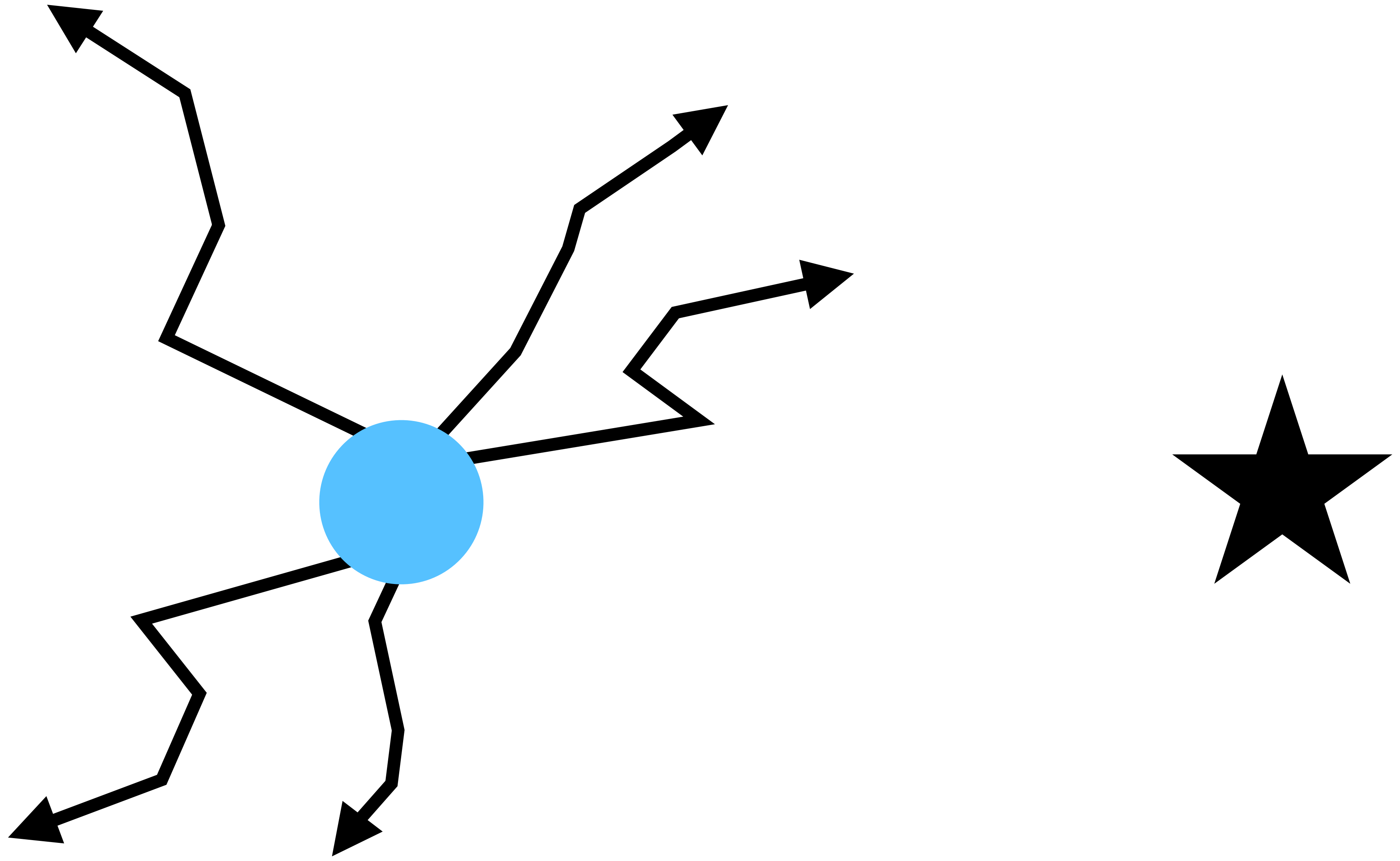

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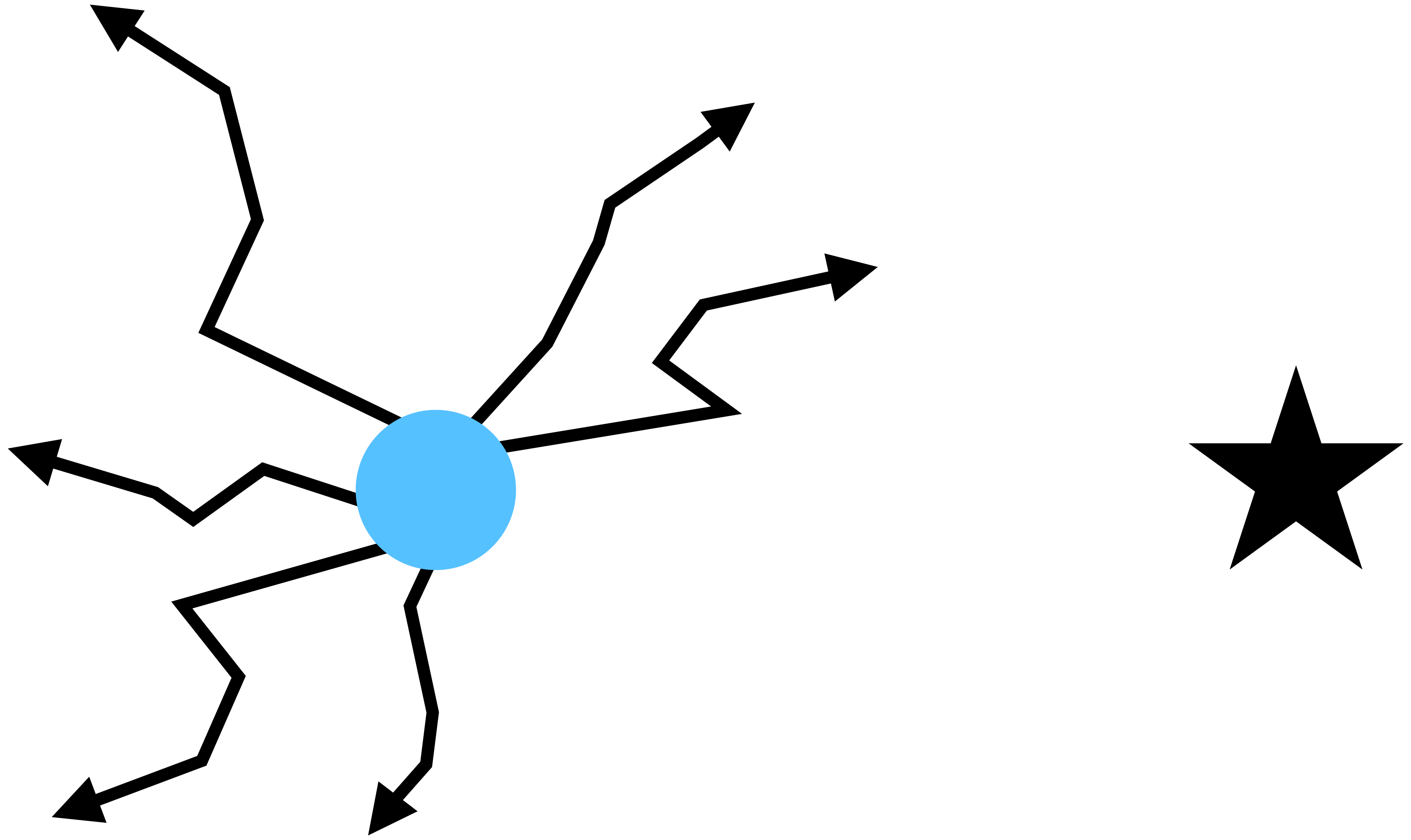

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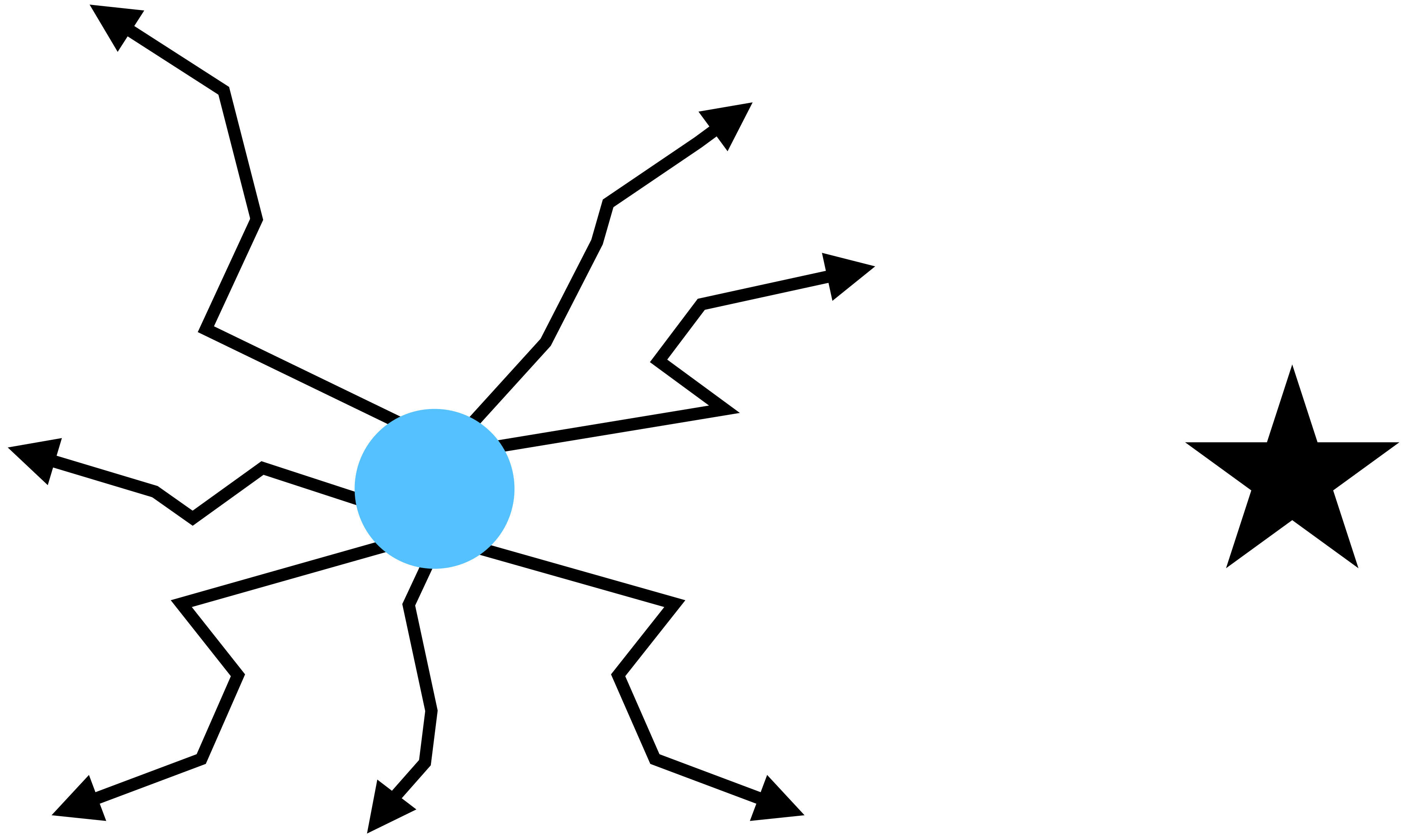

Shooting Methods

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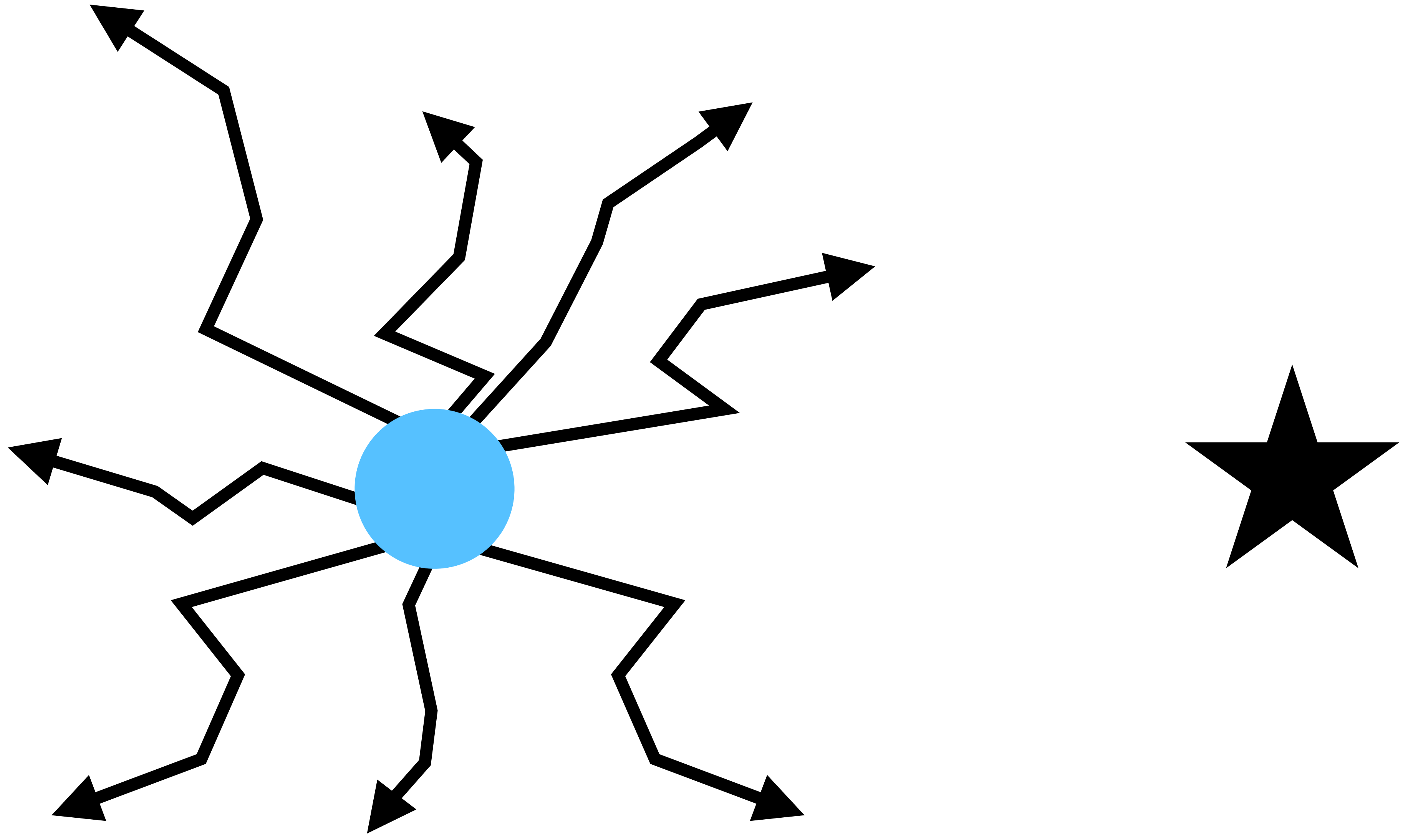
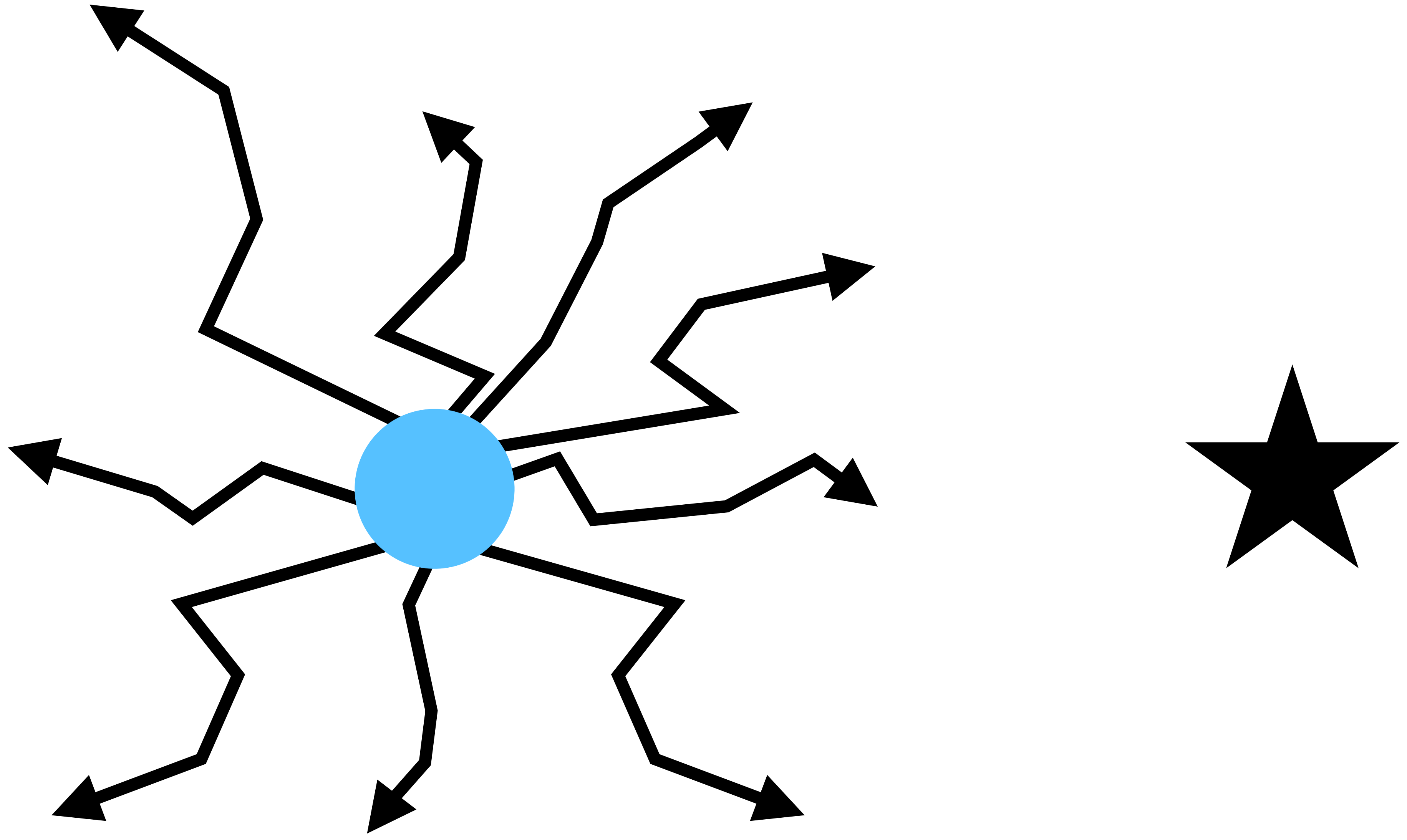
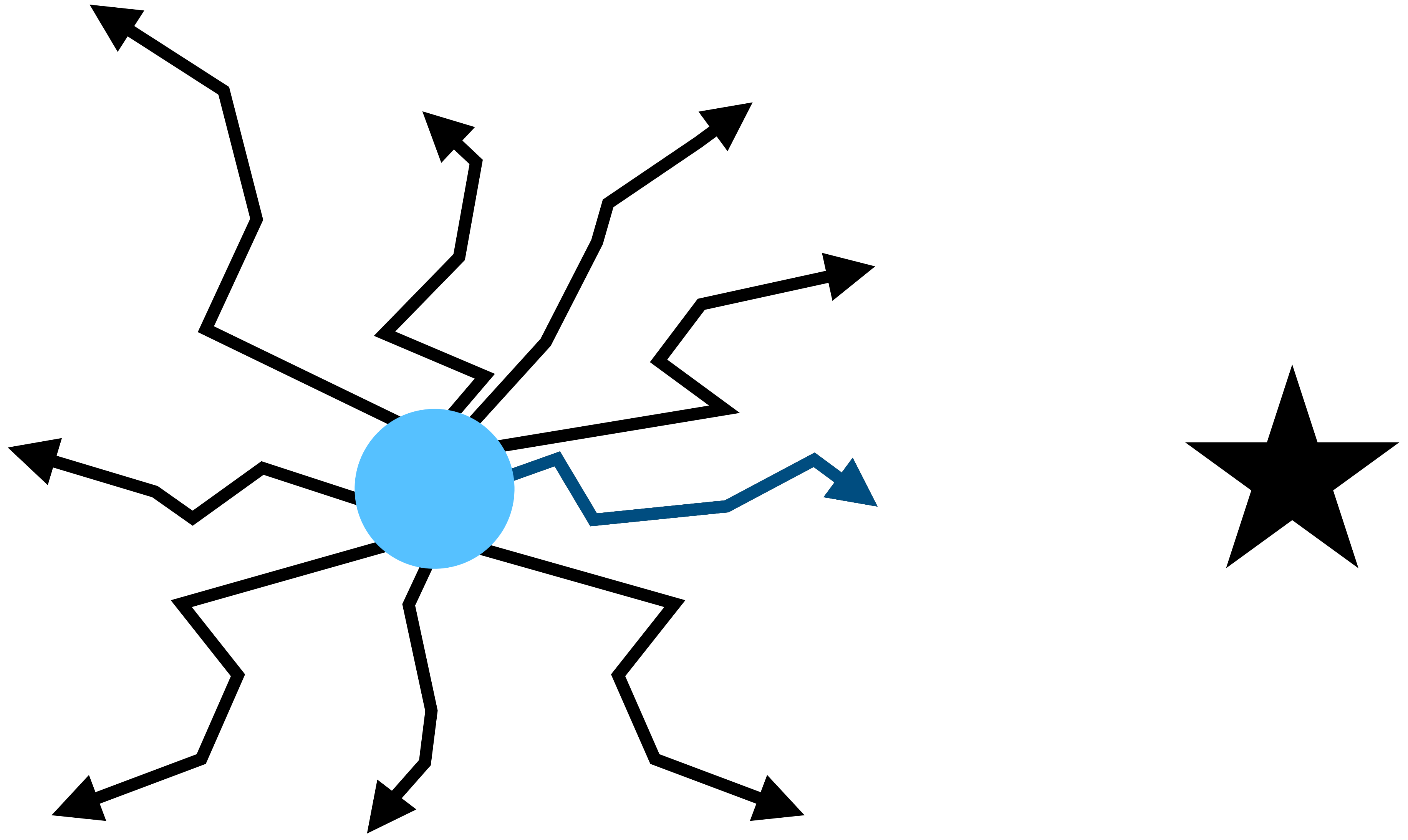


Shooting Methods

Random shooting



Shooting Methods
Random shooting



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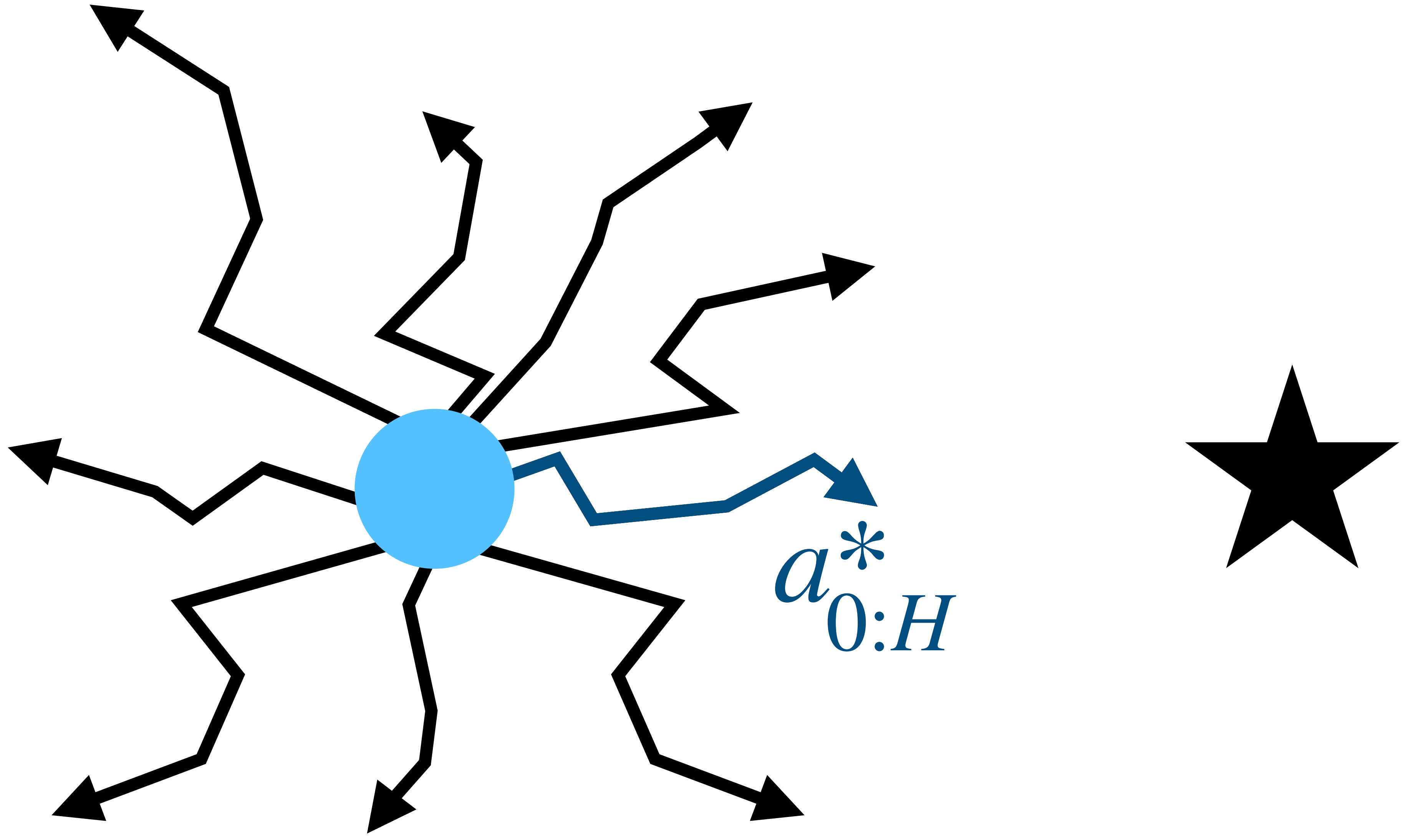
Shooting Methods

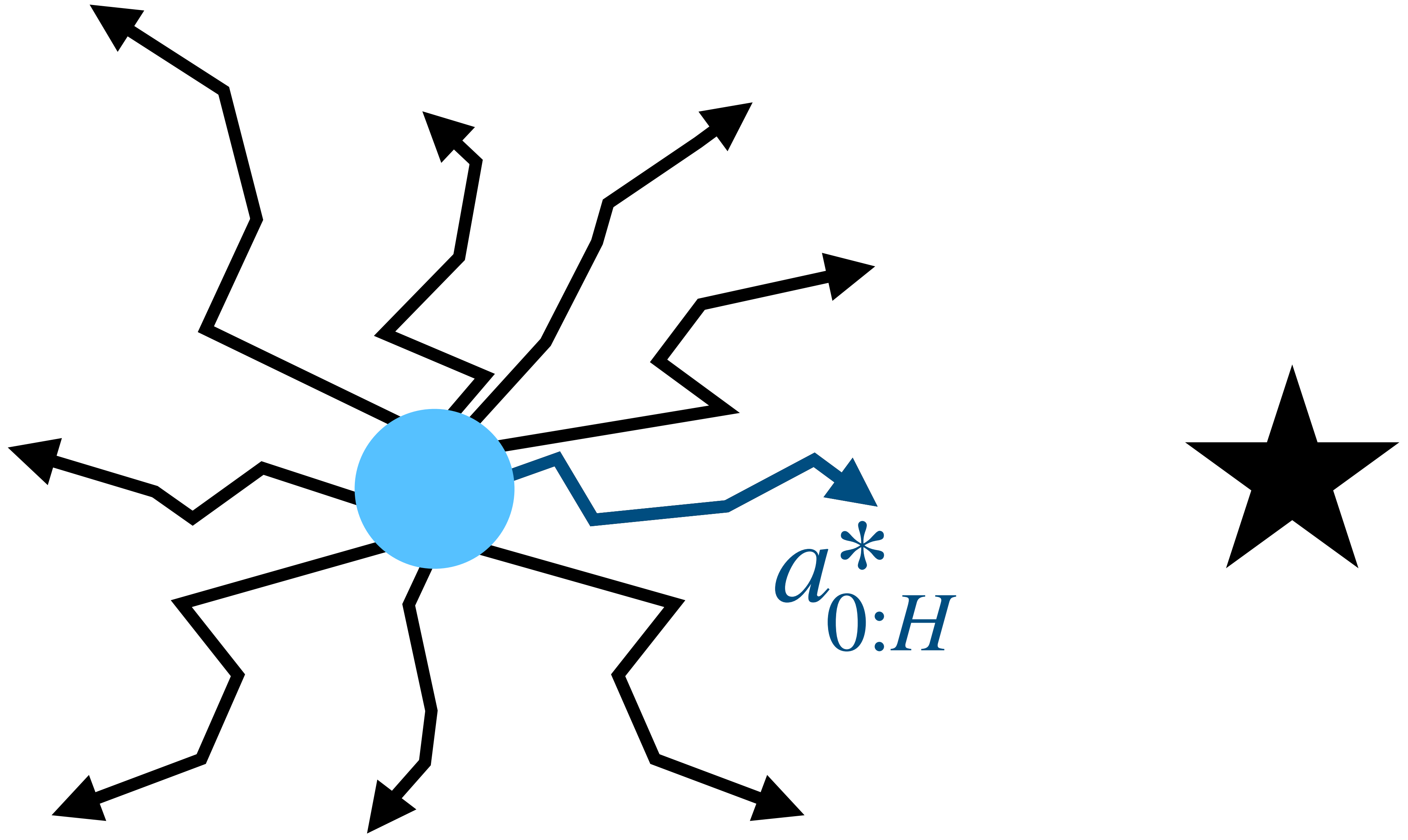
Random shooting



Shooting Methods

Random shooting

Simple

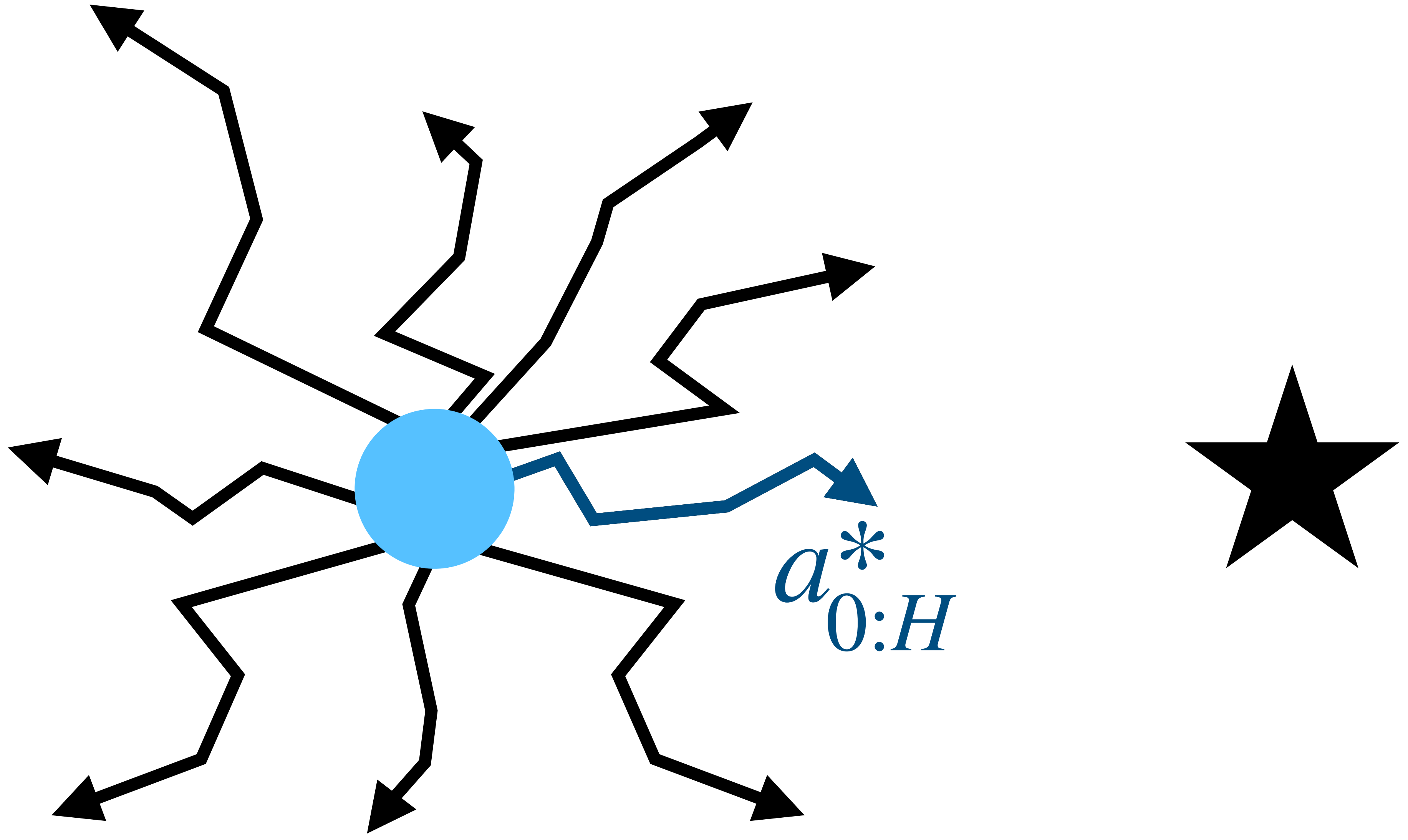


Shooting Methods

Random shooting

Simple

Parallelisable

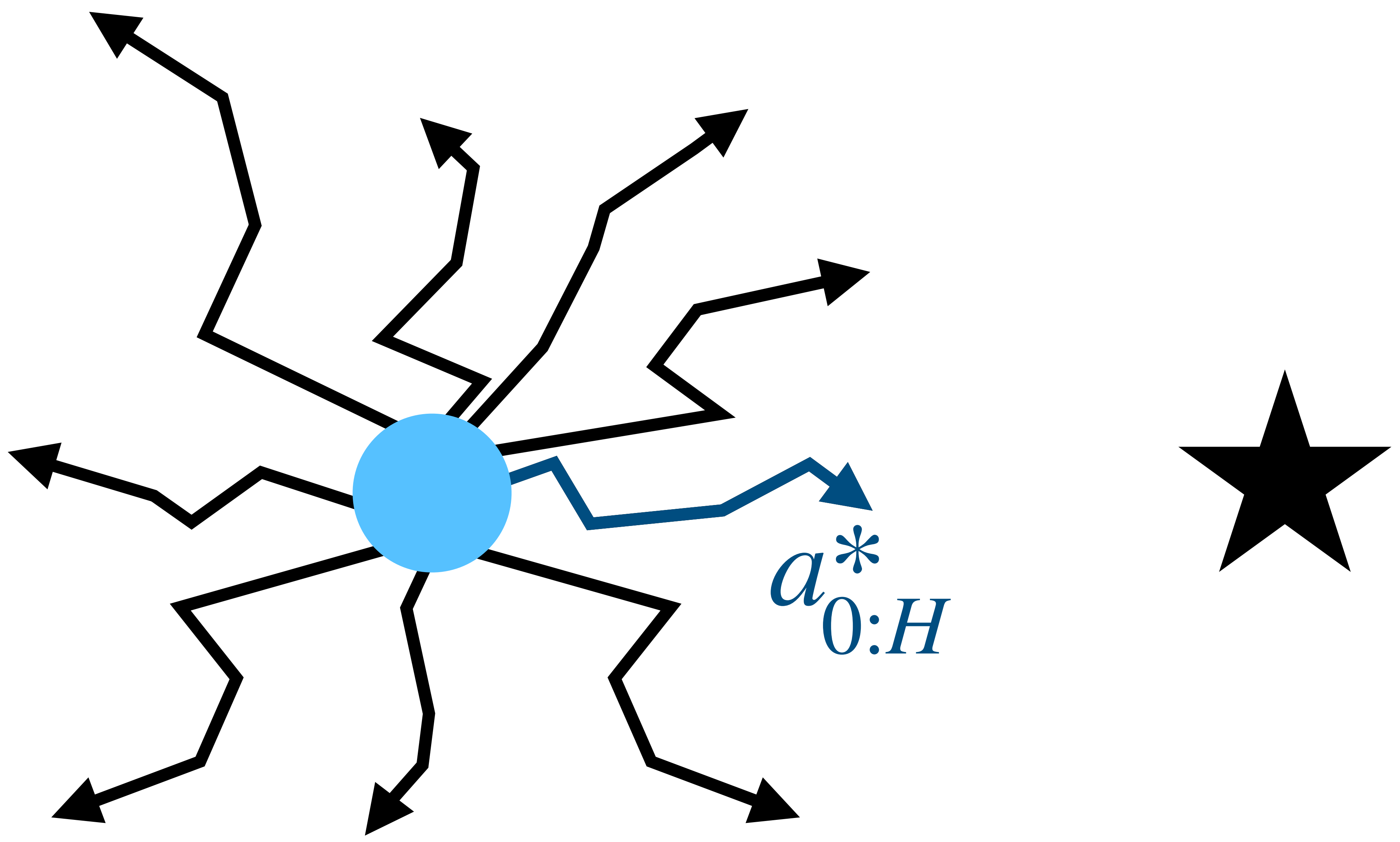


Shooting Methods
Random shooting

Simple

Parallelisable

Sample inefficient



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Shooting Methods

Random shooting

Simple

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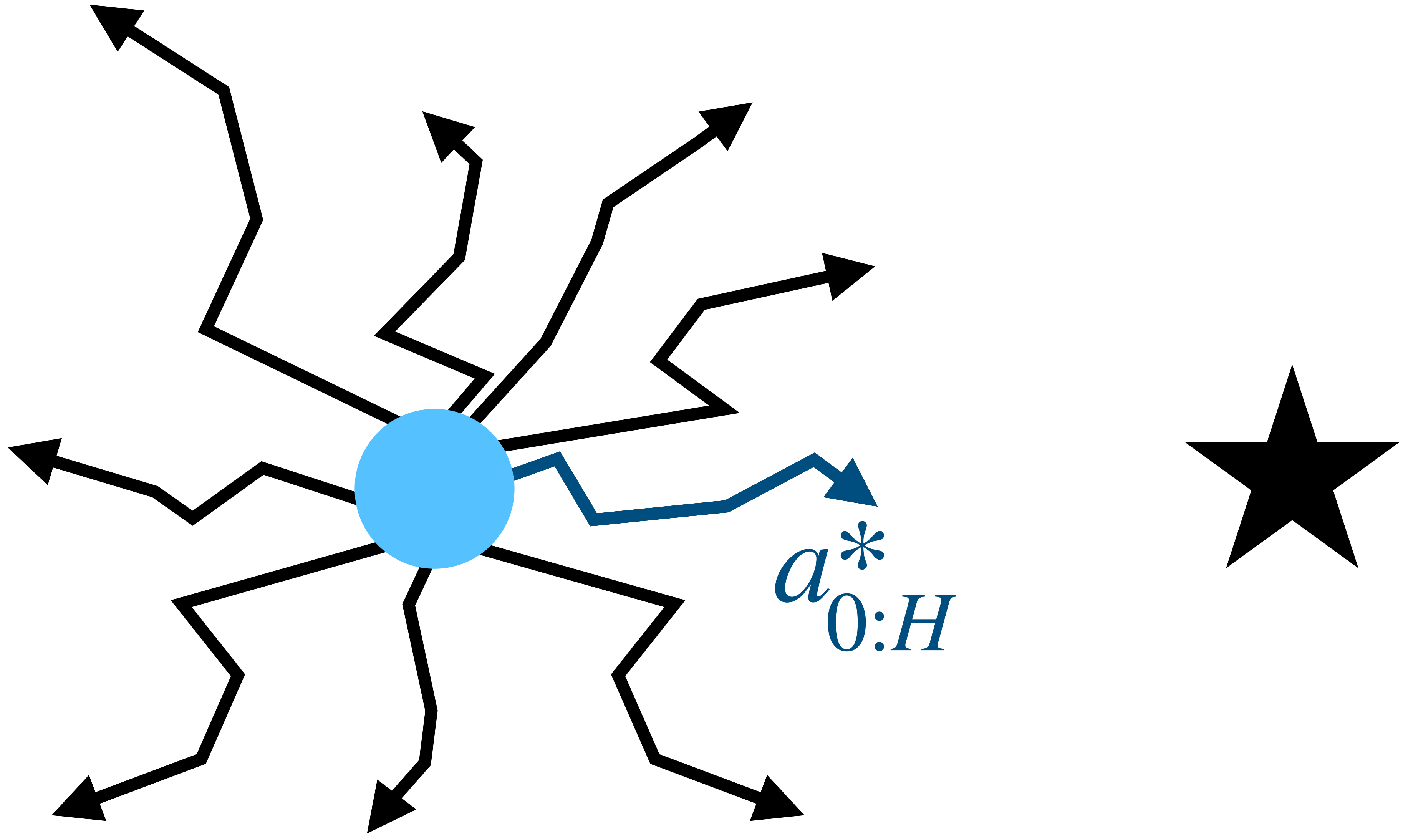
Sample inefficient



Shooting Methods

Cross-Entropy Method

Iteration 1

Shooting Methods

Cross-Entropy Method

Iteration 1

Initialise action sequence sampling distribution $\{a_t \sim \mathcal{N}(\mu_t, \sigma_t^2)\}_{t=0}^{H}$

Shooting Methods

Cross-Entropy Method

Iteration 1

Initialise action sequence sampling distribution $\{a_t \sim \mathcal{N}(\mu_t, \sigma_t^2)\}_{t=0}^{H}$

For each iteration

Shooting Methods

Cross-Entropy Method

Iteration 1

Initialise action sequence sampling distribution $\{a_t \sim \mathcal{N}(\mu_t, \sigma_t^2)\}_{t=0}^{H}$

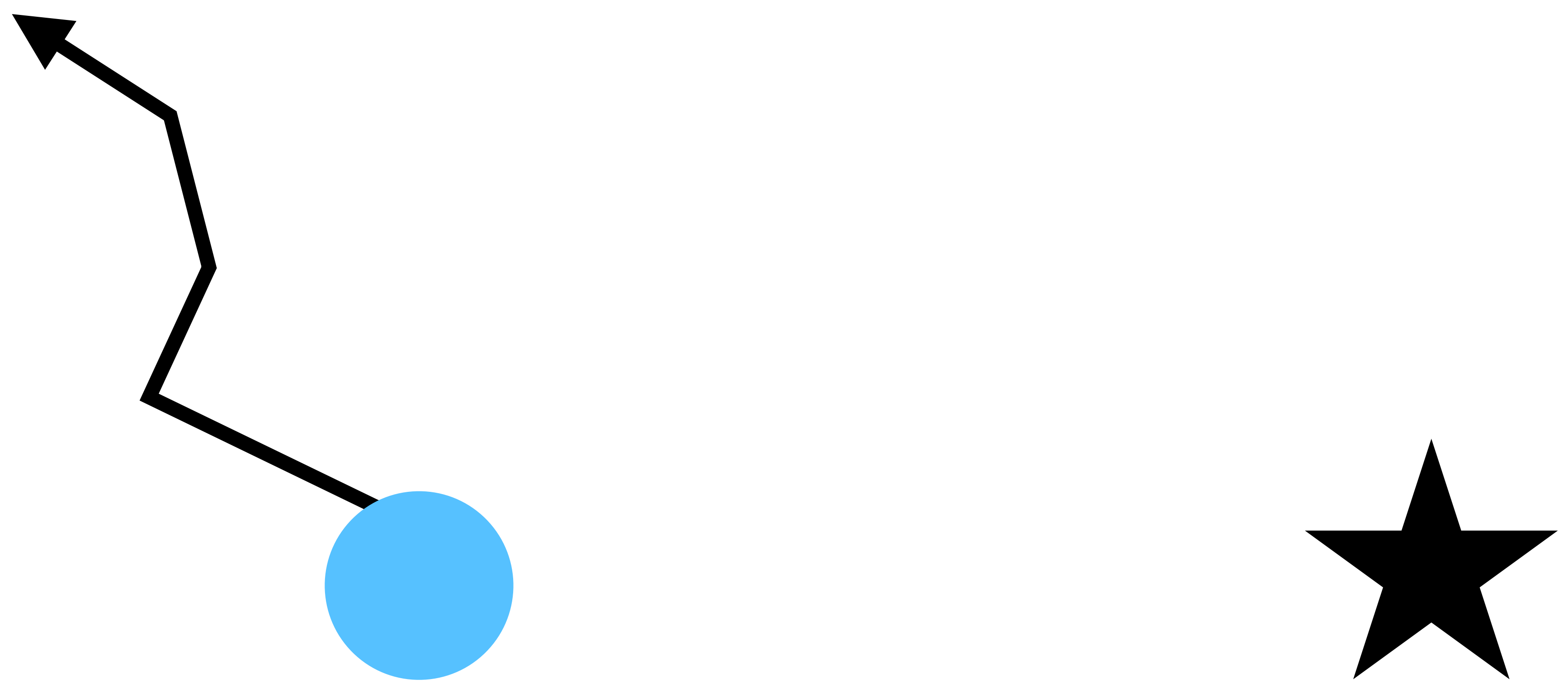
For each iteration

  Sample N action sequences $\{a_{0:H}^i\}_{i=1}^{N}$ from sampling distribution

Shooting Methods

Cross-Entropy Method

Iteration 1



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Shooting Methods

Cross-Entropy Method

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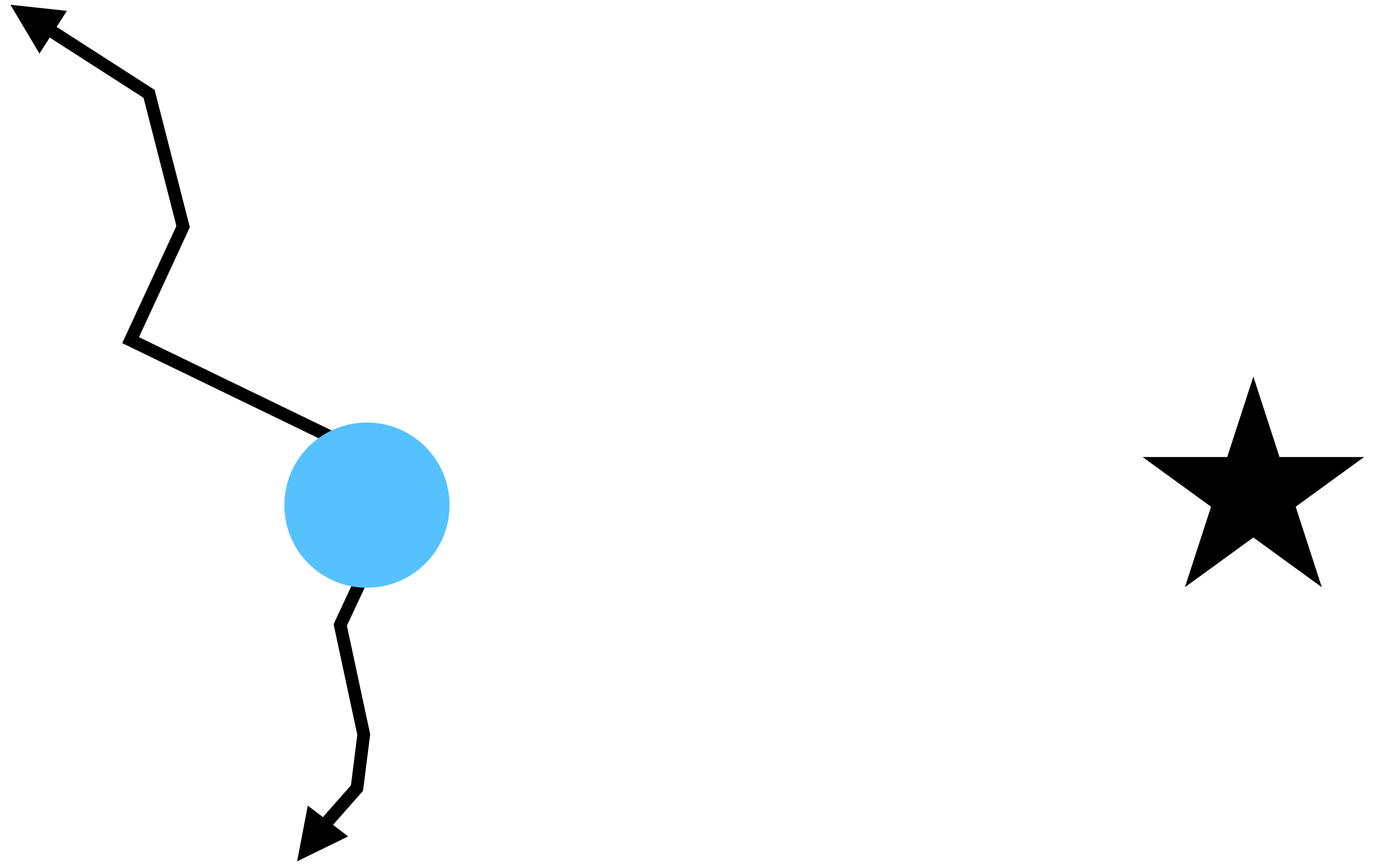

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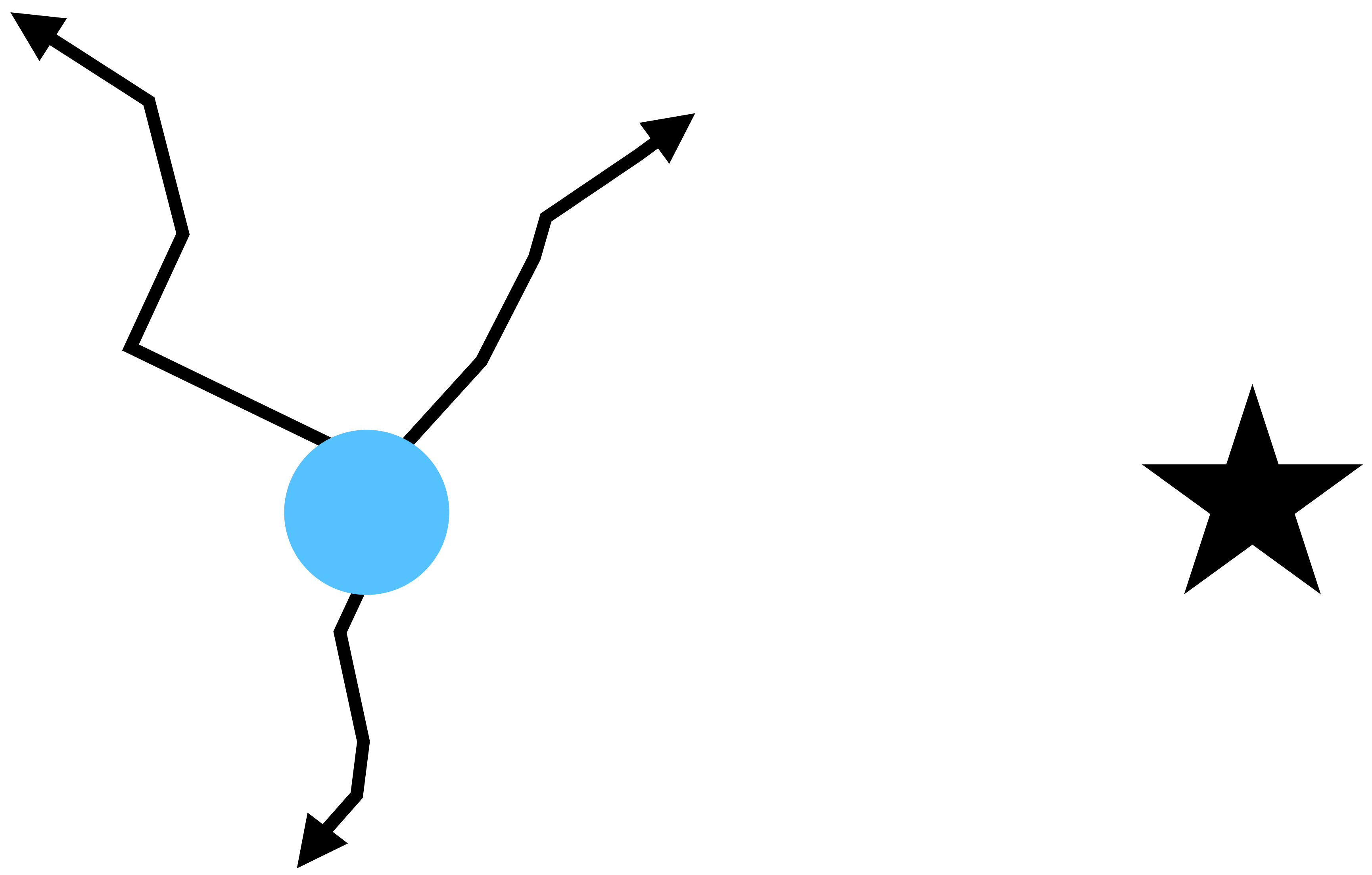

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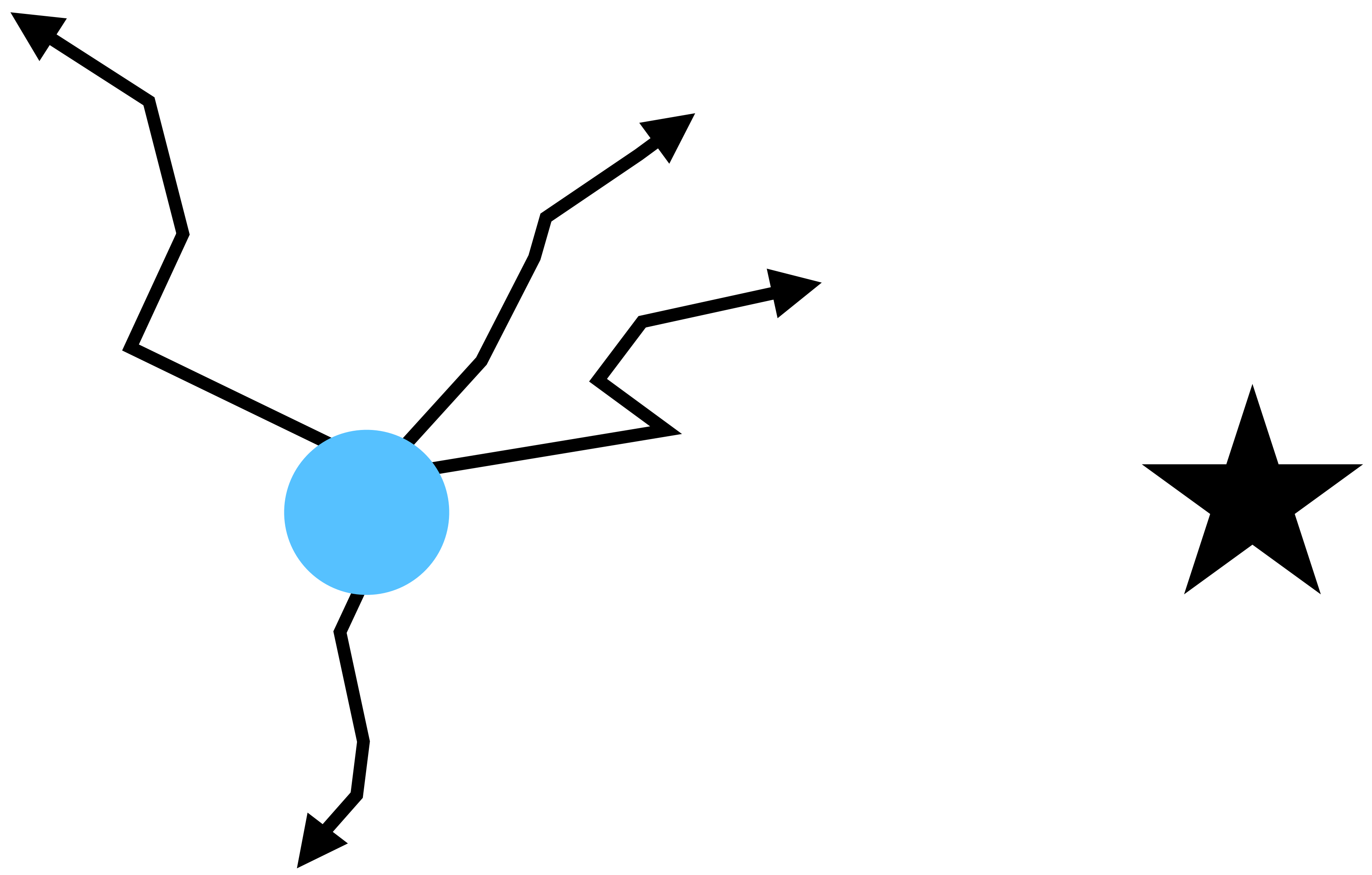

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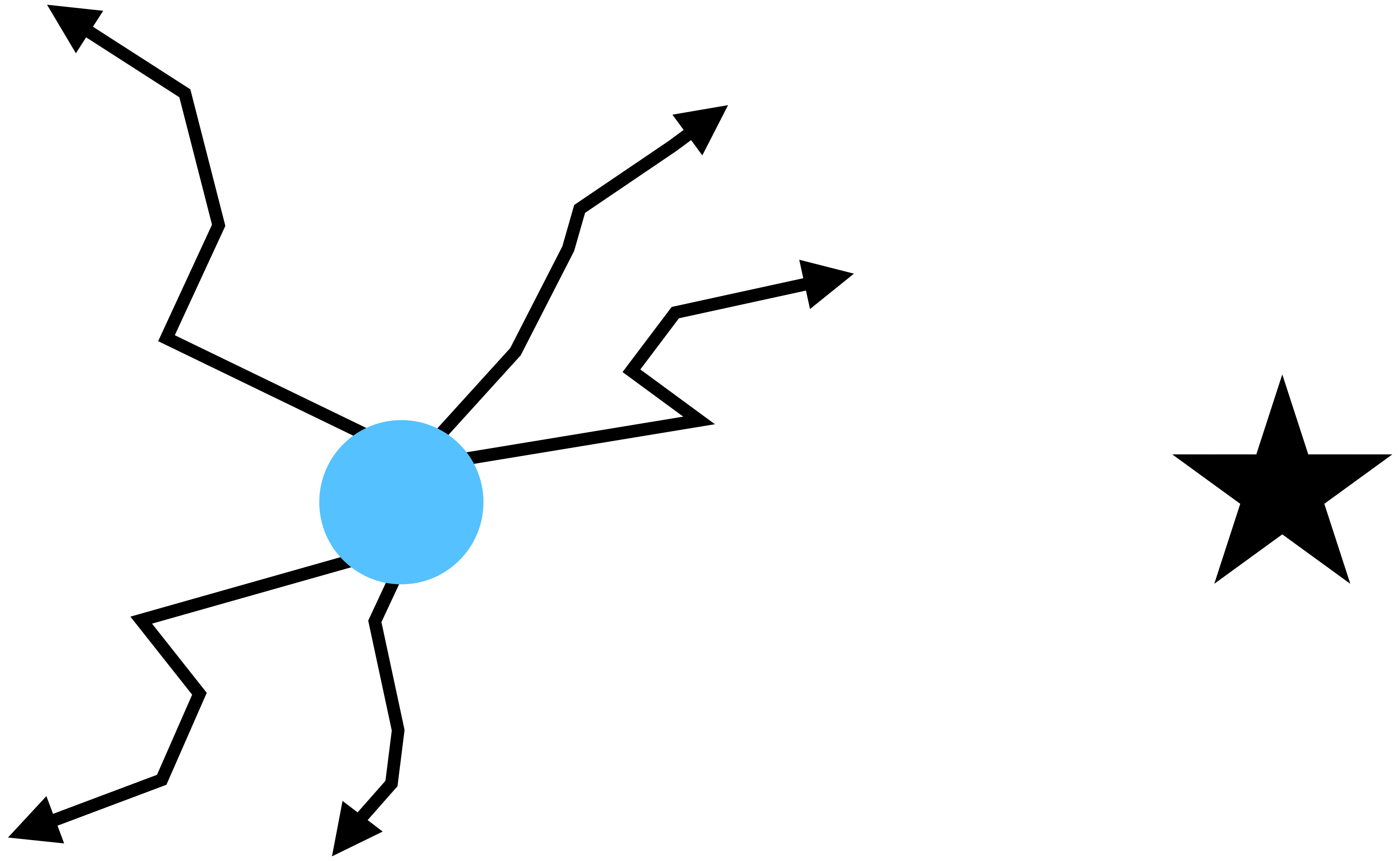
Shooting Methods

Cross-Entropy Method

Iteration 1



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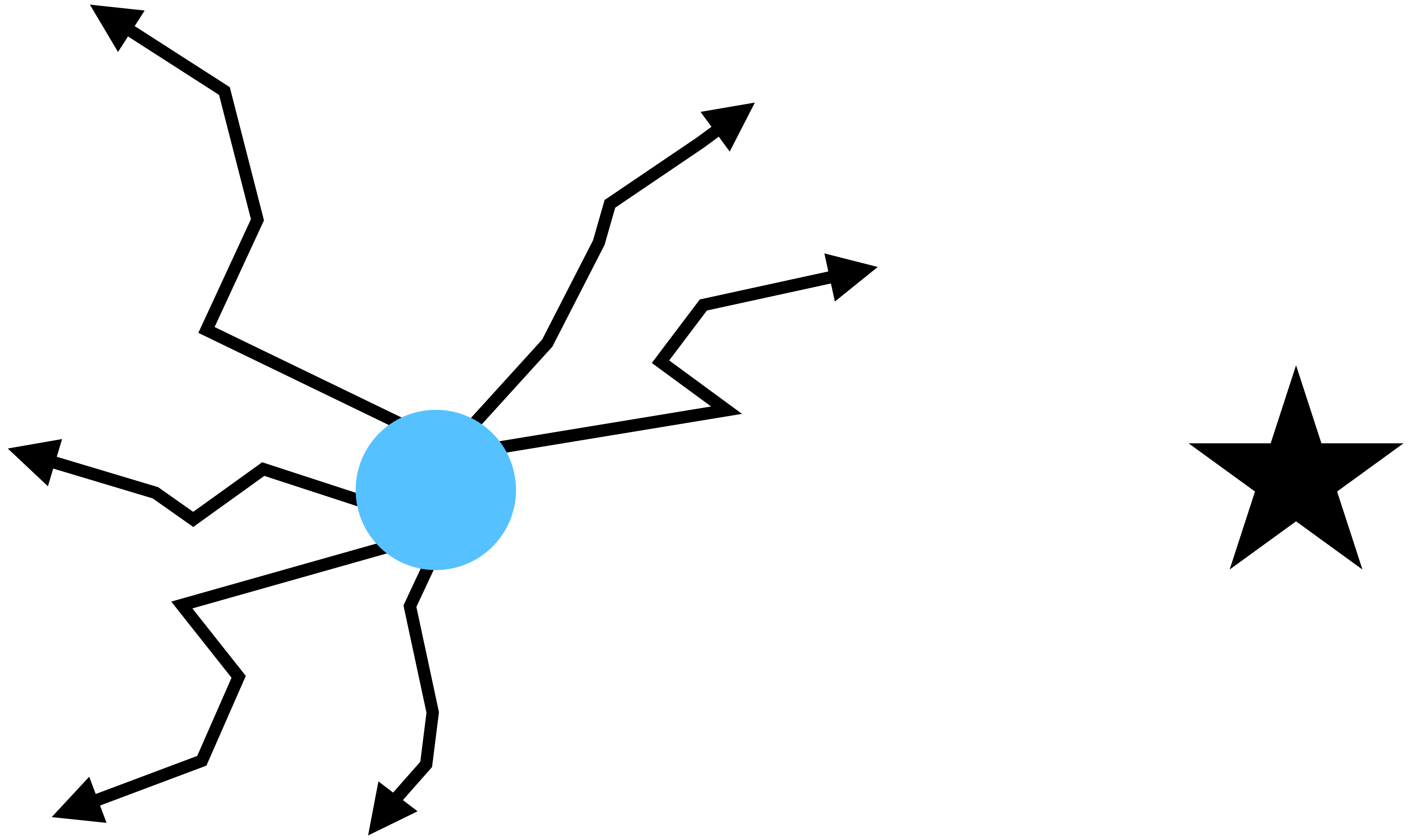

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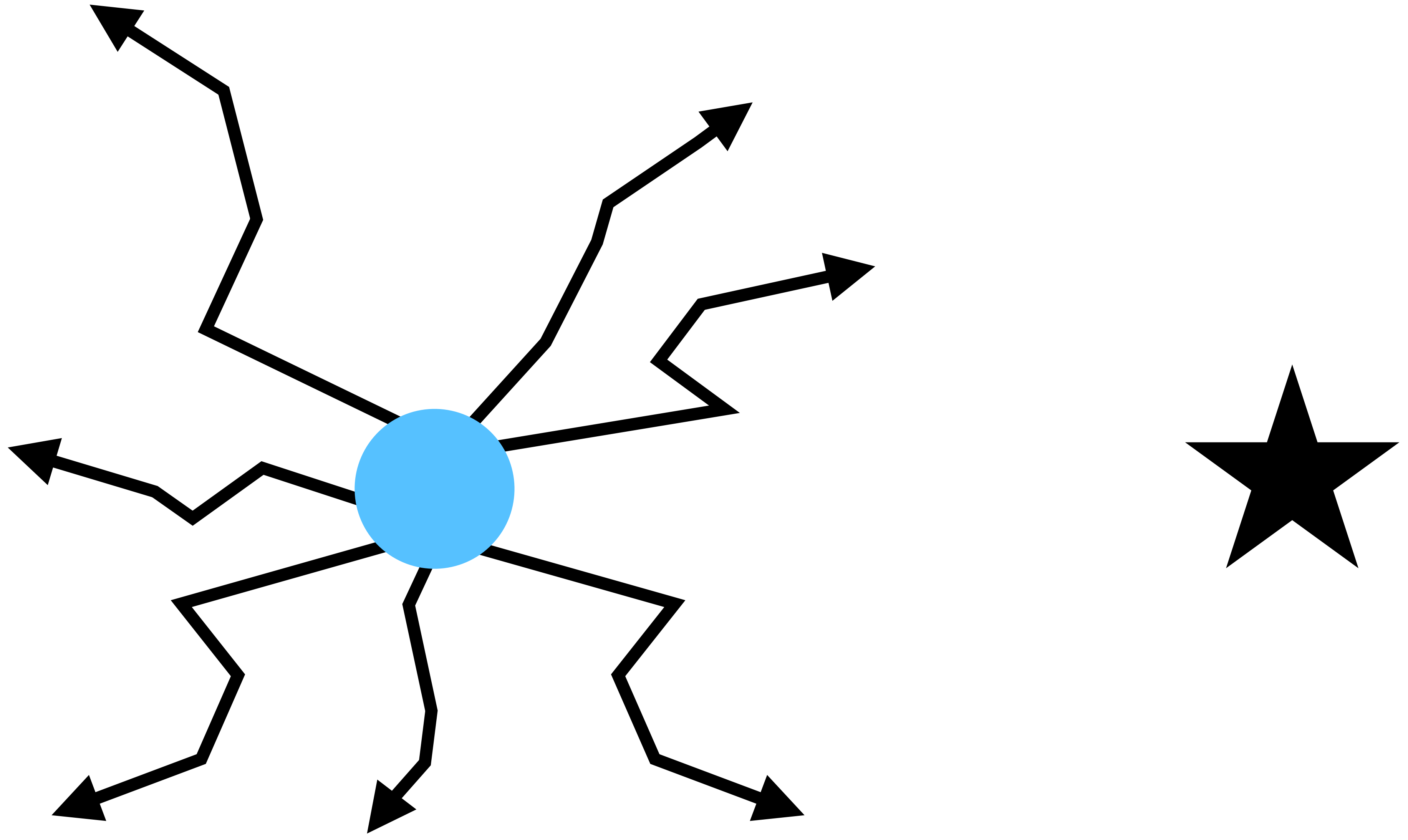

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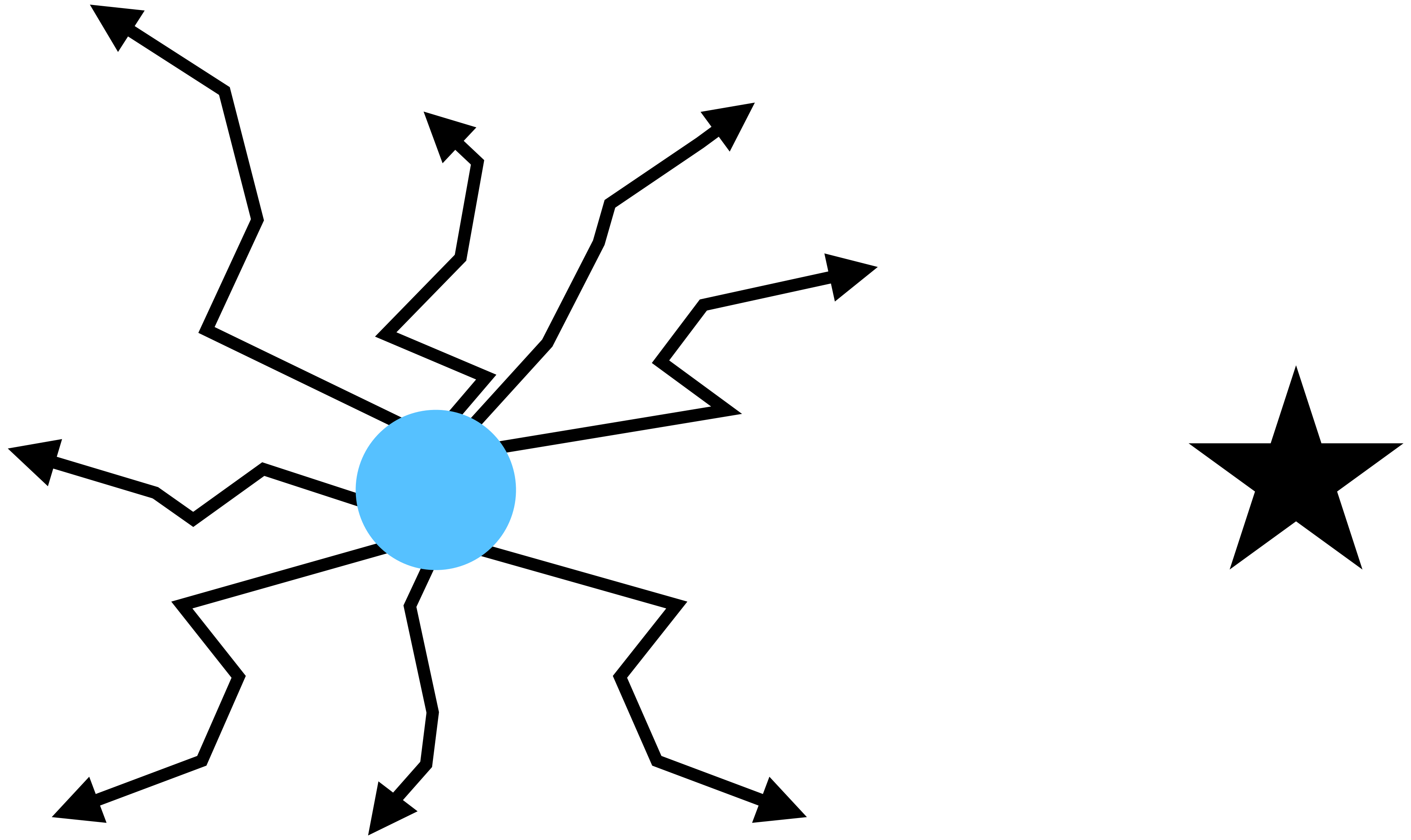

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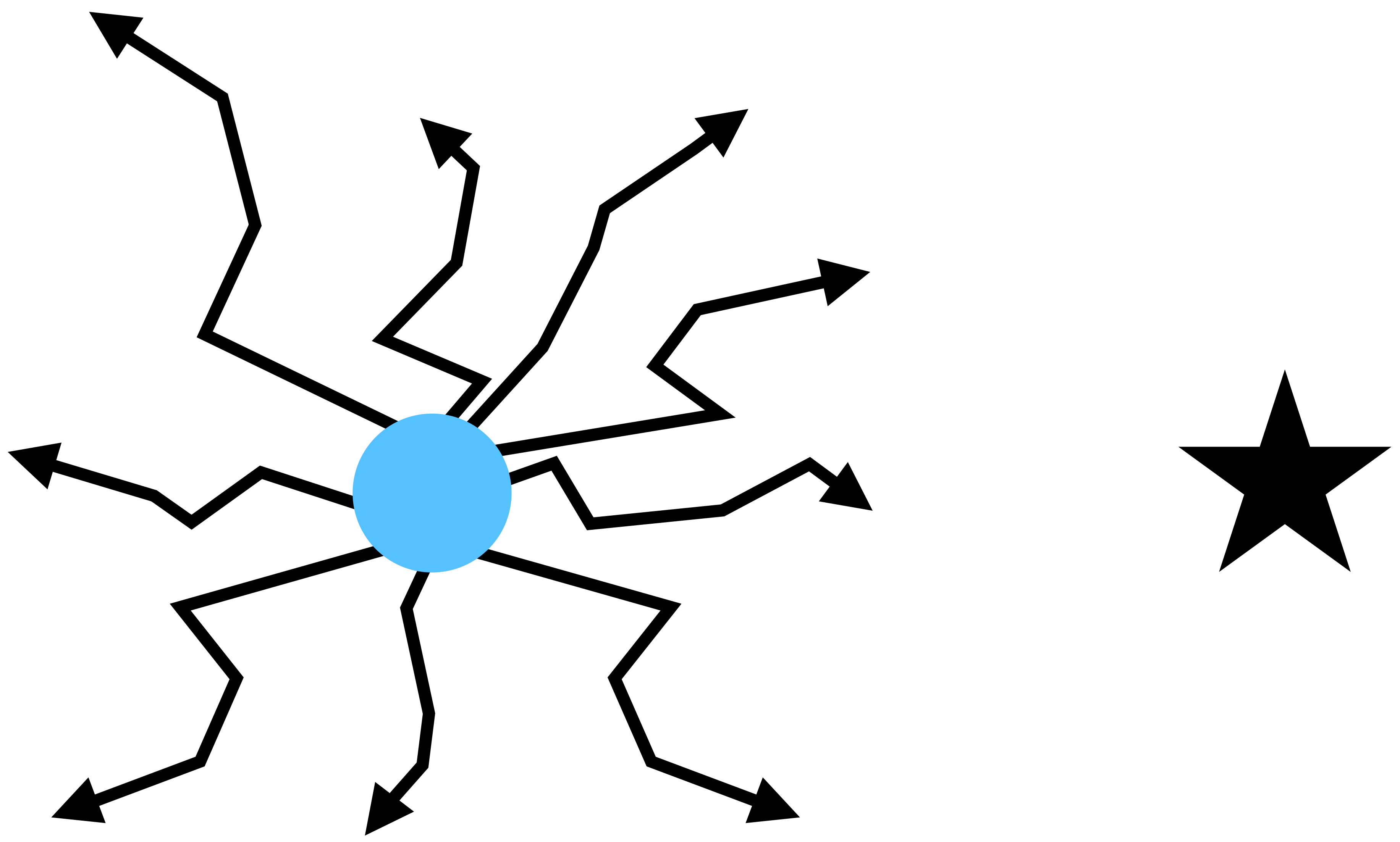

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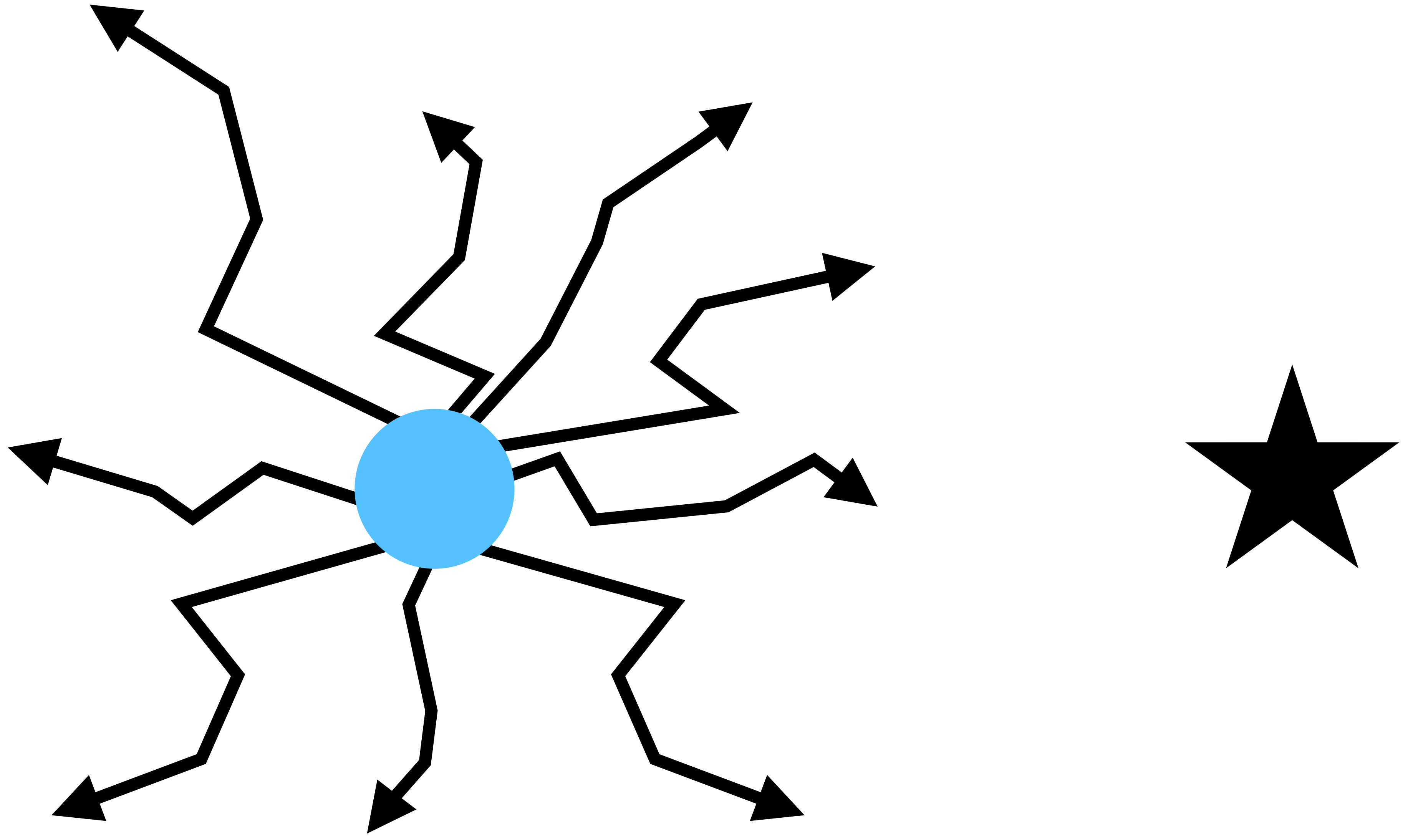

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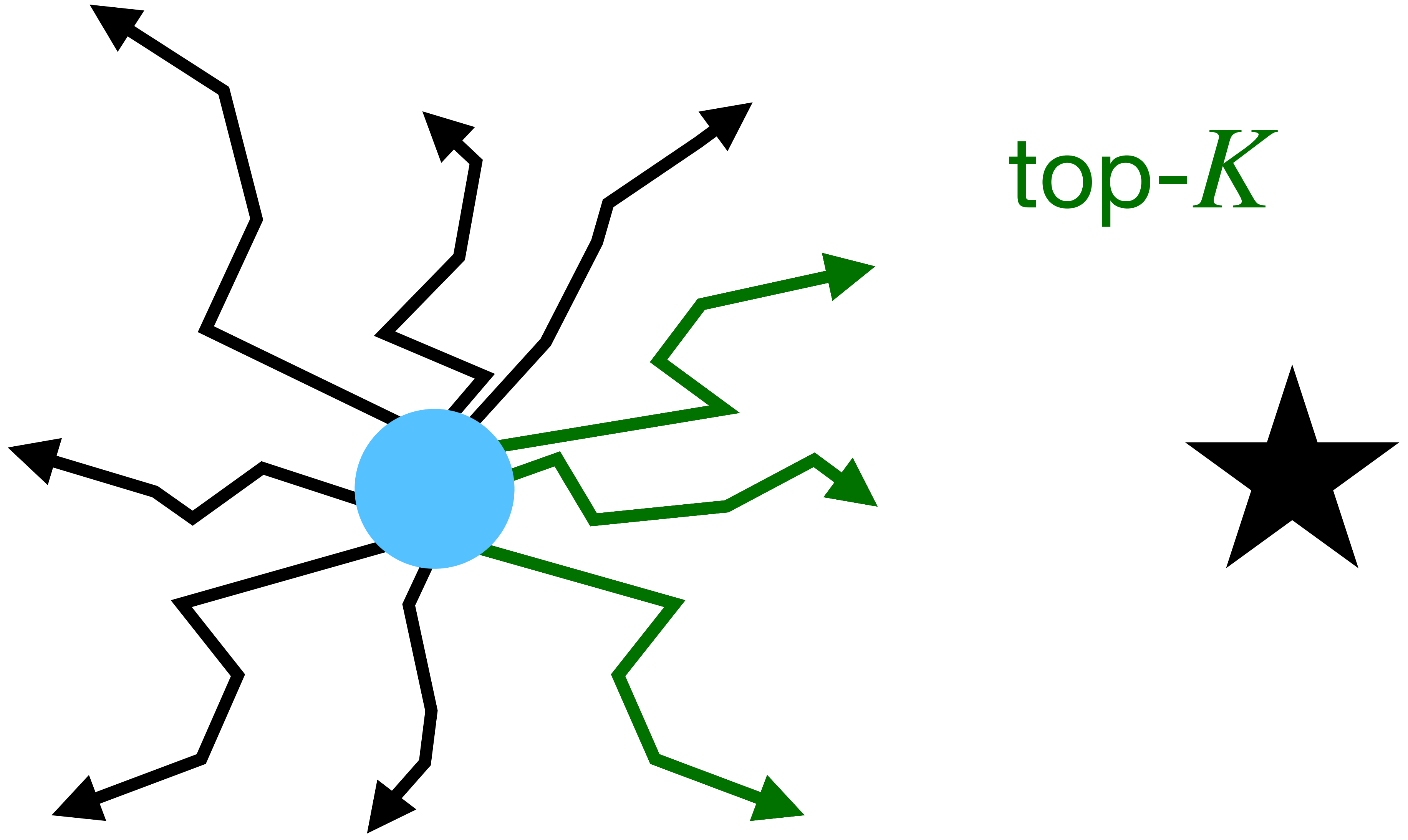
Evaluate objective $J(a_{0:H}^i) = \sum_{t=0}^{H} \gamma^t r(s_t, a_t^i)$ for each sample

Shooting Methods

Cross-Entropy Method

Iteration 1



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Evaluate objective $J(a_{0:H}^i) = \sum_{t=0}^{H} \gamma^t r(s_t, a_t^i)$ for each sample

Select top K performing samples, i.e. highest value $J(a_{0:H}^i)$

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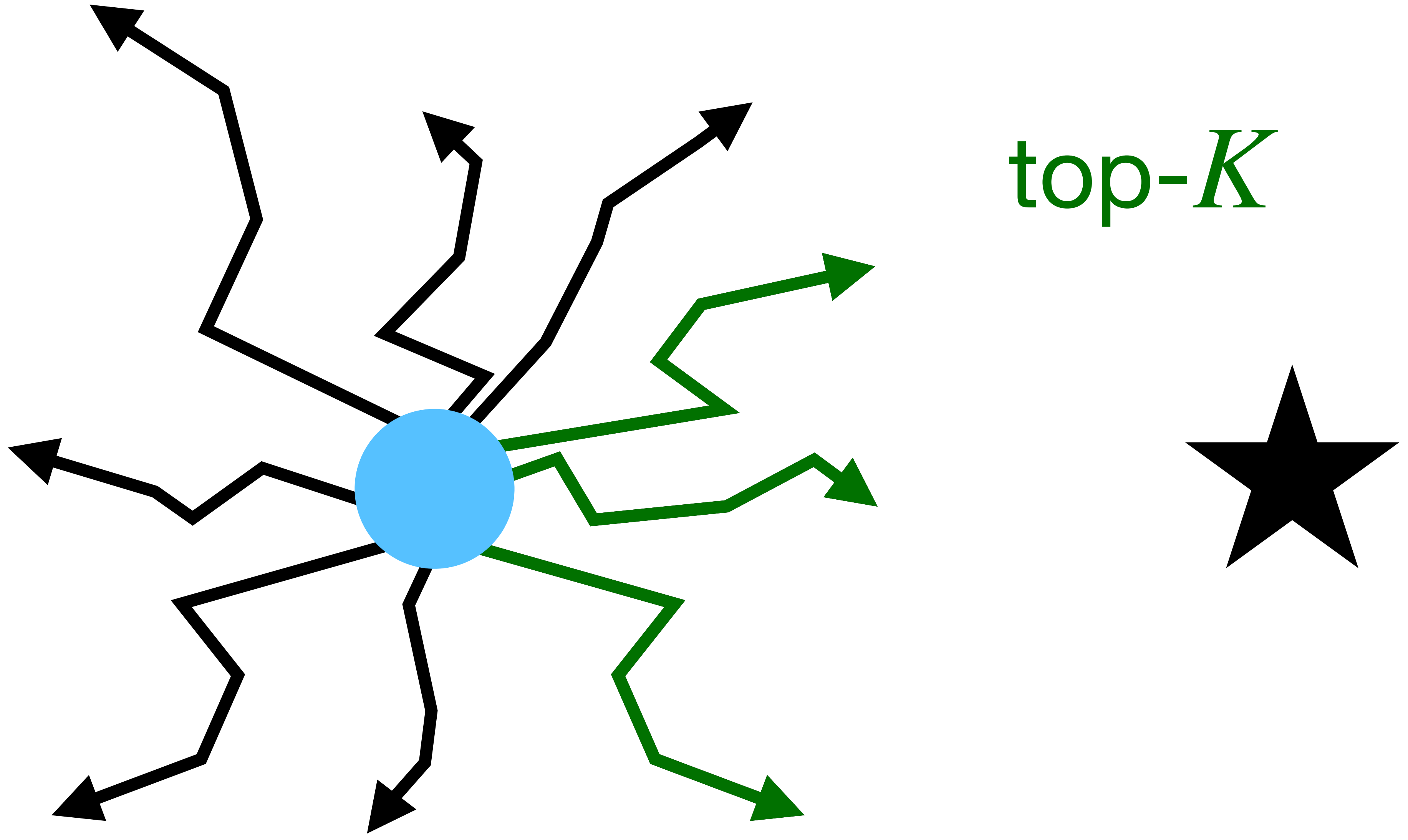
Shooting Methods

Cross-Entropy Method

Iteration 1



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For each iteration

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Evaluate objective $J(a_{0:H}^i) = \sum_{t=0}^{H} \gamma^t r(s_t, a_t^i)$ for each sample

Select top K performing samples, i.e. highest value $J(a_{0:H}^i)$

Update parameters $\{\mu_t, \sigma_t^2\}_{t=0}^{H}$ of action dist. using top K samples

Shooting Methods

Cross-Entropy Method

Iteration 2

Initialise action sequence sampling distribution $\{a_t \sim \mathcal{N}(\mu_t, \sigma_t^2)\}_{t=0}^{H}$

For each iteration

- **Sample** N action sequences $\{a_{0:H}^i\}_{i=1}^N$ from sampling distribution
- **Evaluate** objective $J(a_{0:H}^i) = \sum_{t=0}^{H} \gamma^t r(s_t, a_t^i)$ for each sample
- **Select** top K performing samples, i.e. highest value $J(a_{0:H}^i)$
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Shooting Methods

Cross-Entropy Method

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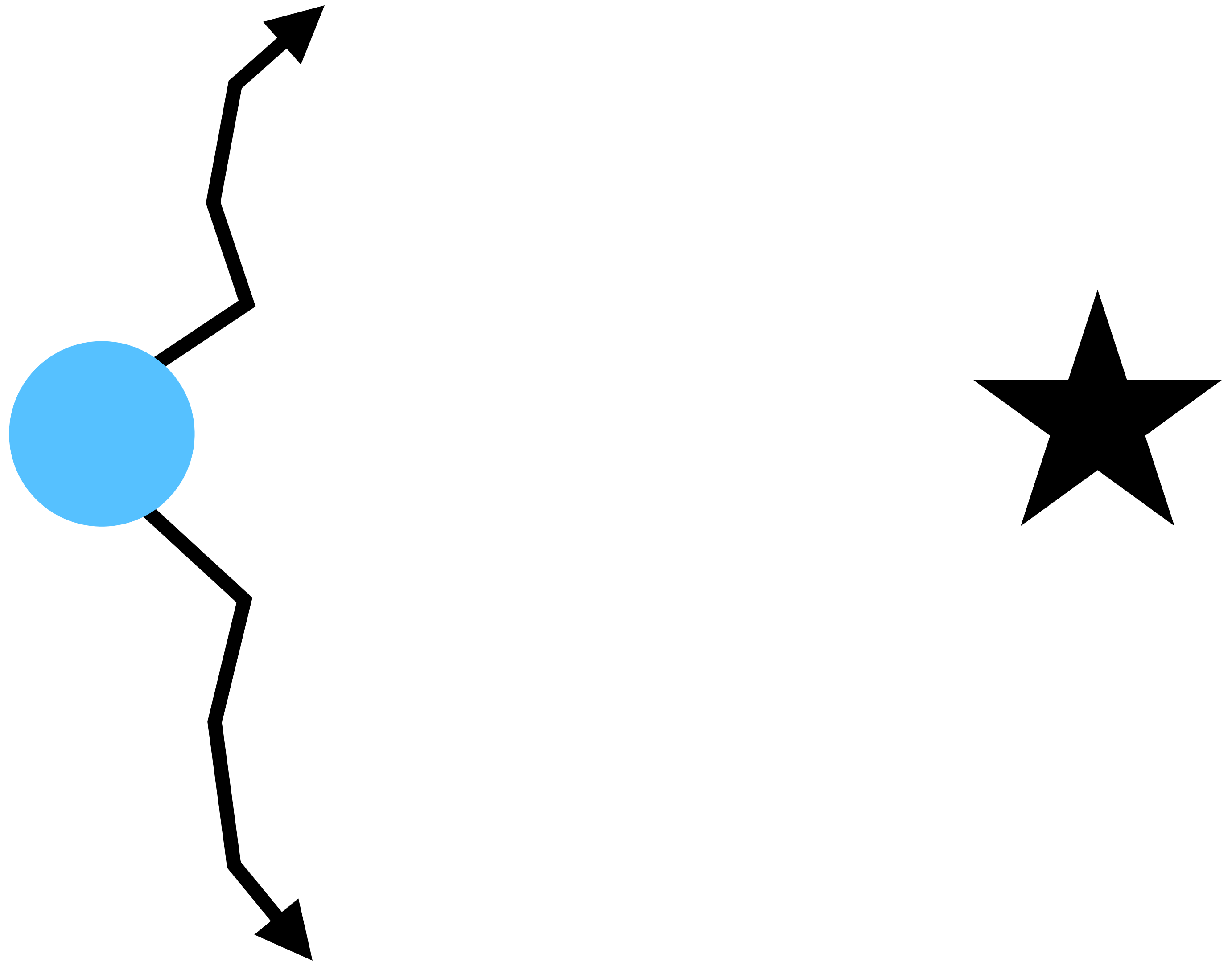
Shooting Methods

Cross-Entropy Method

Iteration 2



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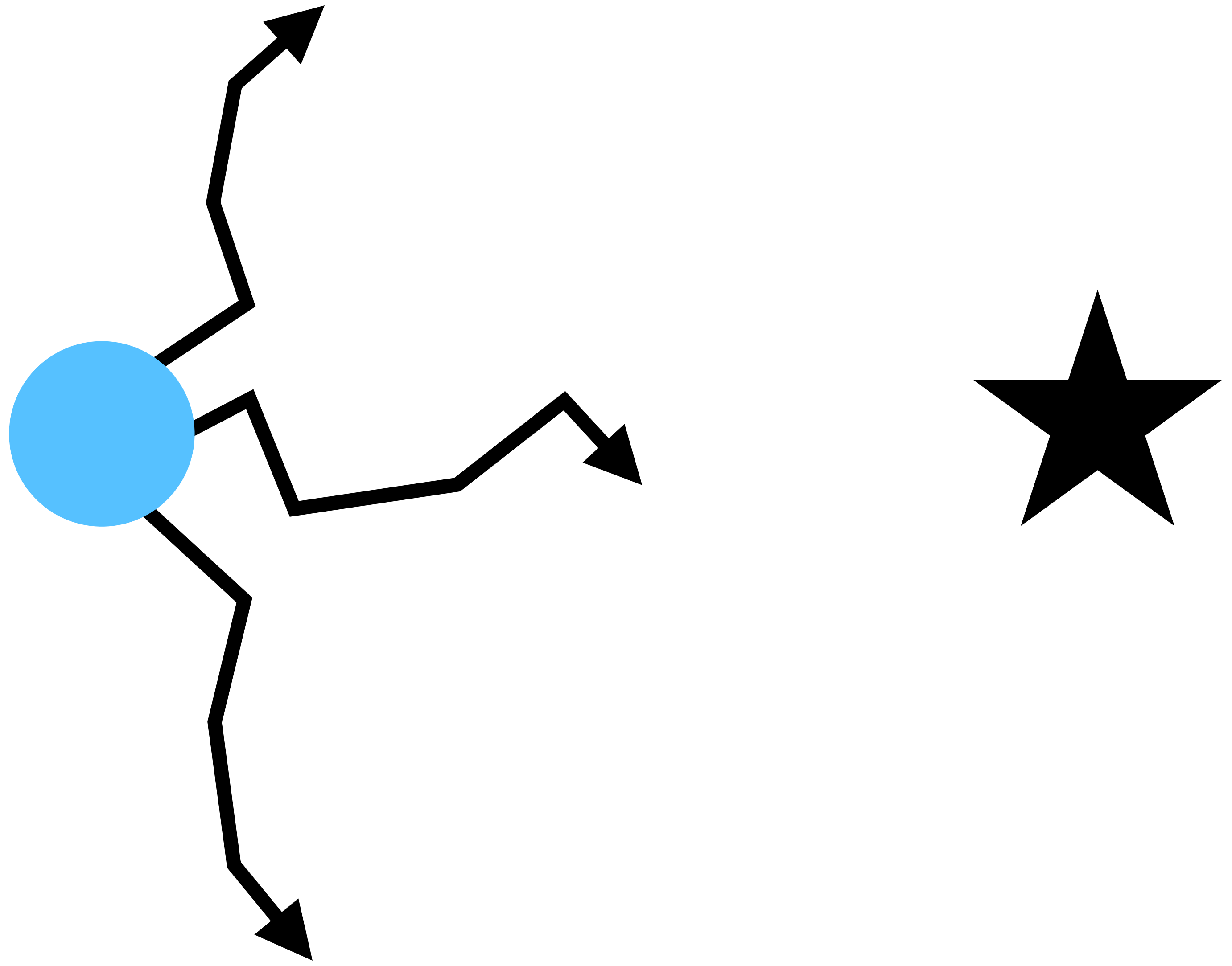
Shooting Methods

Cross-Entropy Method

Iteration 2



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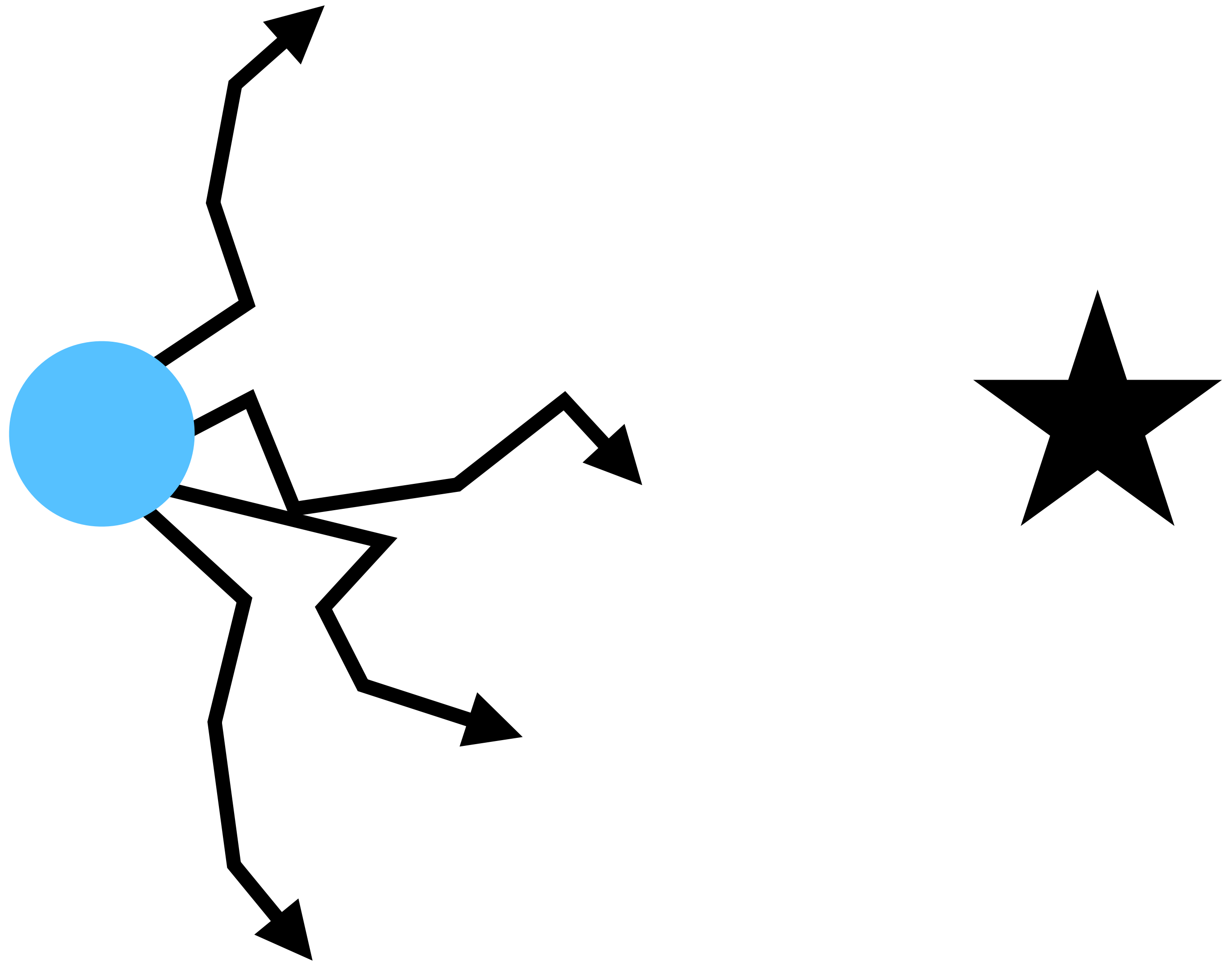
Shooting Methods

Cross-Entropy Method

Iteration 2



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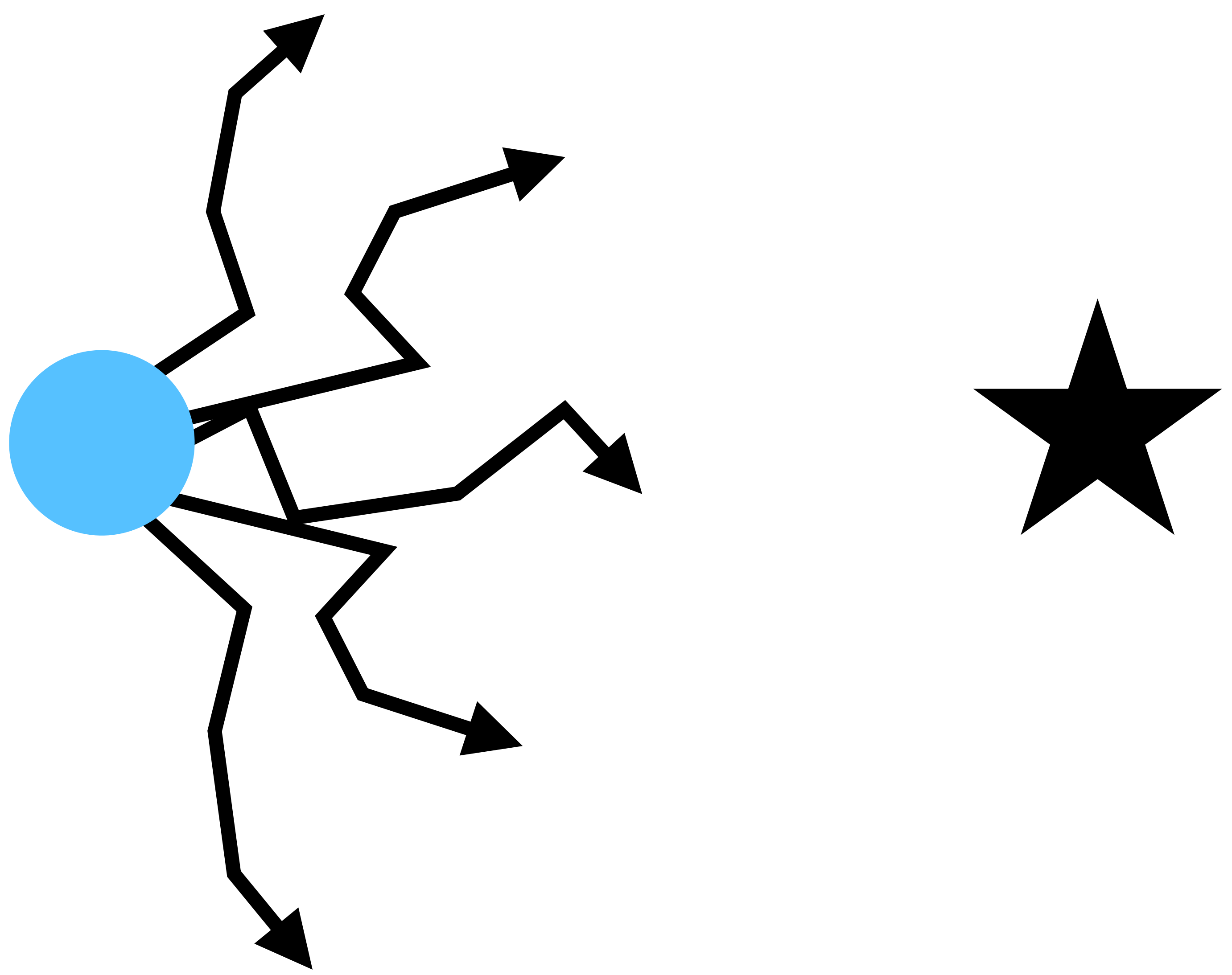
Cross-Entropy Method

Iteration 2



Initialise action sequence sampling distribution $\{a_t \sim \mathcal{N}(\mu_t, \sigma_t^2)\}_{t=0}^{H}$

For each iteration

- **Sample** N action sequences $\{a_{0:H}^i\}_{i=1}^{N}$ from sampling distribution
- **Evaluate** objective $J(a_{0:H}^i) = \sum_{t=0}^{H} \gamma^t r(s_t, a_t^i)$ for each sample
- **Select** top K performing samples, i.e. highest value $J(a_{0:H}^i)$
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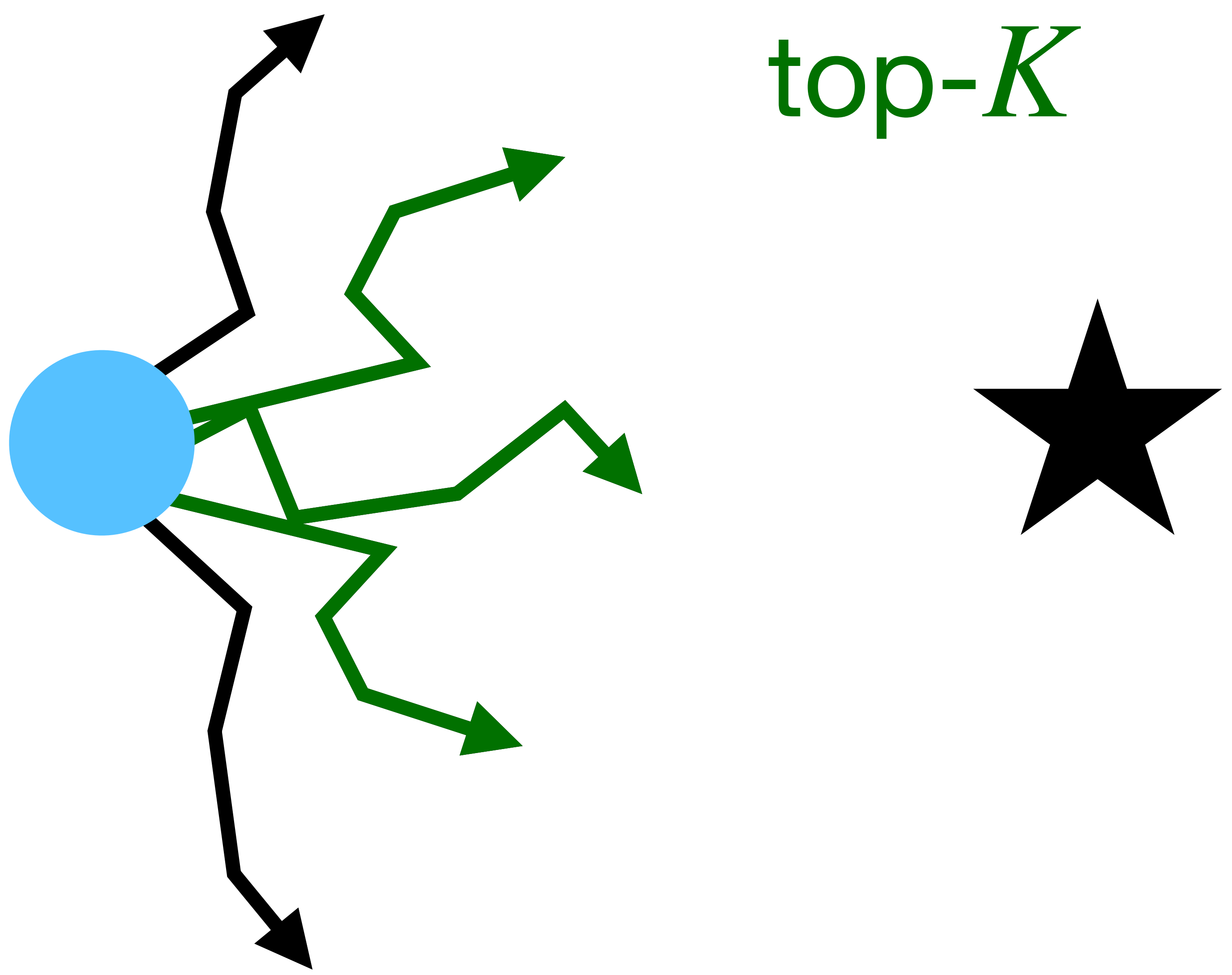
Shooting Methods

Cross-Entropy Method

Iteration 2



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Shooting Methods

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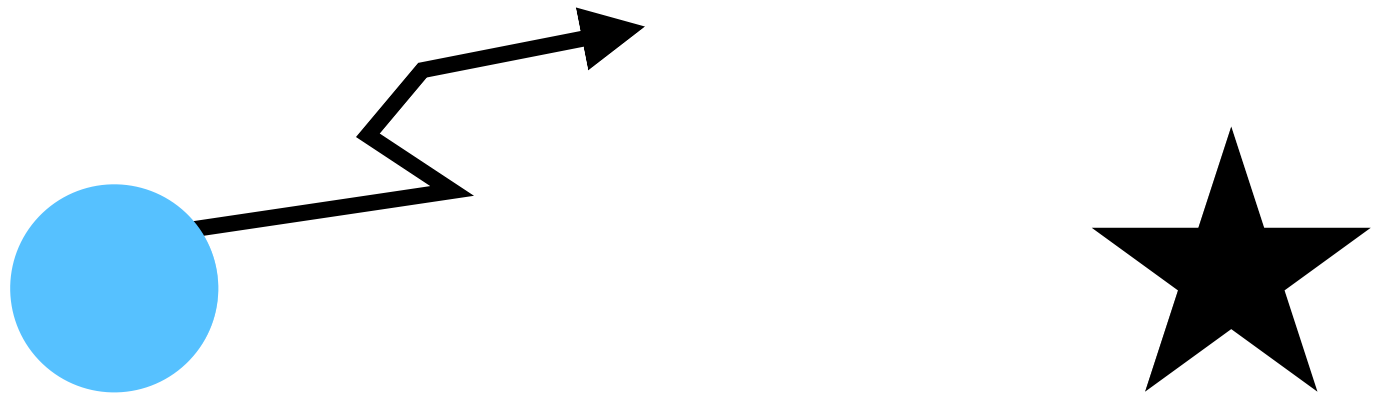

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Shooting Methods

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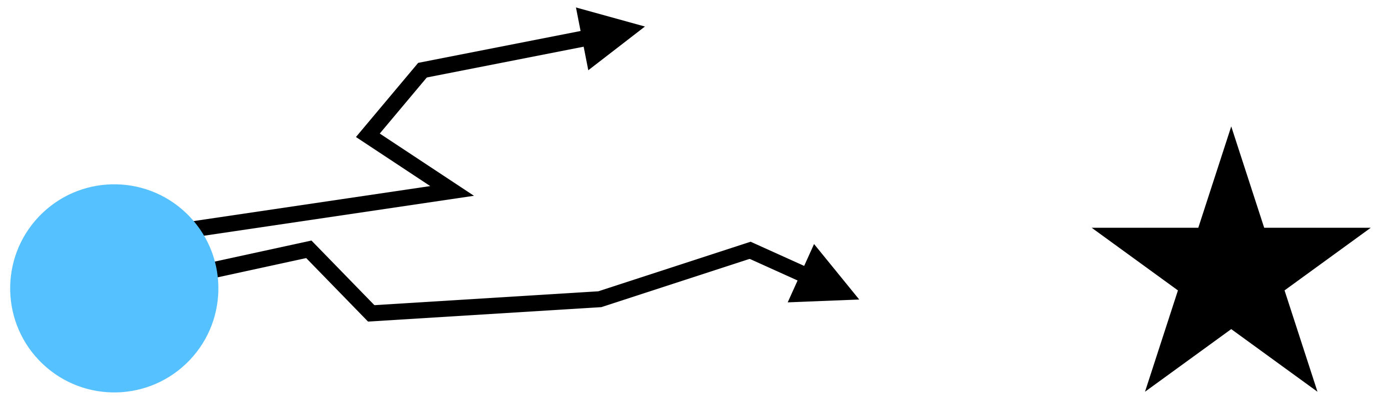

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Shooting Methods

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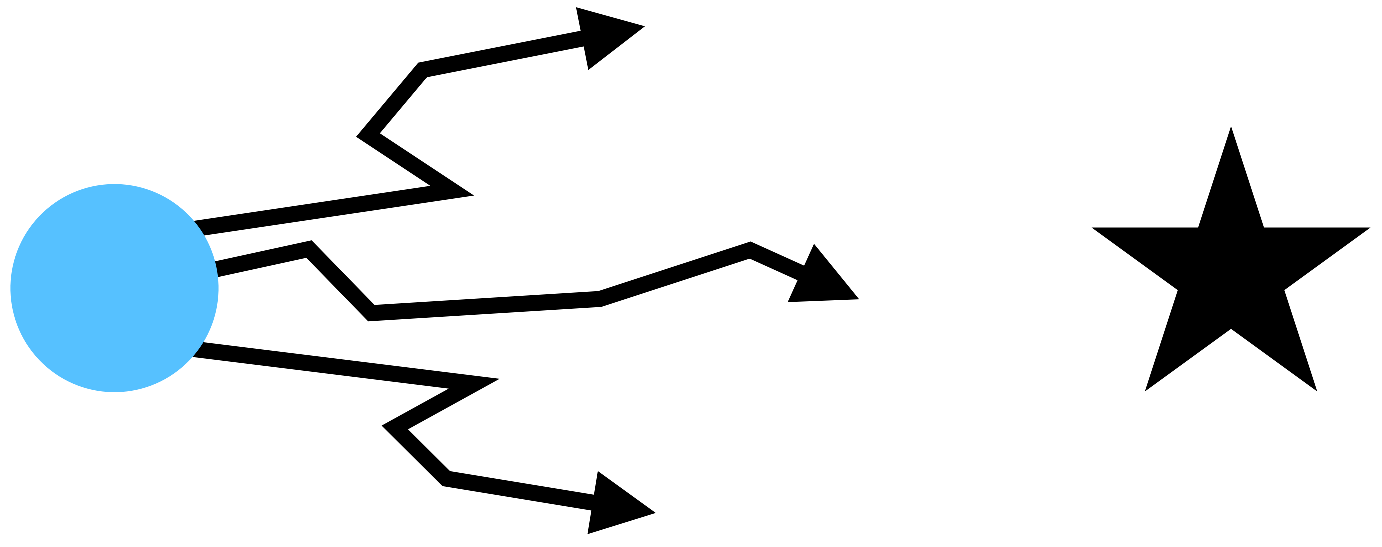

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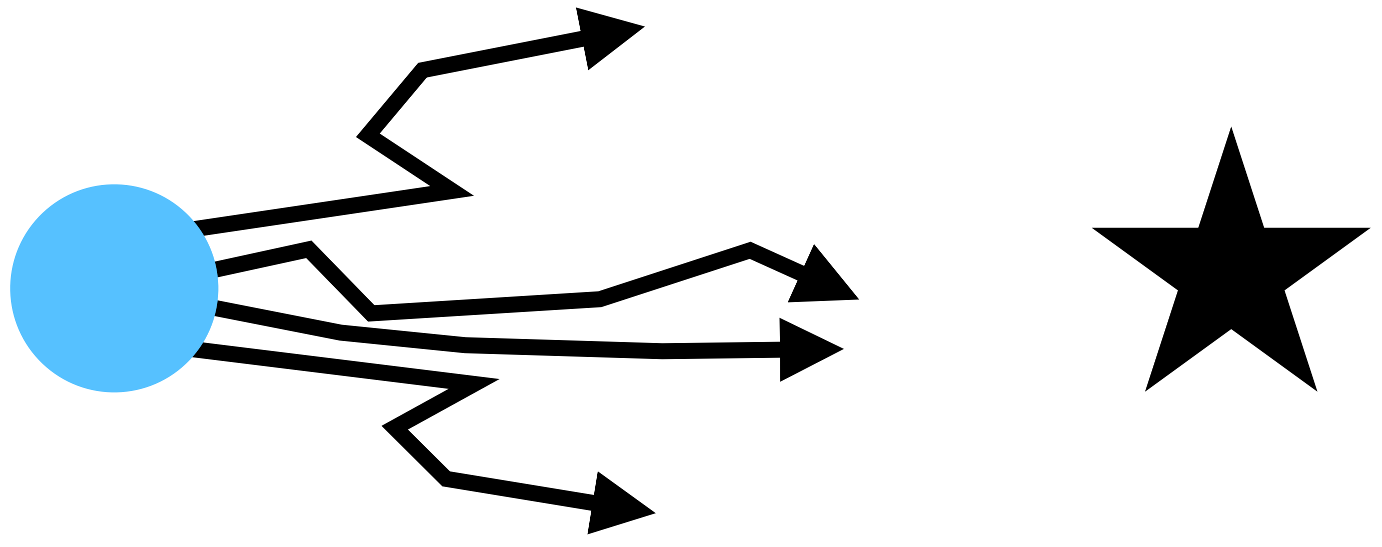

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Shooting Methods

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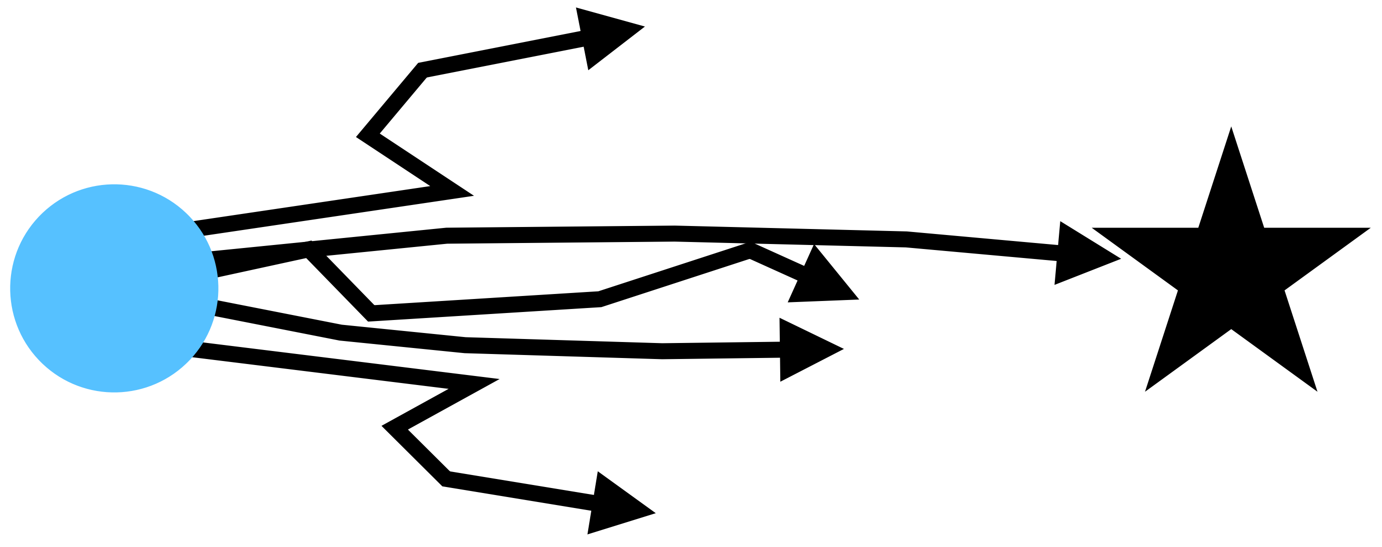

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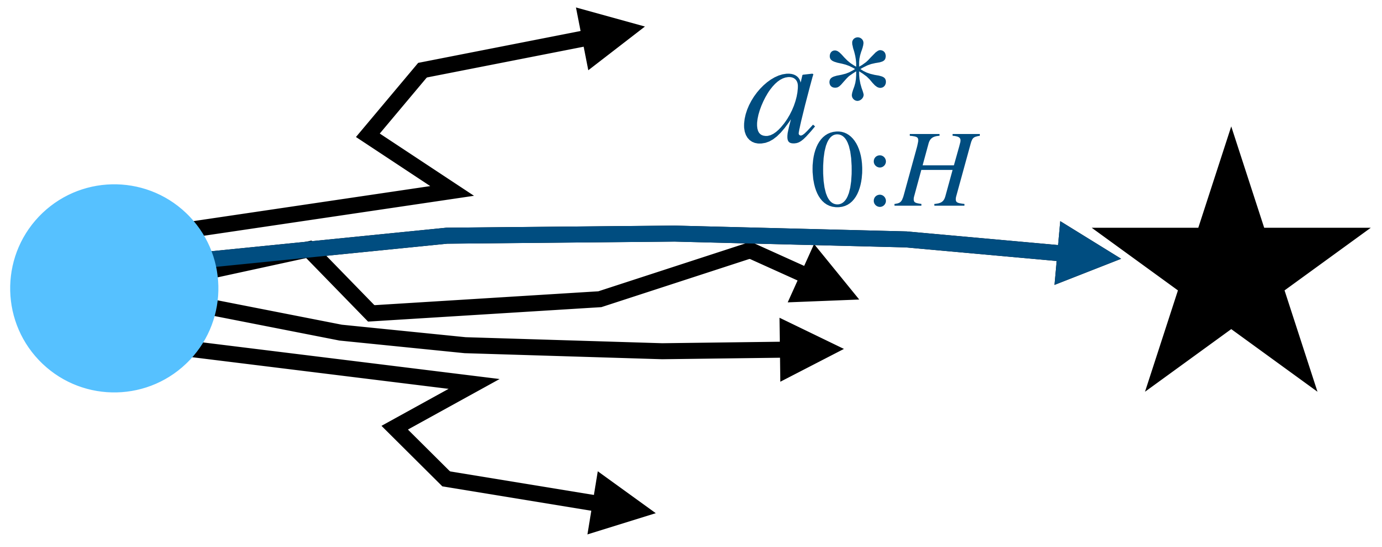

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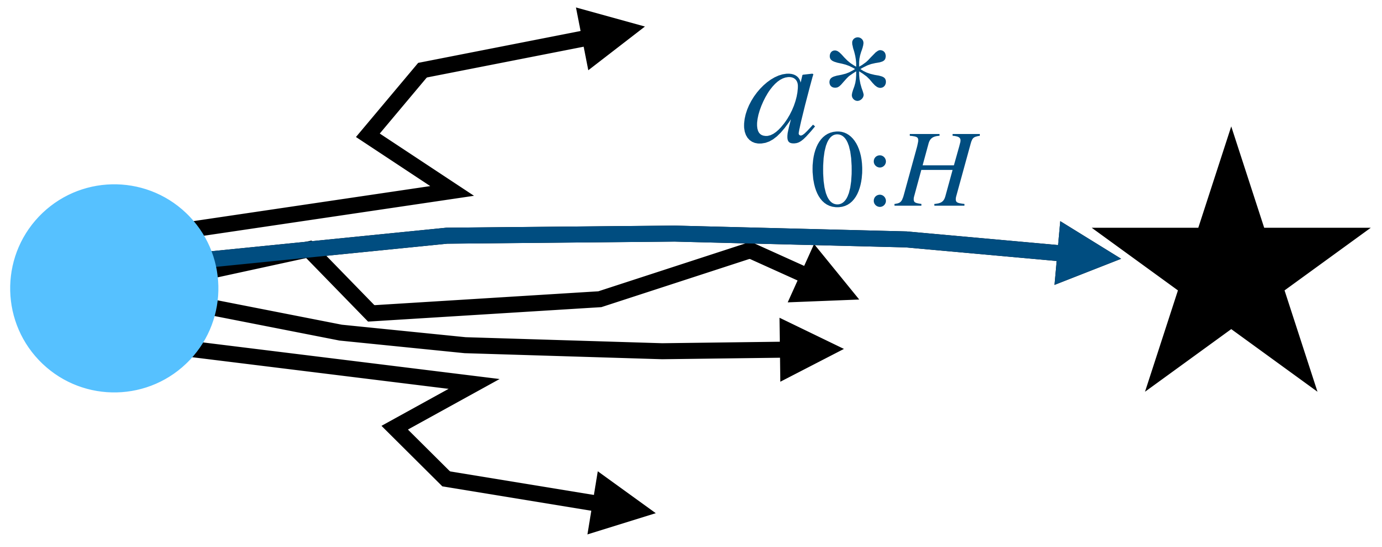
More sample efficient

Shooting Methods

Cross-Entropy Method

Iteration 3



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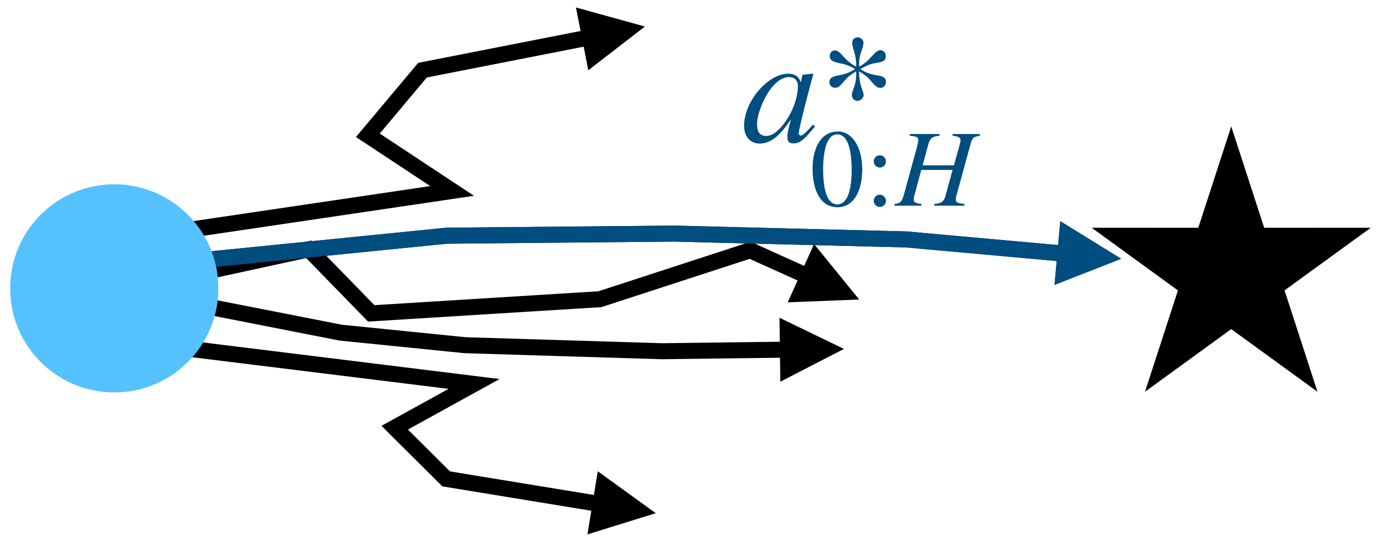
Shooting Methods

Cross-Entropy Method

More sample efficient

Faster convergence

Iteration 3



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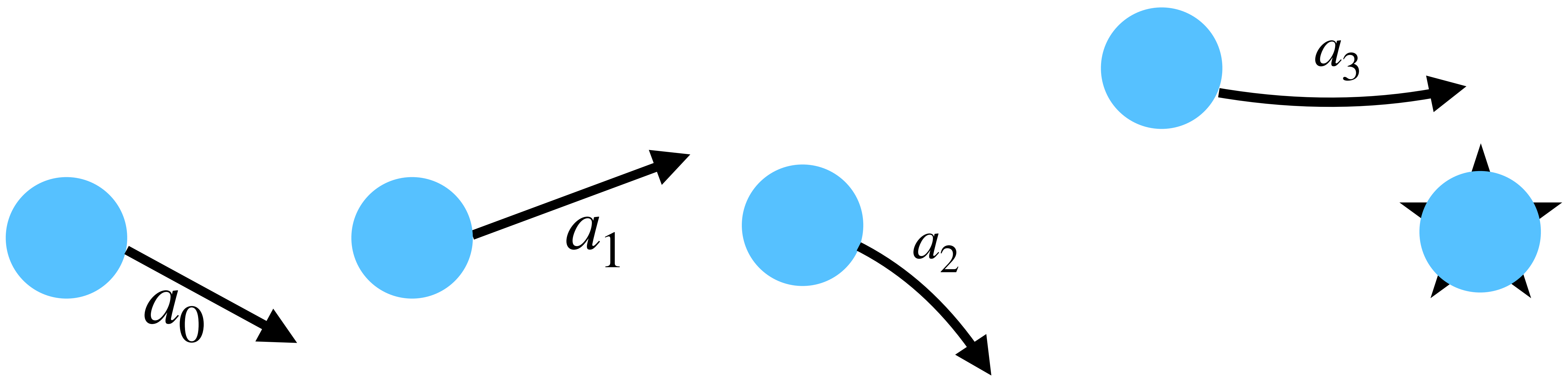
Collocation methods

Illustration

$$J(a_{0:H}, s_{0:H}) = \sum_{t=0}^{H} \gamma^t r(s_t, a_t) \quad \text{s.t. } \|s_{t+1} - f(s_t, a_t)\| = 0$$



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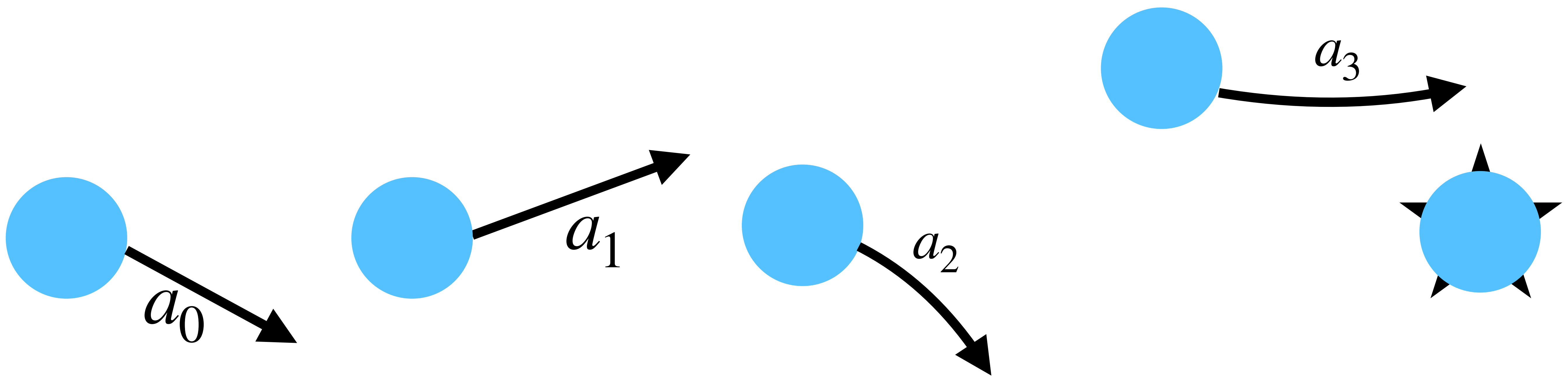
Optimising states and actions



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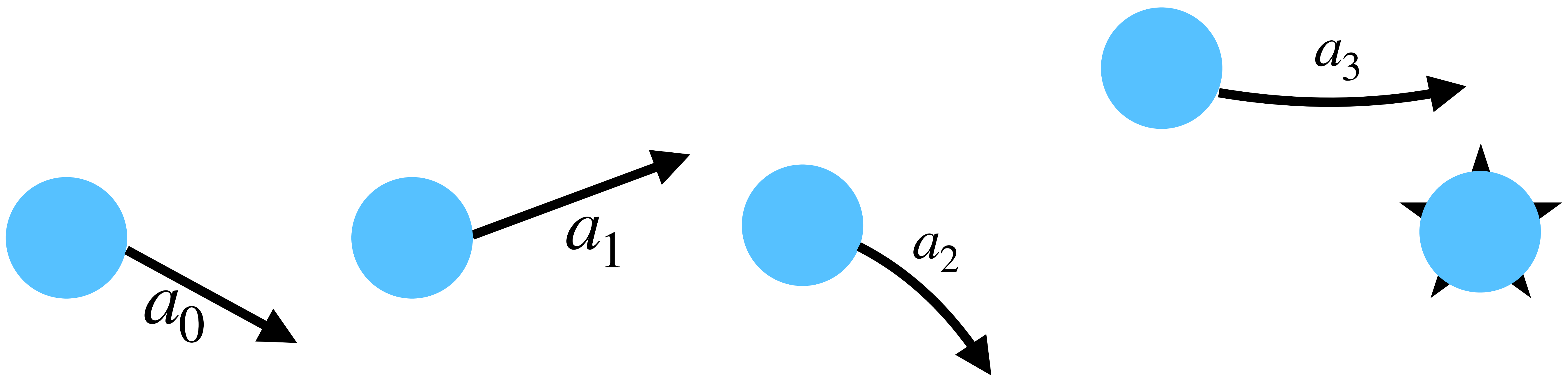

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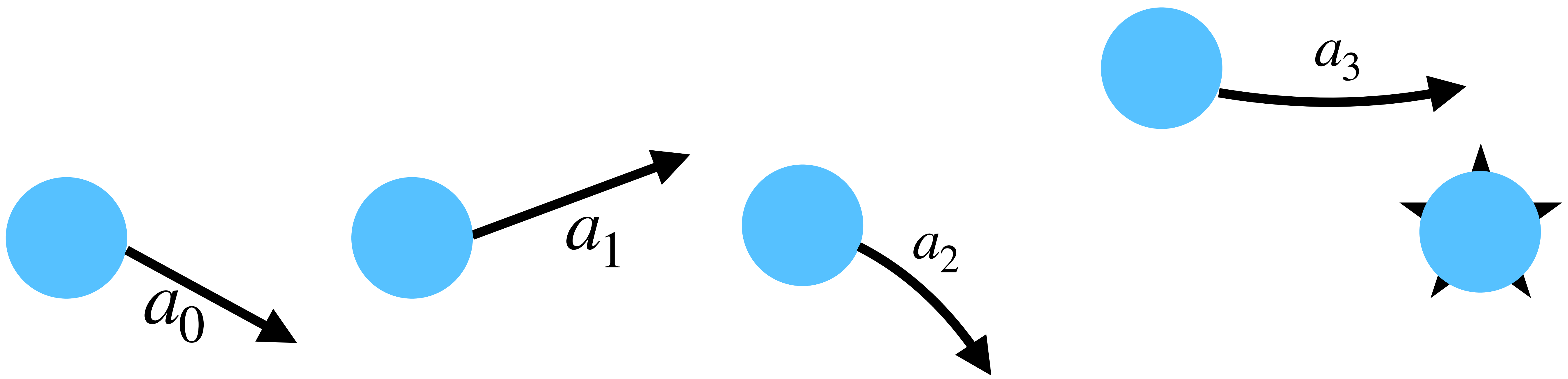
Optimising states and actions

Dynamics constraint



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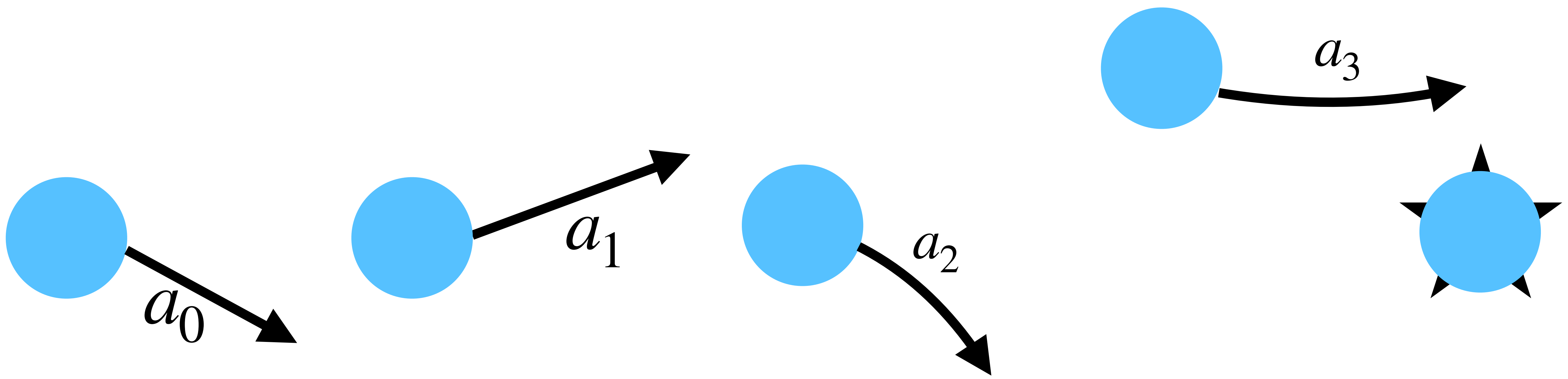
Optimising states and actions

Dynamics constraint

No dynamics rollout



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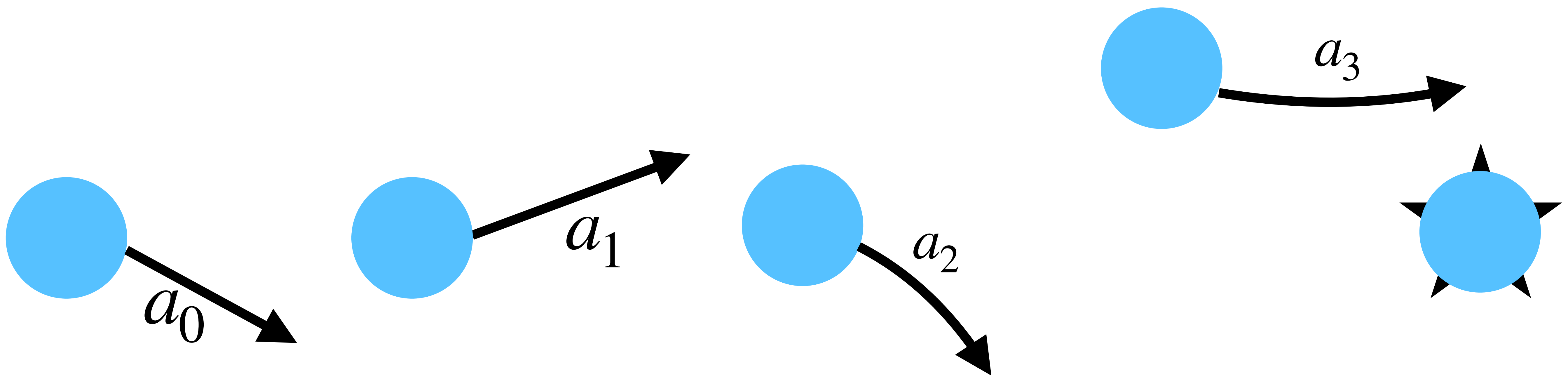

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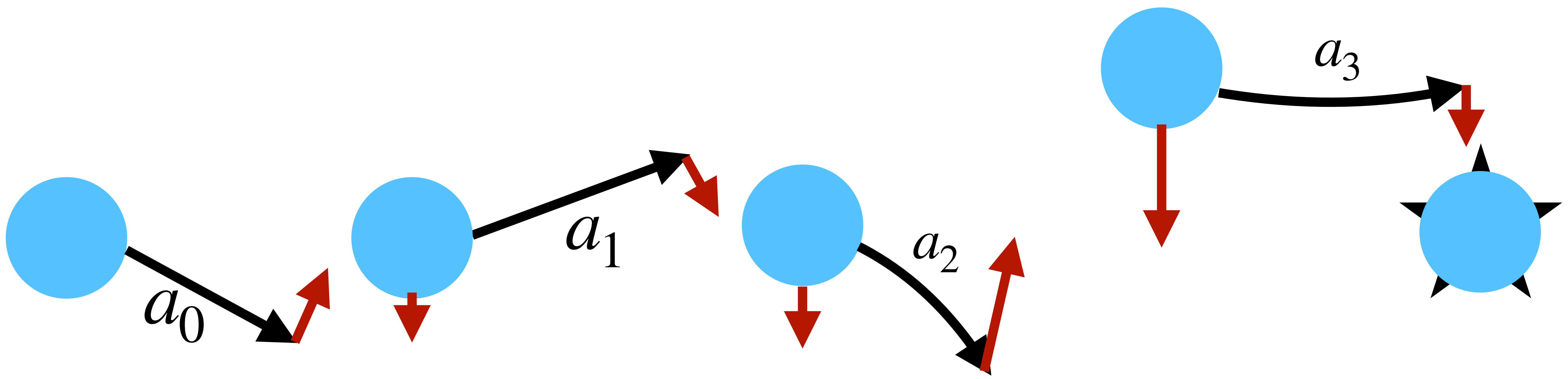

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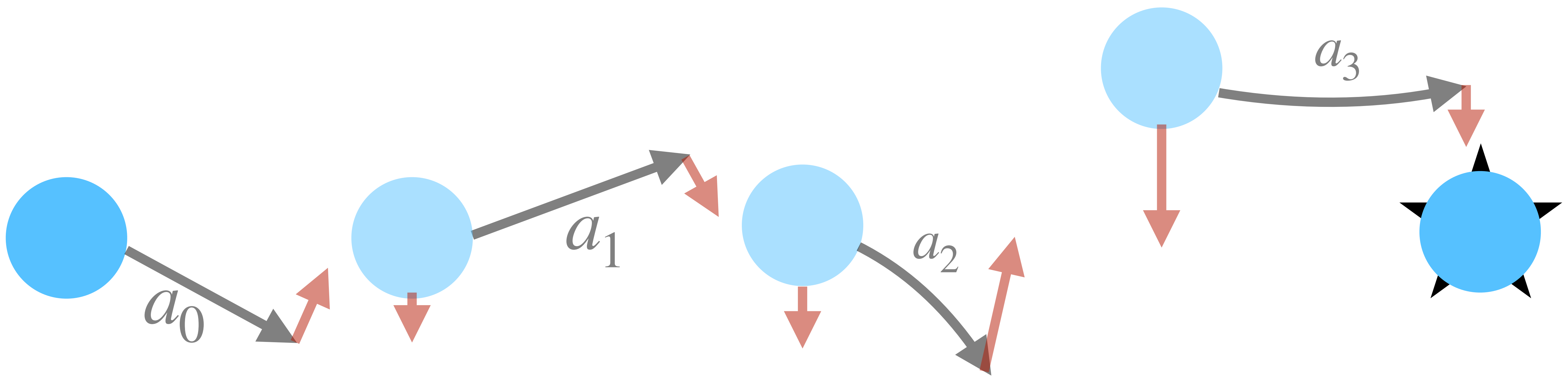

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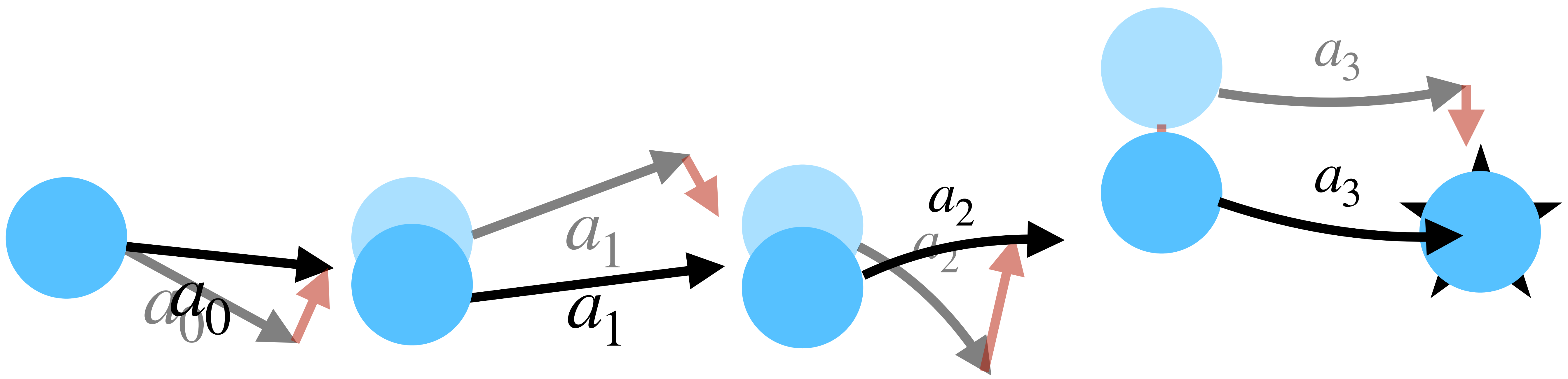

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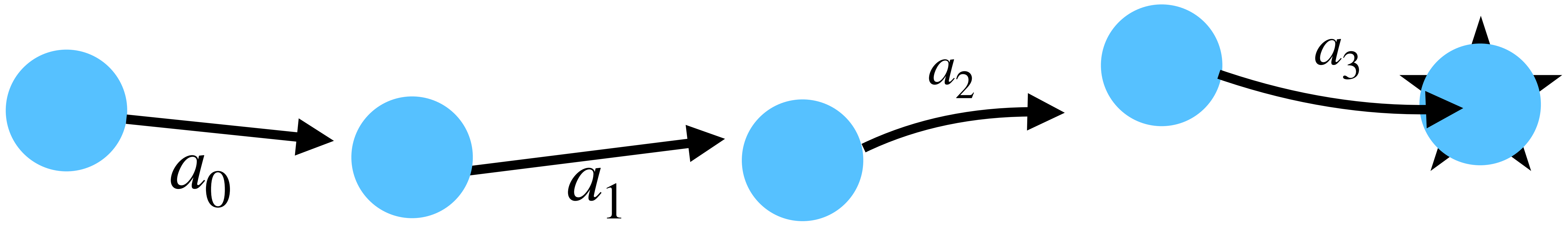

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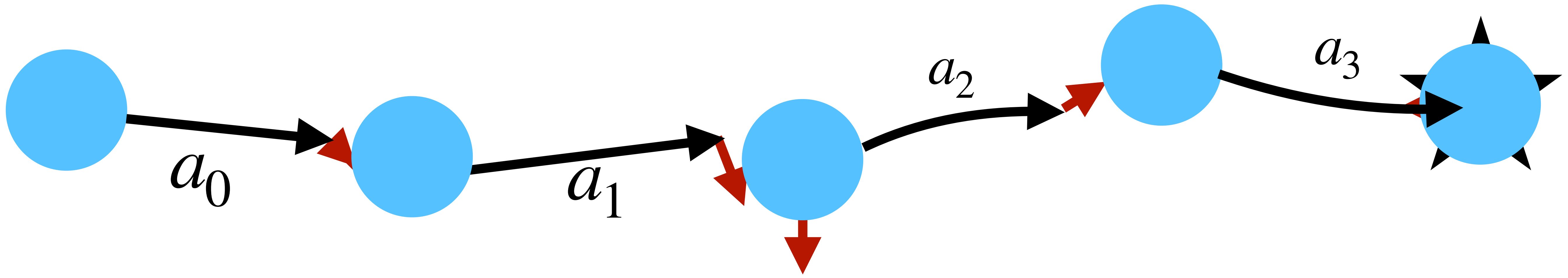

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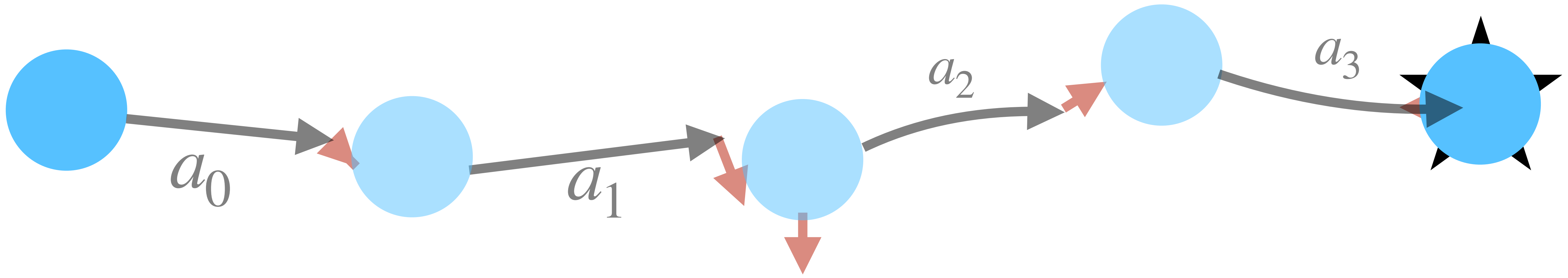

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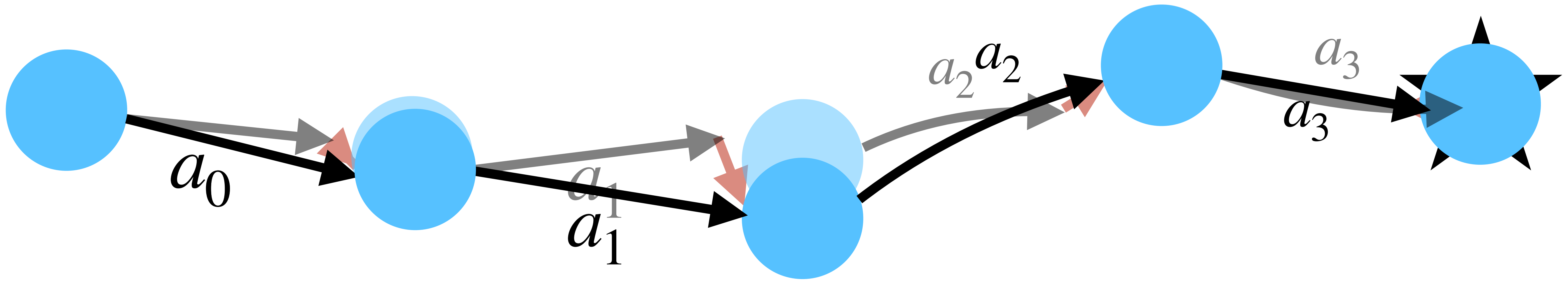

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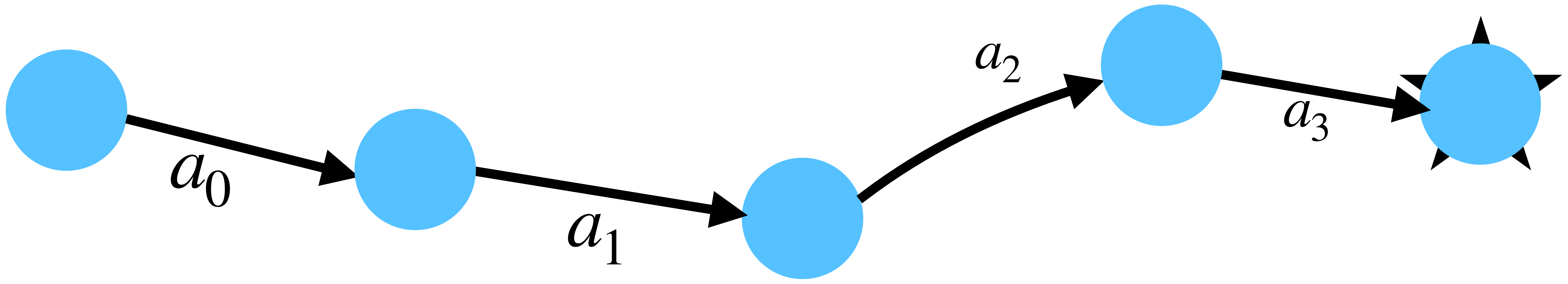

Dynamics constraint satisfied!

Finite Horizon Planning has Limitations

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$$\sum_{t=0}^{H-1} \gamma^t r(s_t, a_t)$$

Finite Horizon Planning has Limitations

$$\sum_{t=0}^{H-1} \gamma^t r(s_t, a_t) + \gamma^H Q_\theta(s_H, a_H)$$

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Approximate infinite horizon return using learned Q-function

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Finite Horizon Planning has Limitations

$$\sum_{t=0}^{\infty} \gamma^t Q(s_t, a_t) \approx \sum_{t=0}^{H-1} \gamma^t r(s_t, a_t) + \boxed{\gamma^H Q_\theta(s_H, a_H)}$$

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Approximate infinite horizon return using learned Q-function

Learned Q-function is common in model-free RL

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Approximate infinite horizon return using learned Q-function

Learned Q-function is common in model-free RL

Best of both worlds!

Trajectory optimisation methods are open loop.

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We can do better.

Decision-time Planning

Model Predictive Control (MPC)

Decision-time Planning

Model Predictive Control (MPC)

For each environment step

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Decision-time Planning

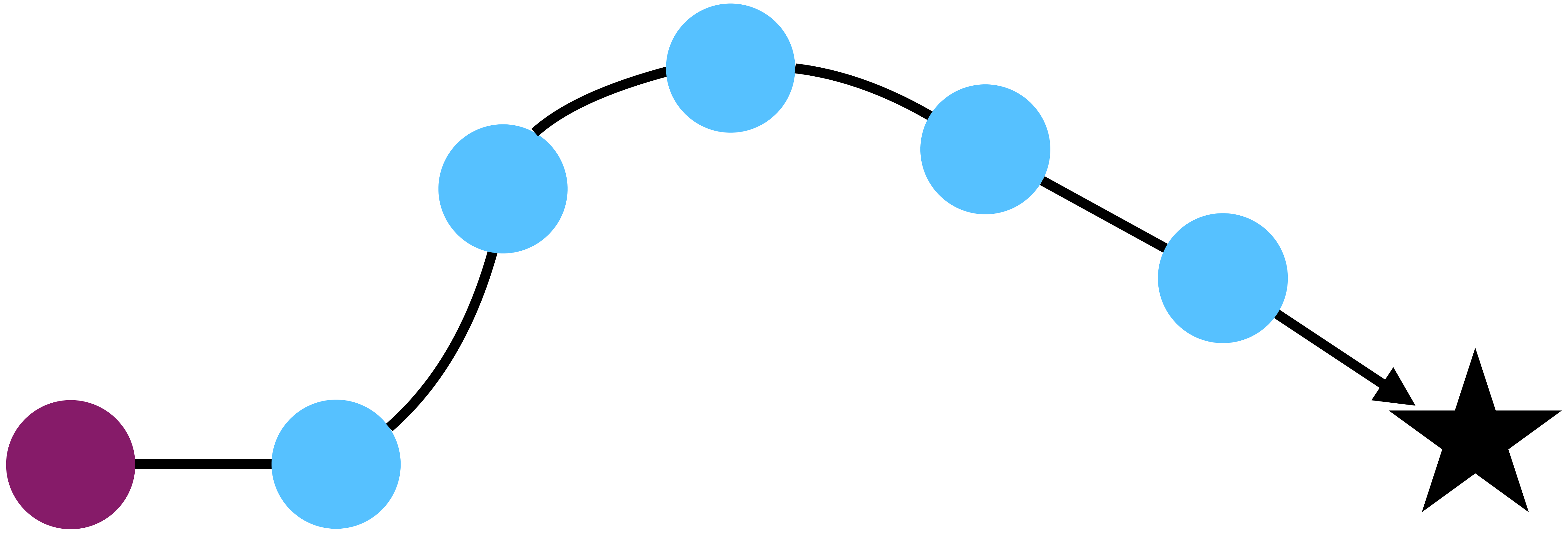
Model Predictive Control (MPC)

For each environment step

Observe state s

Decision-time Planning

Model Predictive Control (MPC)



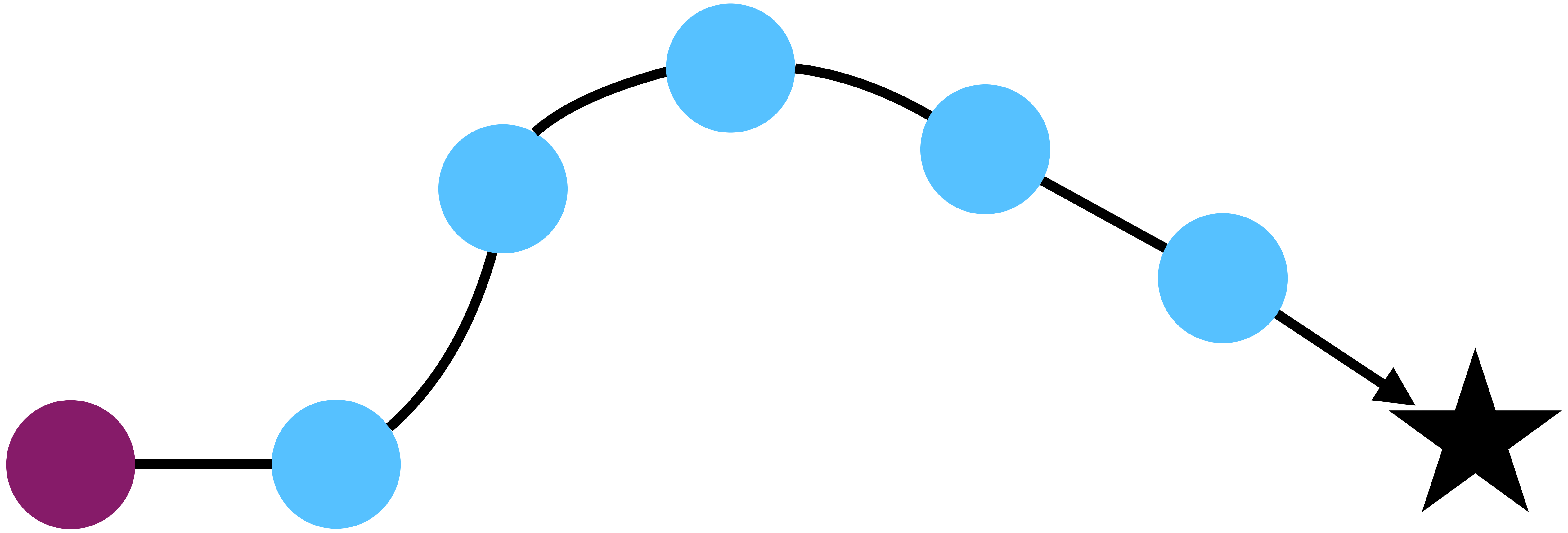
For each environment step

Observe state s

Plan $a_{0:H}$ to maximise return $\sum_{t=0}^{H-1} \gamma^t r(s_t, a_t) + \gamma^H Q_\theta(s_H, a_H)$

Decision-time Planning

Model Predictive Control (MPC)



For each environment step

Observe state s

Any trajectory optimisation method

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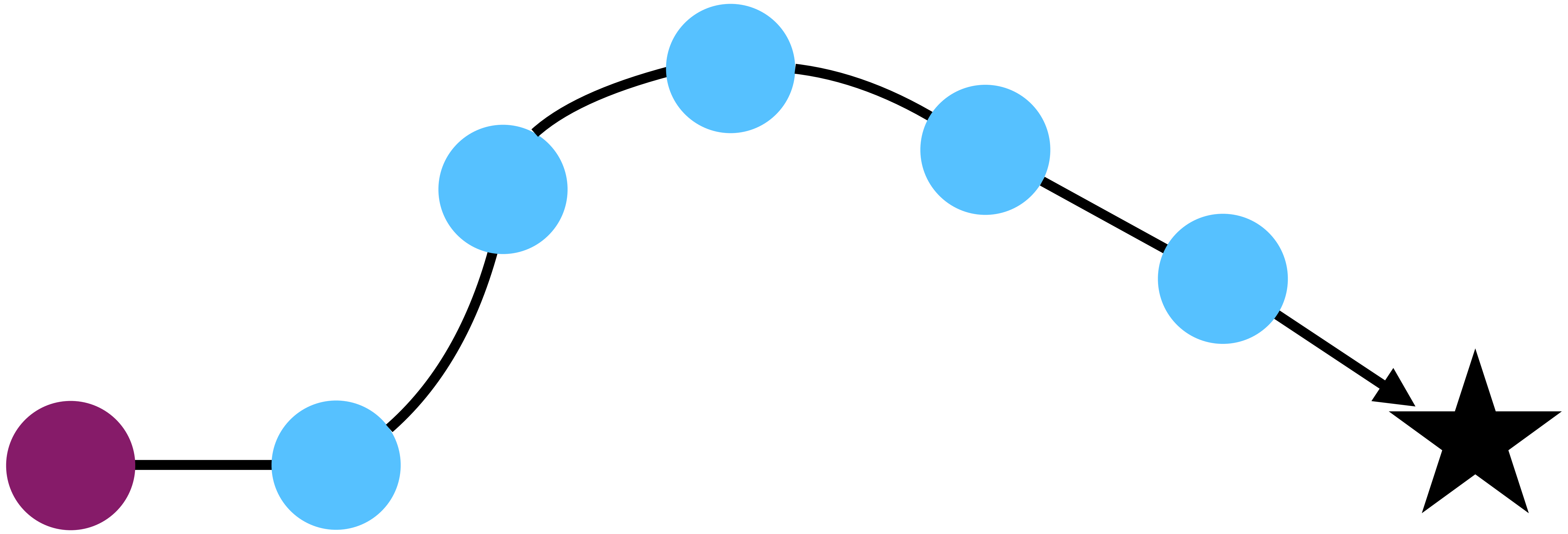

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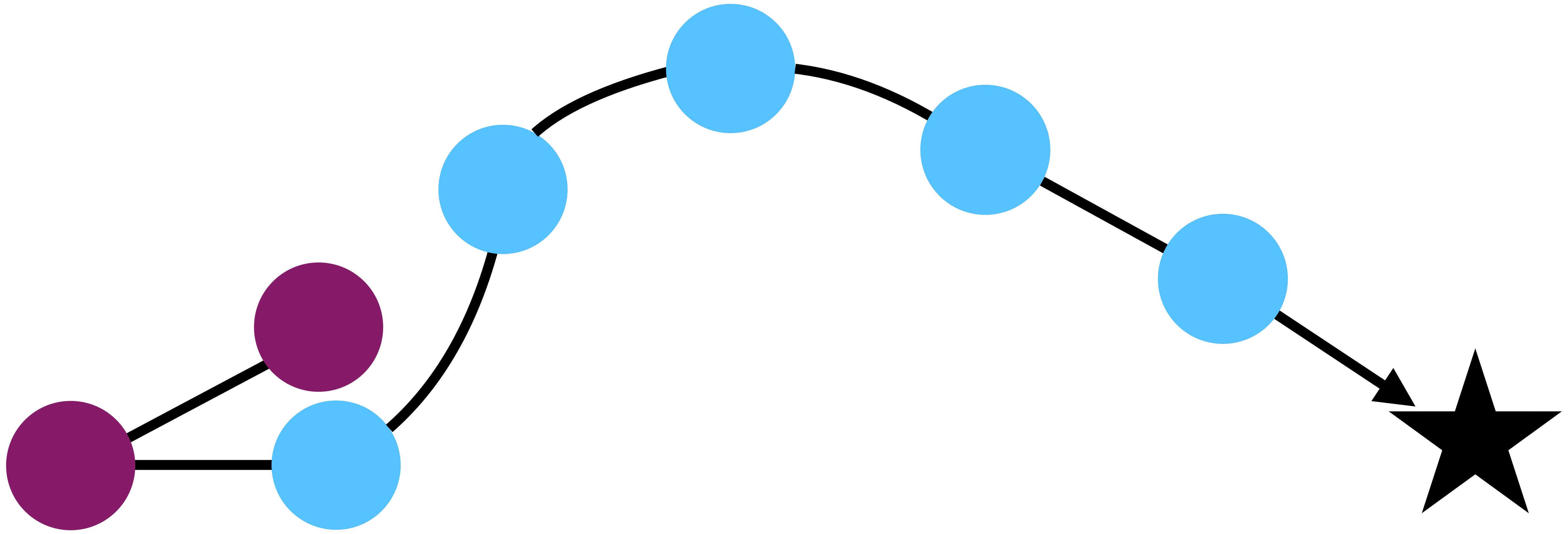
Execute a_0 and discard $a_1, \ldots, a_H$

Decision-time Planning

Model Predictive Control (MPC)



For each environment step

Observe state s

Any trajectory optimisation method

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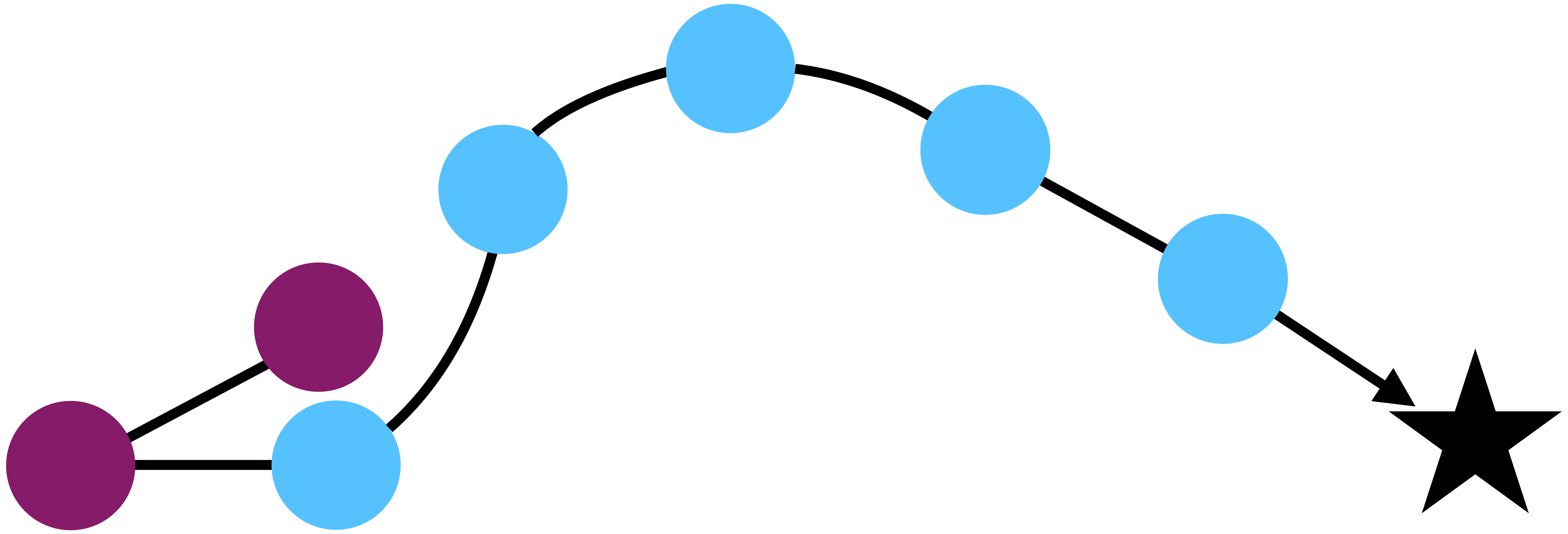


Diverged from planned trajectory…

For each environment step

Observe state s

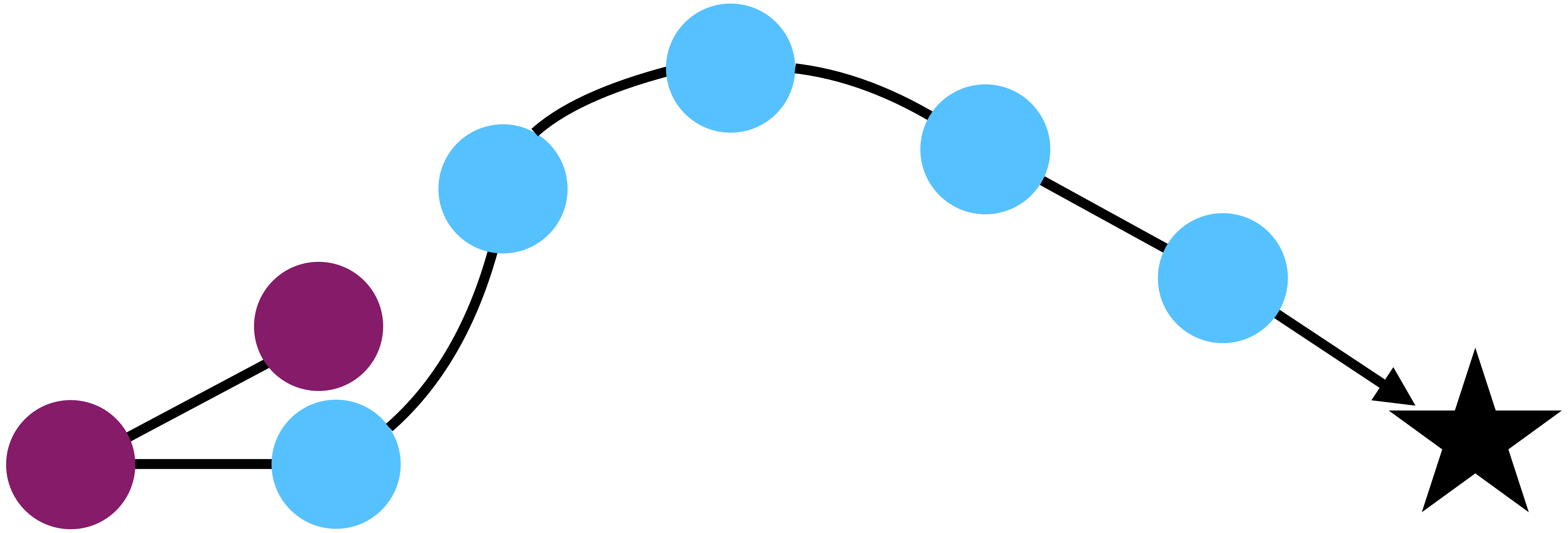
Any trajectory optimisation method

Plan $\boxed{a_{0:H}}$ to maximise return $\sum_{t=0}^{H-1} \gamma^t r(s_t, a_t) + \gamma^H Q_\theta(s_H, a_H)$

Execute a_0 and discard $a_1, \ldots, a_H$

Decision-time Planning

Model Predictive Control (MPC)



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For each environment step

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Any trajectory optimisation method

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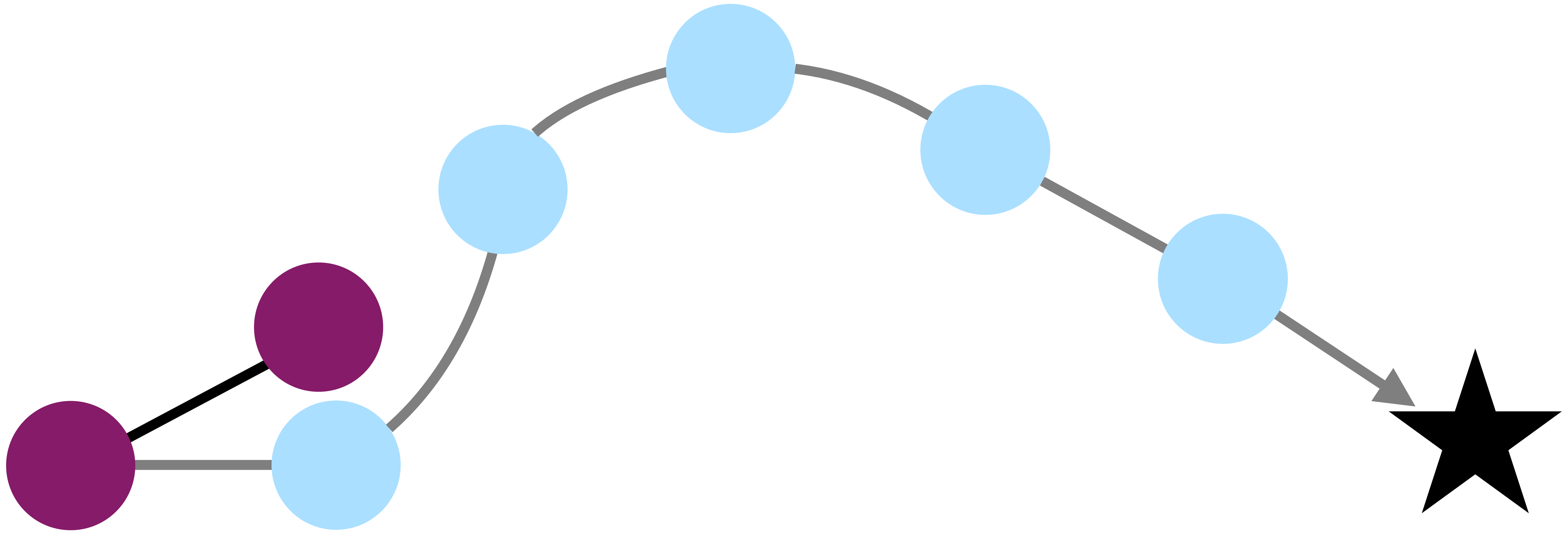

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For each environment step

Observe state s

Any trajectory optimisation method

Plan $\boxed{a_{0:H}}$ to maximise return $\sum_{t=0}^{H-1} \gamma^t r(s_t, a_t) + \gamma^H Q_\theta(s_H, a_H)$

Execute a_0 and discard $a_1, \ldots, a_H$

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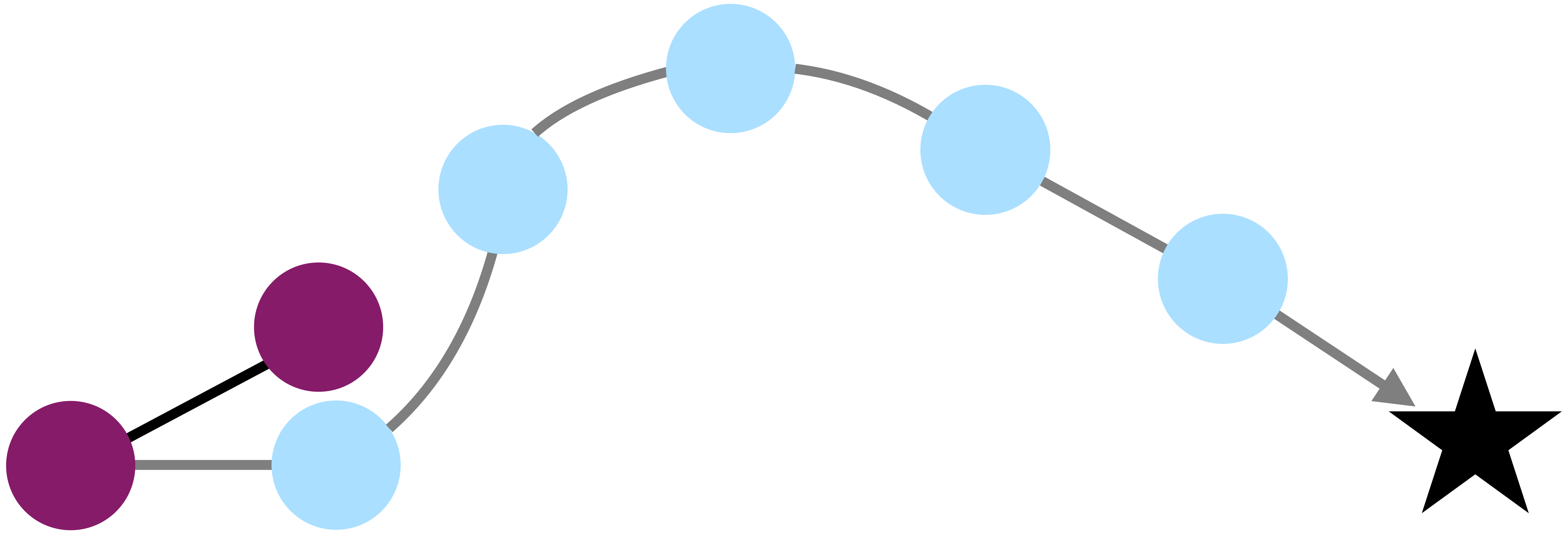
Decision-time Planning

Model Predictive Control (MPC)



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So let's replan.

For each environment step

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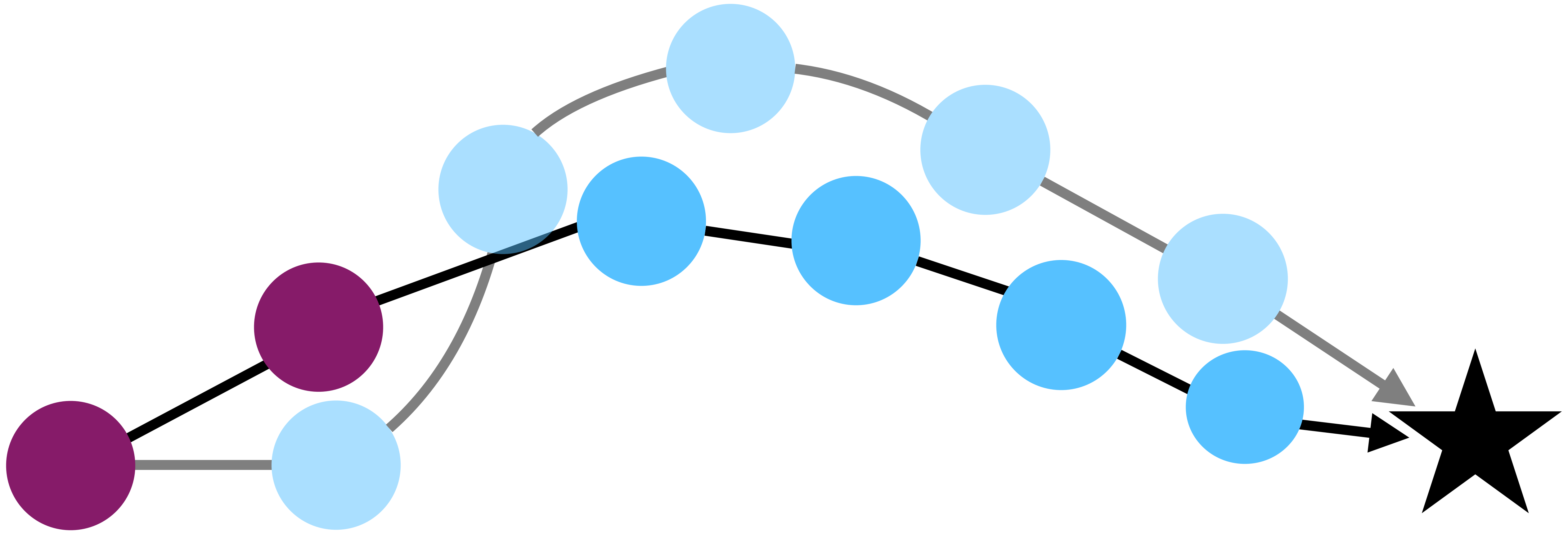

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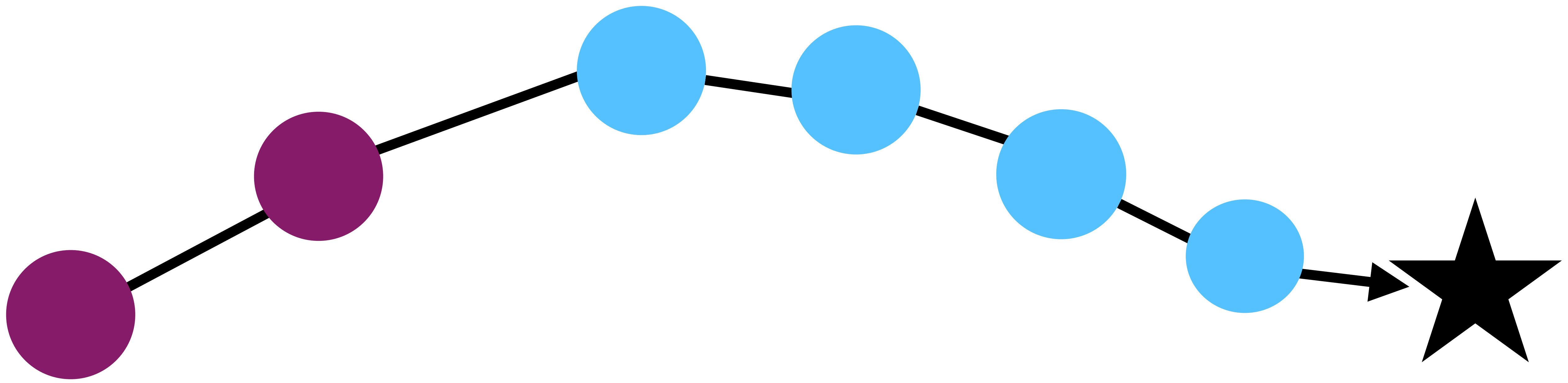

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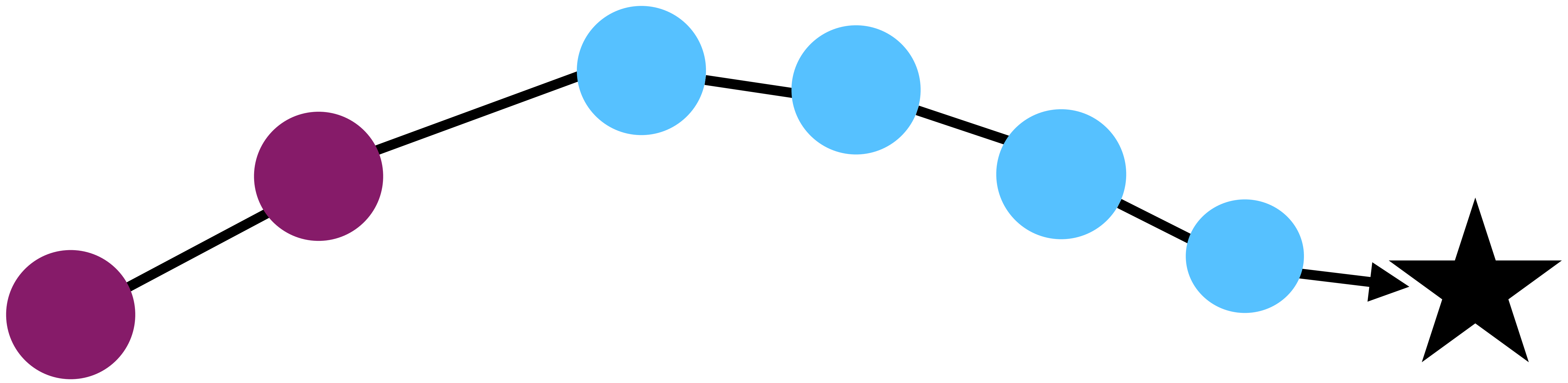

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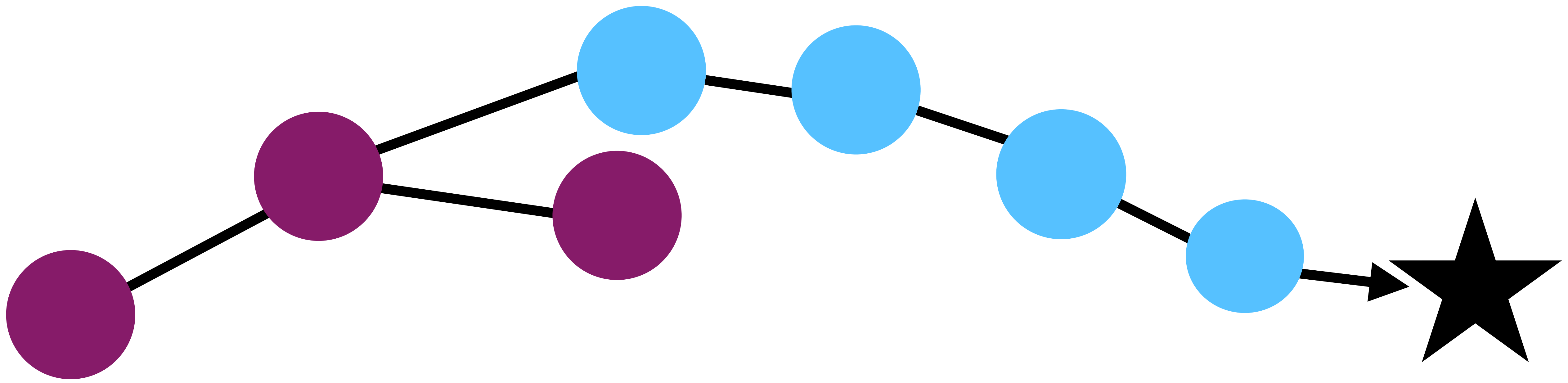

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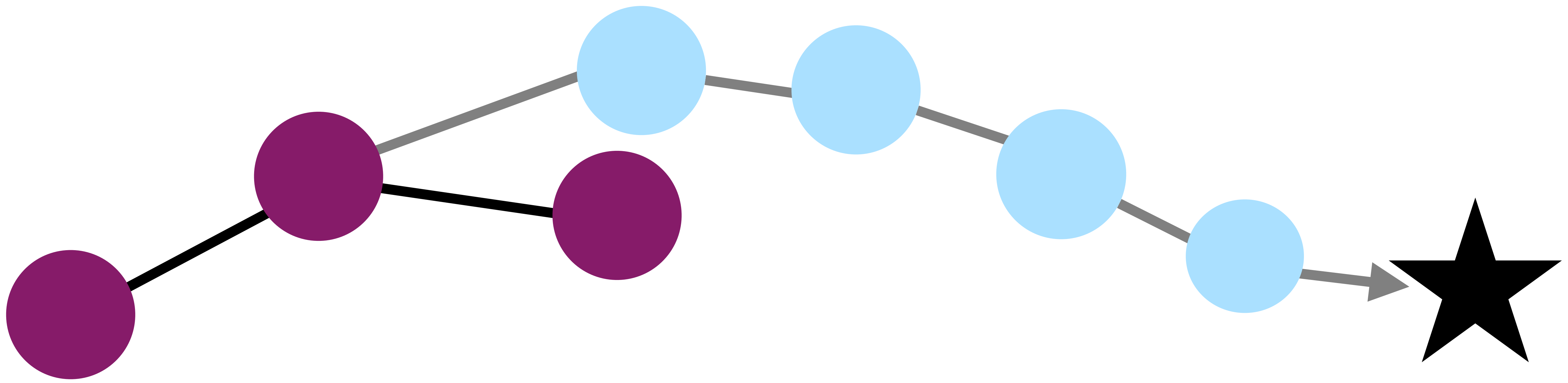

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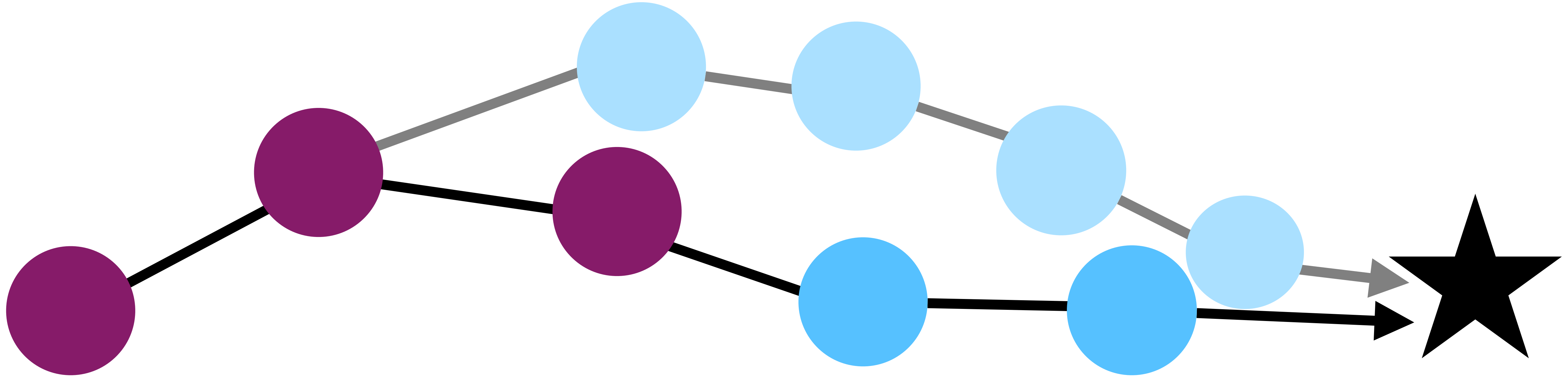

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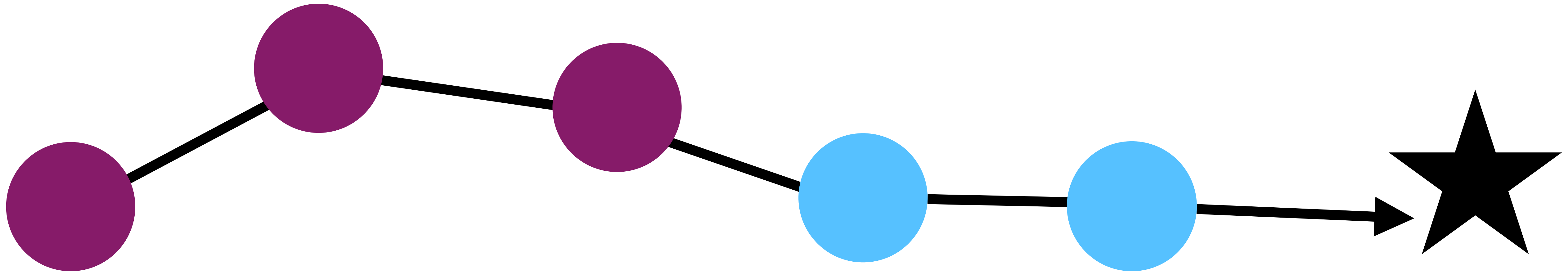

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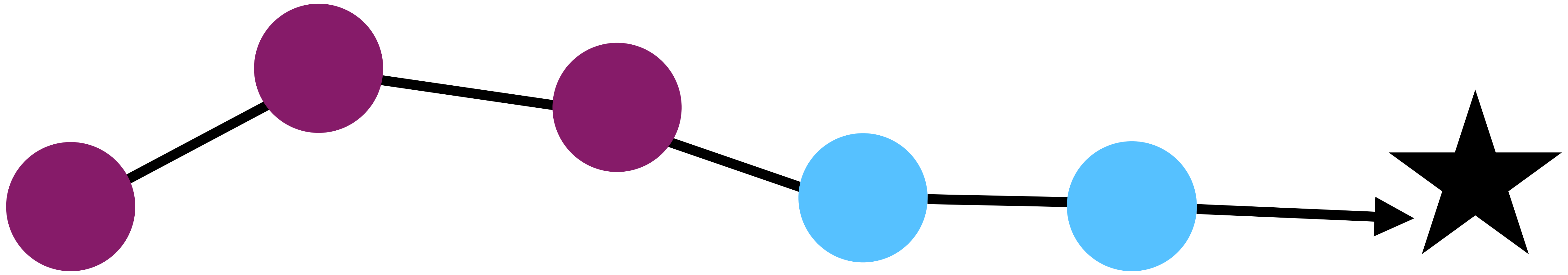

And so on…

For each environment step

Observe state s

Any trajectory optimisation method

Plan $\boxed{a_{0:H}}$ to maximise return $\sum_{t=0}^{H-1} \gamma^t r(s_t, a_t) + \gamma^H Q_\theta(s_H, a_H)$

Execute a_0 and discard $a_1, \ldots, a_H$

Decision-time Planning

Model Predictive Control (MPC)

$$\pi_{\mathrm{MPC}}(s; f, r, Q_\theta) = \arg\max_{a_0} \max_{a_1,\ldots,a_{H-1}} = \sum_{t=0}^{H-1} \gamma^t r(s_t, a_t) + \gamma^H Q_\theta(s_H, a_H) \qquad \text{s.t.} \quad s_{t+1} = f(s_t, a_t)$$

$$s_0 = s$$

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Decision-time Planning

Main Takeaways

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Common to use CEM

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Use MPC to make CEM closed loop

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Common to use CEM

- Avoids local optima
- Can handle deterministic and stochastic dynamics
- Avoids exploding/vanishing gradients

Use MPC to make CEM closed loop

Consider infinite horizon via learned $Q_\theta(s, a)$

Learning Objectives

Understand

1. ~~What a “model” is in model-based RL~~
2. ~~How a “model” can aid decision making~~
3. ~~Differences between background and decision-time planning~~
4. ~~Decision-time planning strategies for continuous actions~~
5. Sources of uncertainty in model-based RL
6. Rationale and insights for decision-making under uncertainty

Sources of Uncertainty in Model-Based RL

Sources of Uncertainty

Learning From Limited Data

$$s_{t+1} = f_{env}(s_t, a_t)$$

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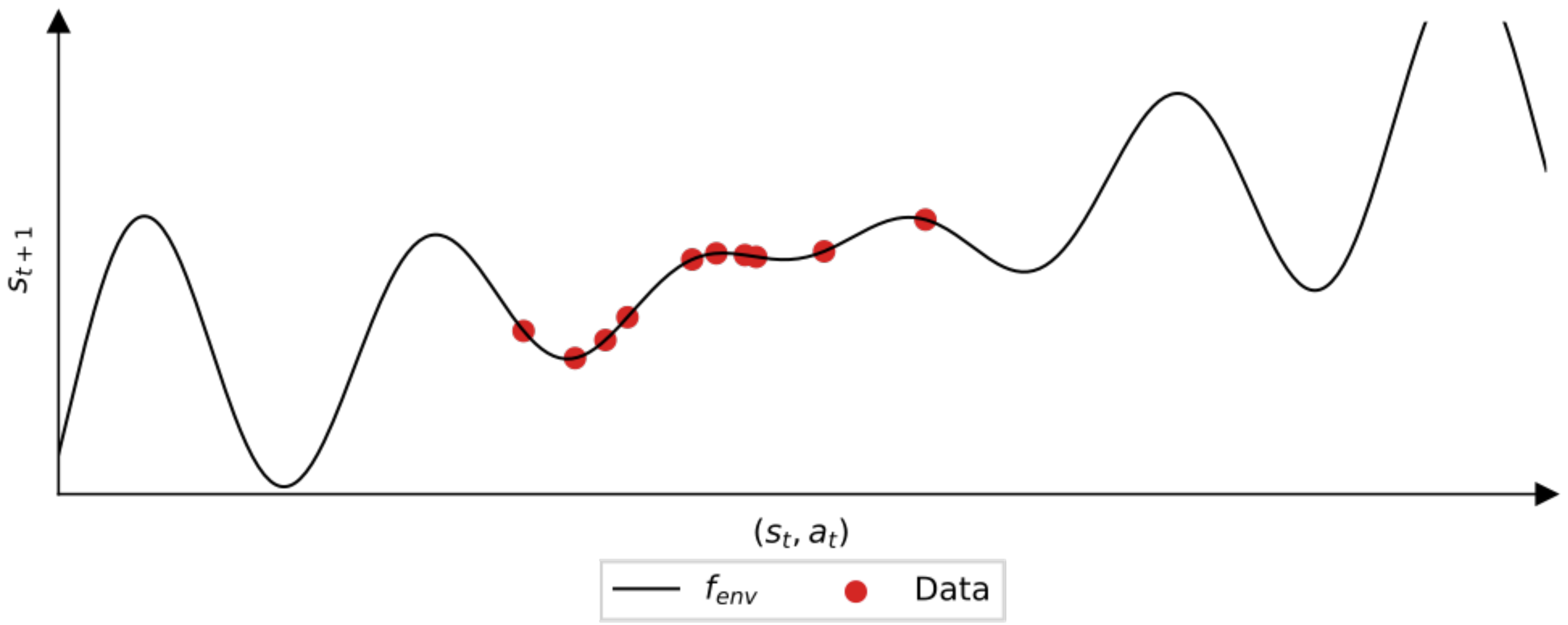

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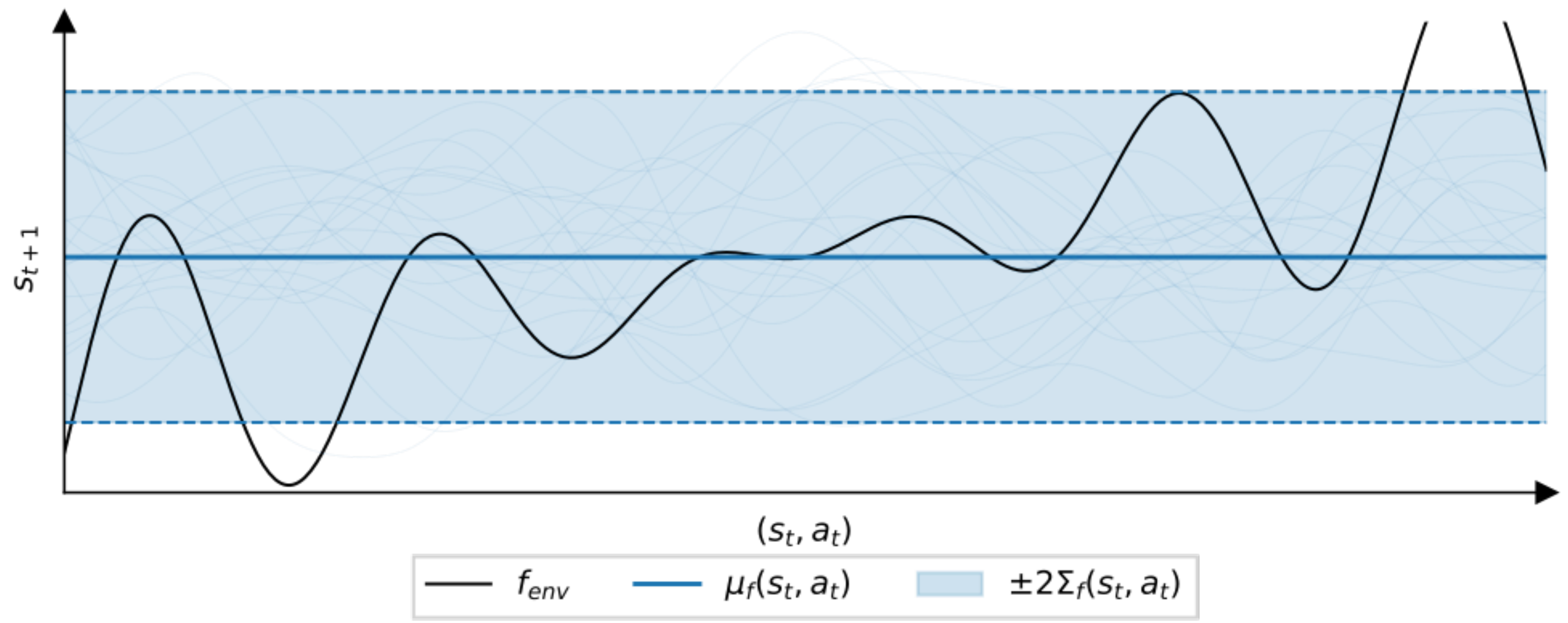

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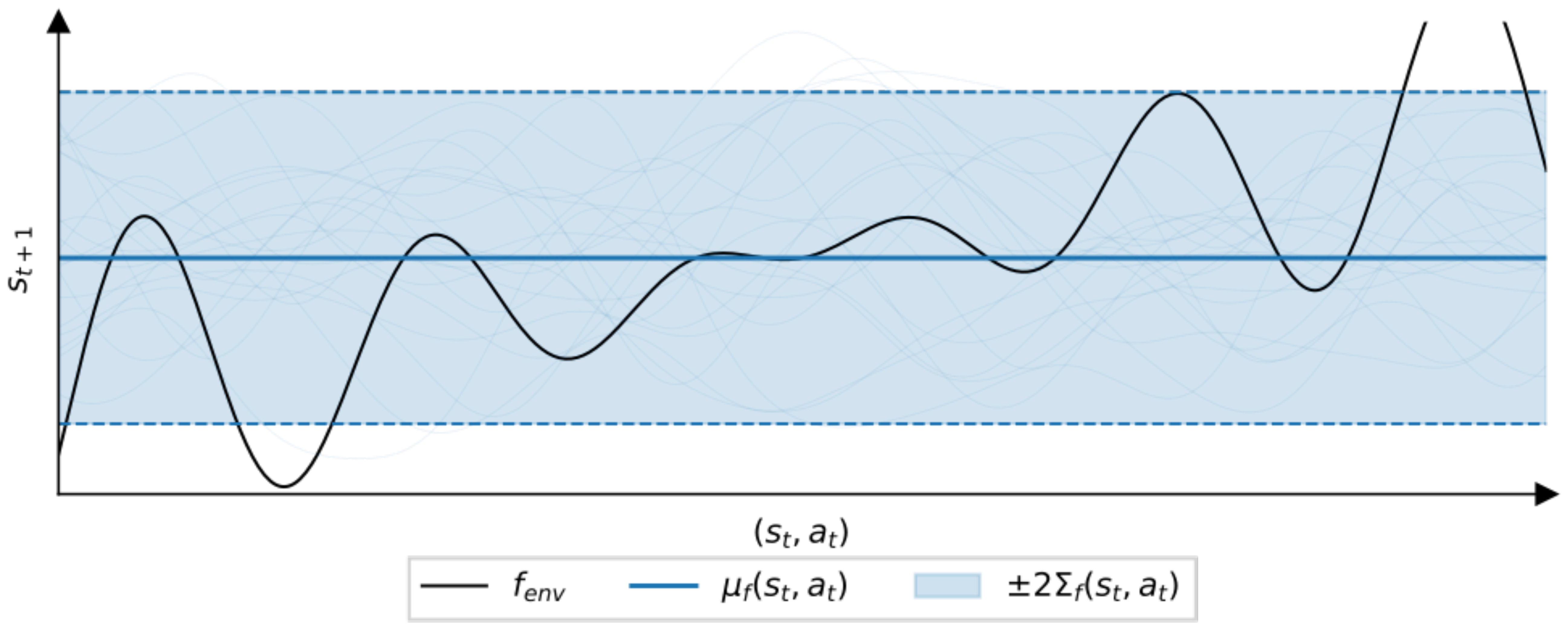
Epistemic uncertainty



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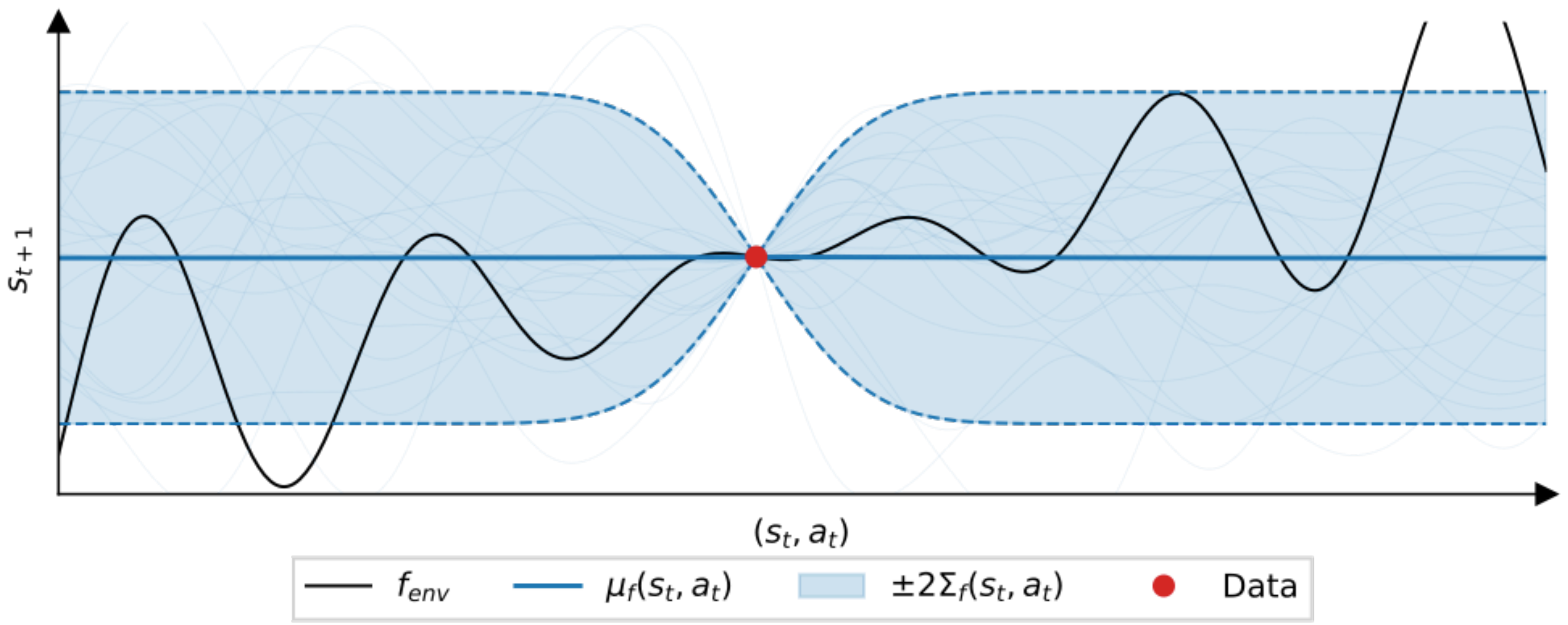

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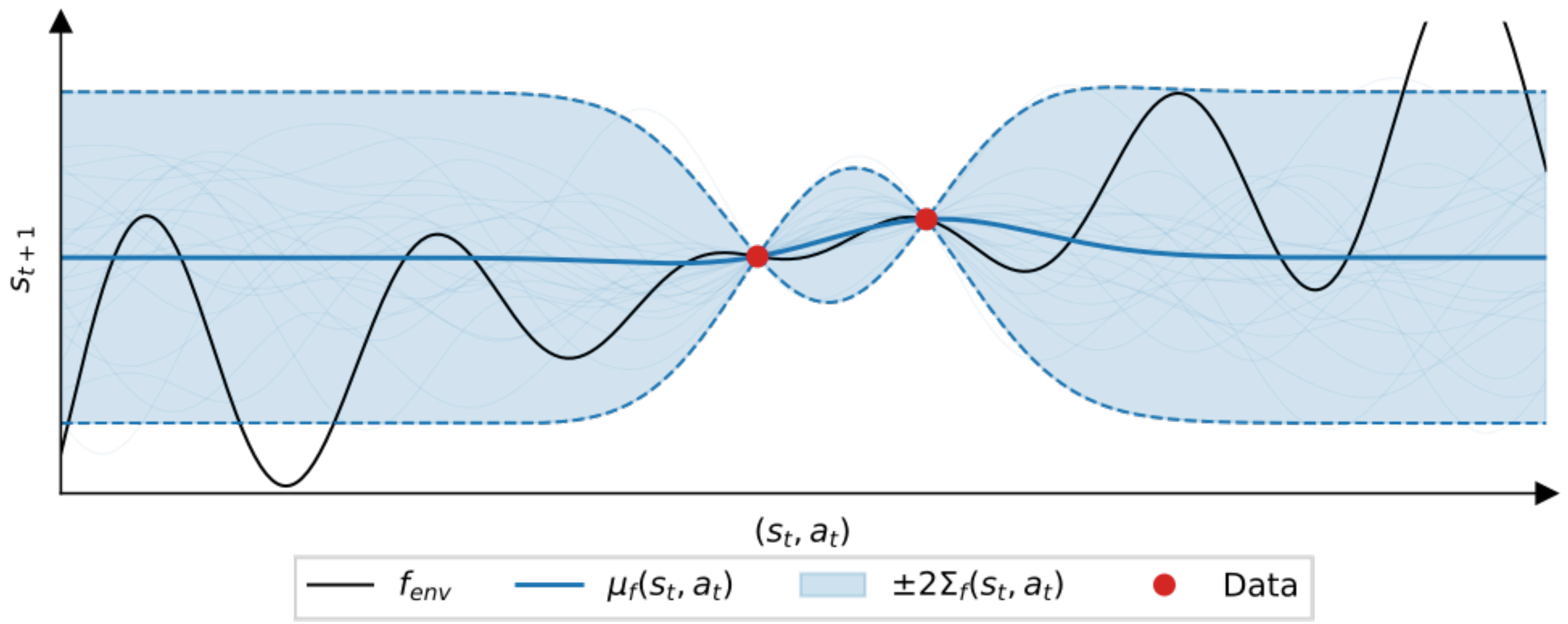

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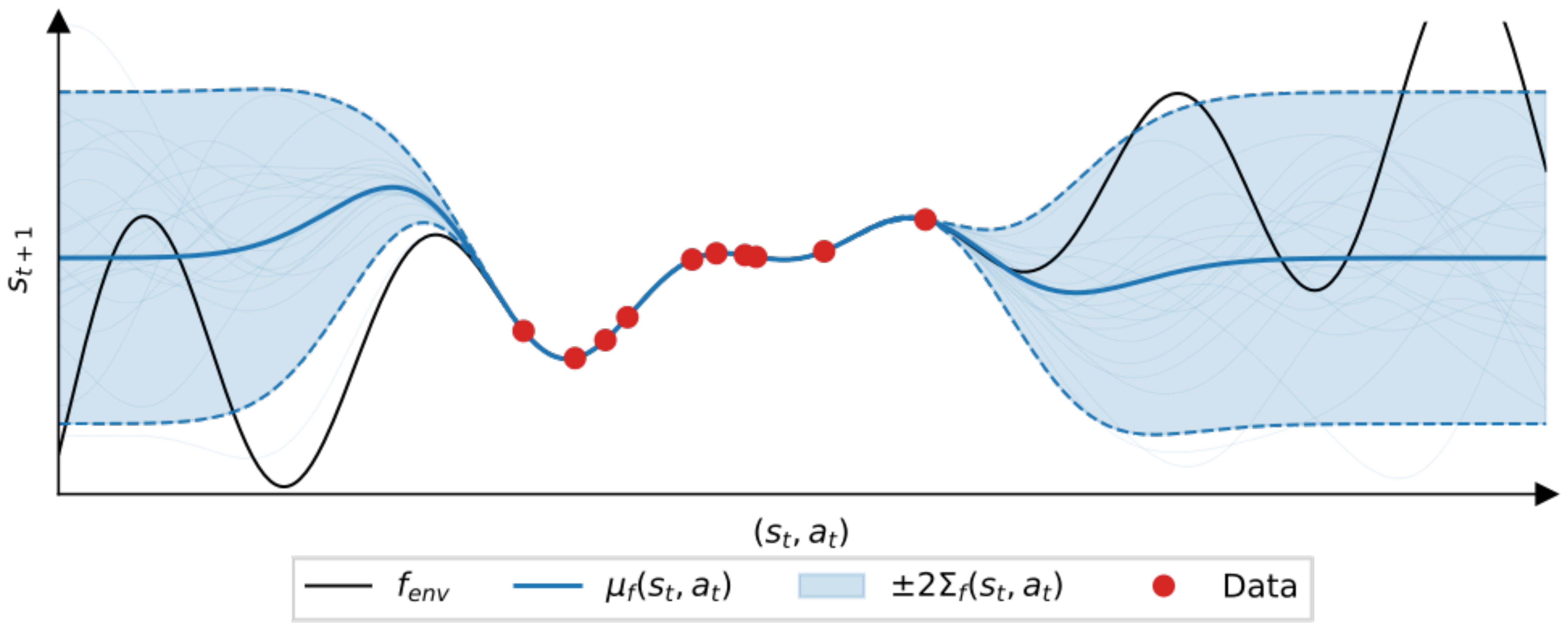

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Sources of Uncertainty

Stochastic Environments

$$s_{t+1} = f_{env}(s_t, a_t) + \epsilon_t \quad \text{where} \quad \mathbb{E}[\epsilon_t] = 0$$

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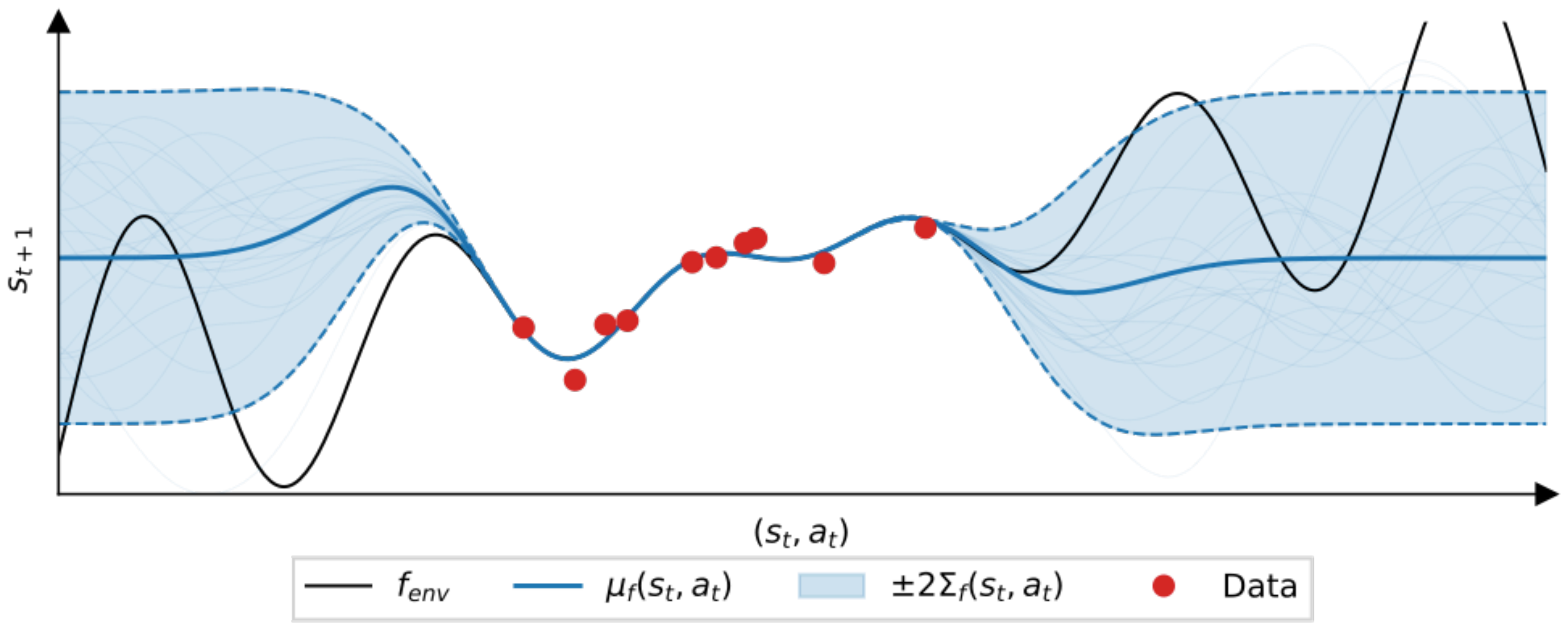

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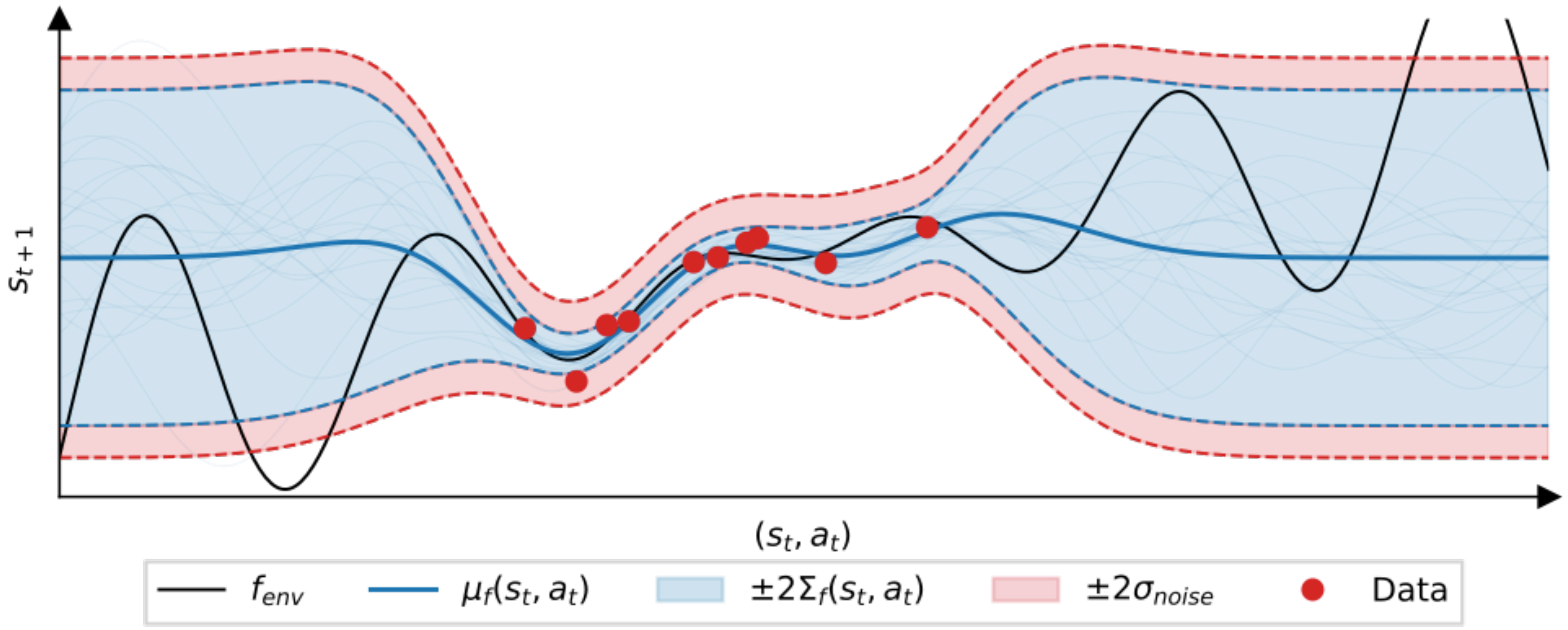

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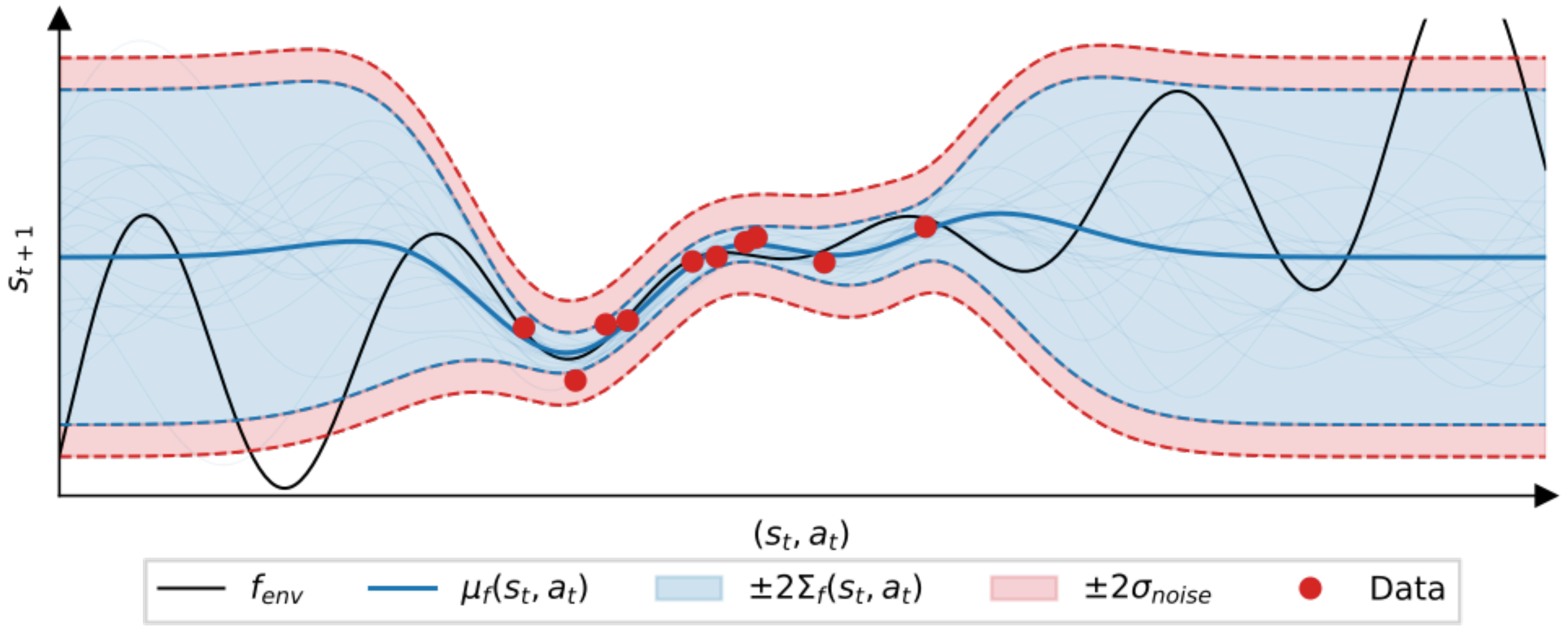
Aleatoric uncertainty



Decision-making Under Uncertainty

Sources of Uncertainty

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RL objective:

Sources of Uncertainty

Decision-making Under Uncertainty

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$$J(\pi; f) = \mathbb{E}_{???}\left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \mid s_{t+1} = f(s_t, a_t) + \epsilon_t, a_t = \pi(s_t)\right]$$

Sources of Uncertainty

Decision-making Under Uncertainty

RL objective:

Return = discounted sum of rewards

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Sources of Uncertainty

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Stochastic dynamics

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RL objective:

Deterministic policy

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What is the expectation over?

Sources of Uncertainty

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Posterior over dynamics models:

Sources of Uncertainty

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Posterior over dynamics models:

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How should we use this?

Model Averaging

$$\pi^{Greedy} = \arg\max_{\pi} \mathbb{E}_{p(f|\mathscr{D})}\Big[J(\pi; f)\Big]$$

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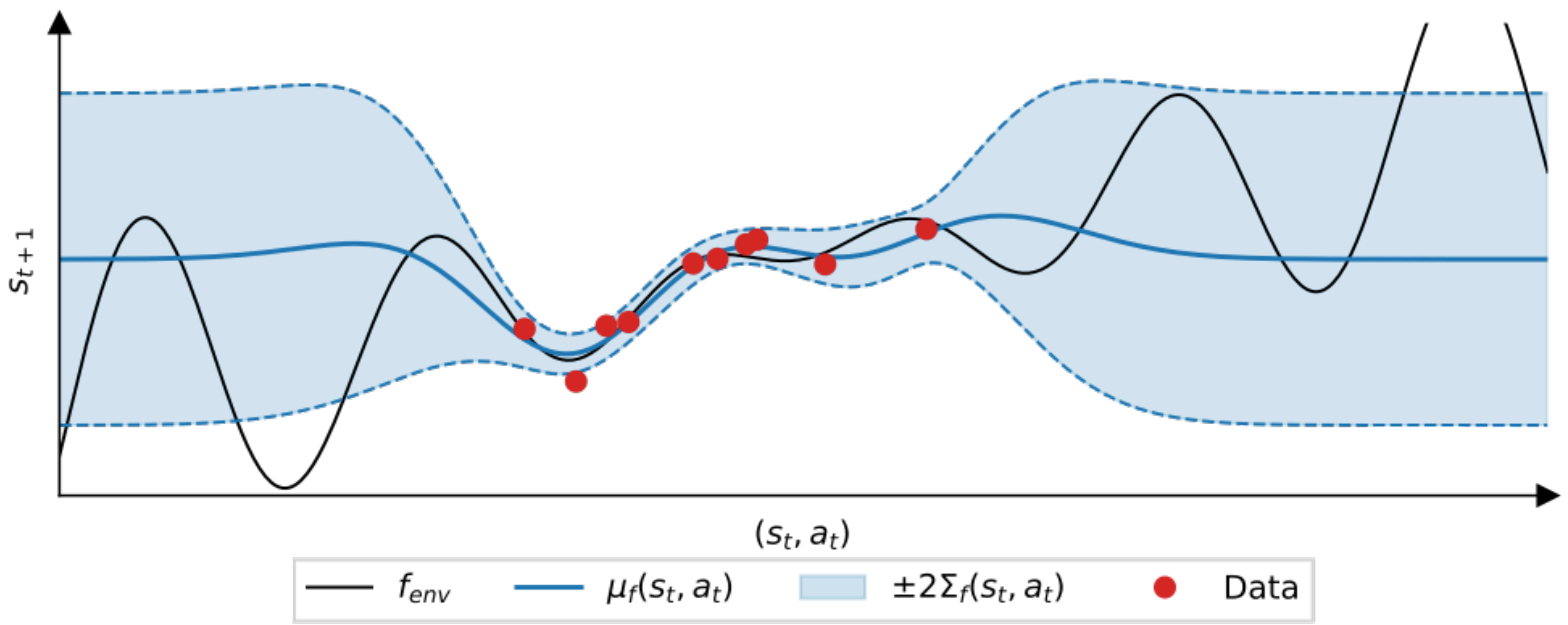

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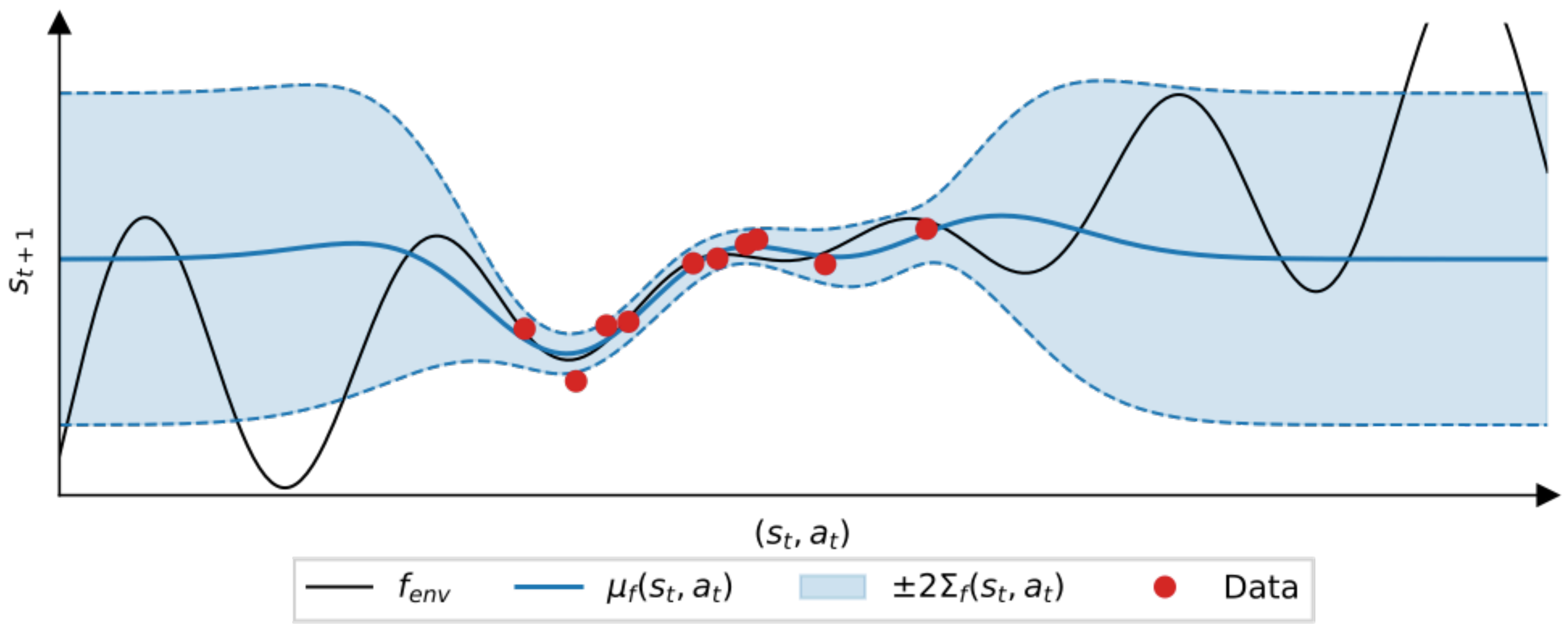

Combats model bias

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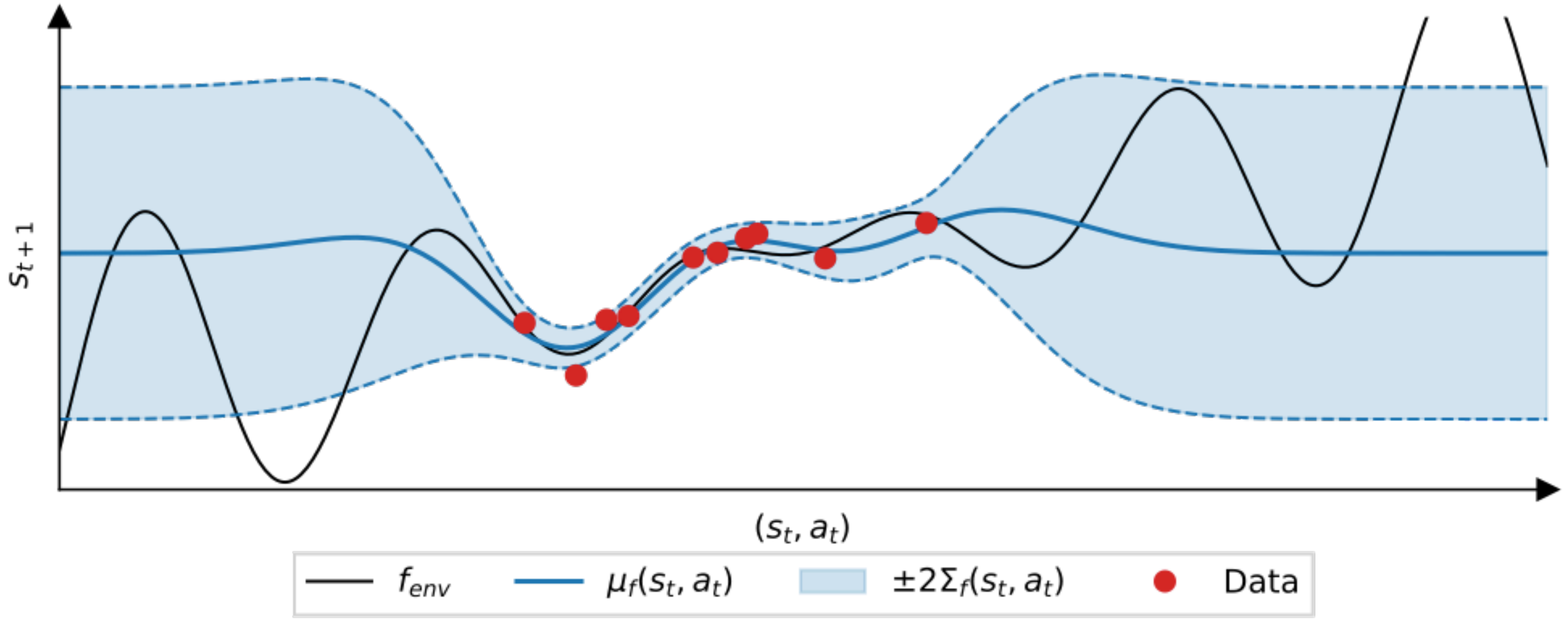
PILCO, PETS, etc

Combats model bias

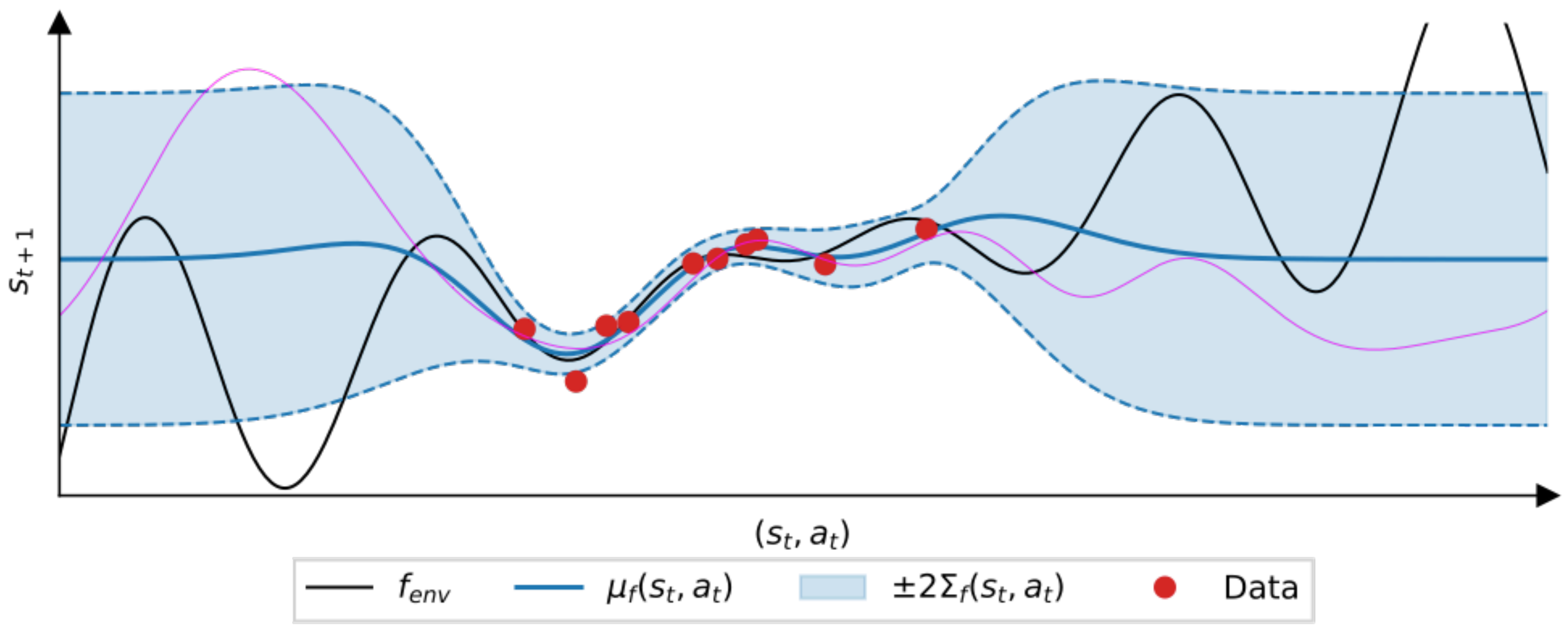


Deisenroth et al. (2011). PILCO: A Model-Based and Data-Efficient Approach to Policy Search. ICML.

Kurtland et al. (2018). Deep Reinforcement Learning in a Handful of Trials using Probabilisitic Dynamics Models. NeurIPS.

Exploration via Posterior Sampling

$$\pi^{PS} = \arg\max_{\pi} J(\pi; \tilde{f}), \quad \tilde{f} \sim p(f \mid \mathcal{D})$$



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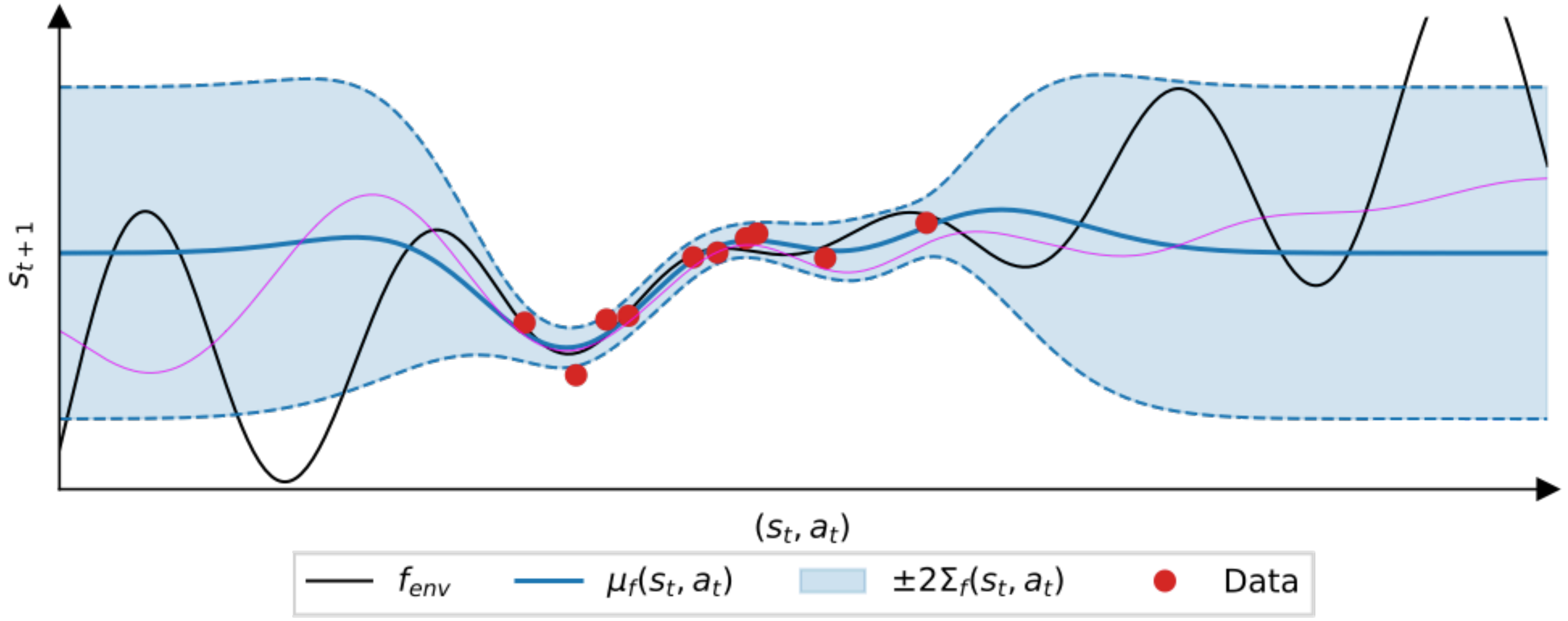

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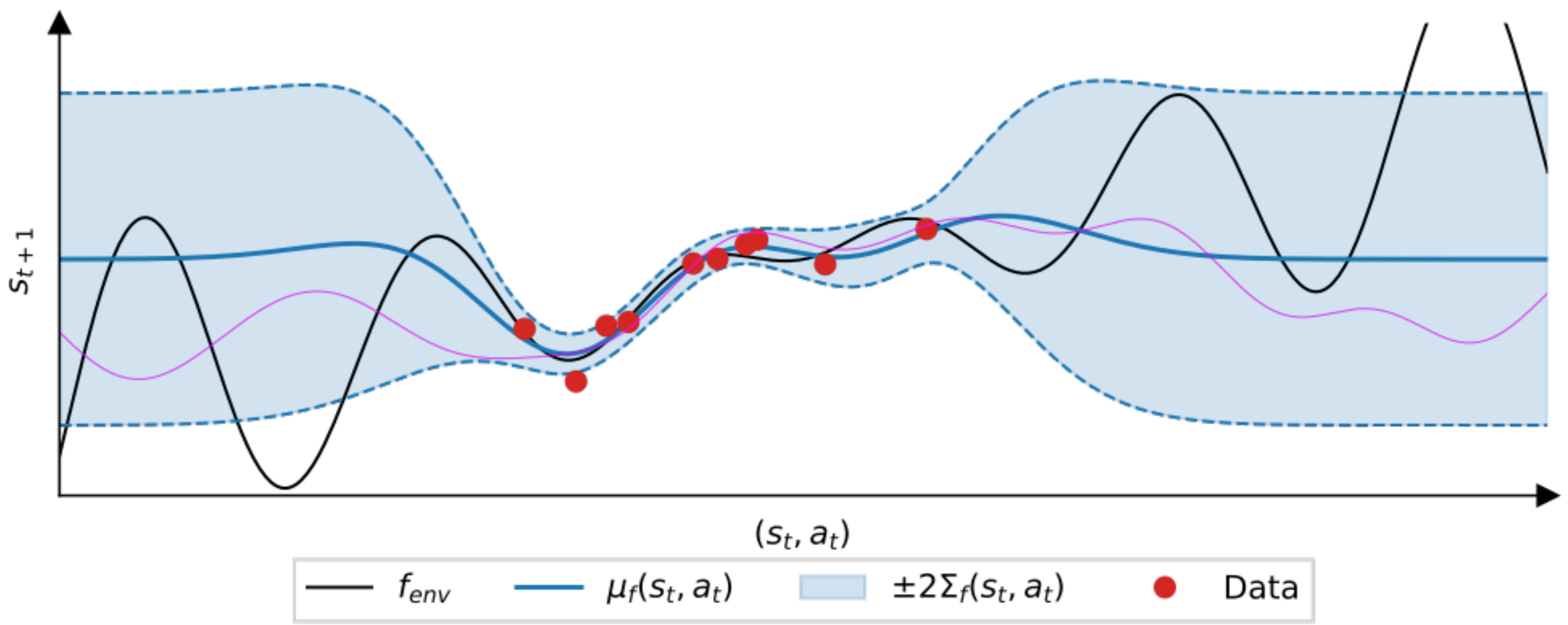

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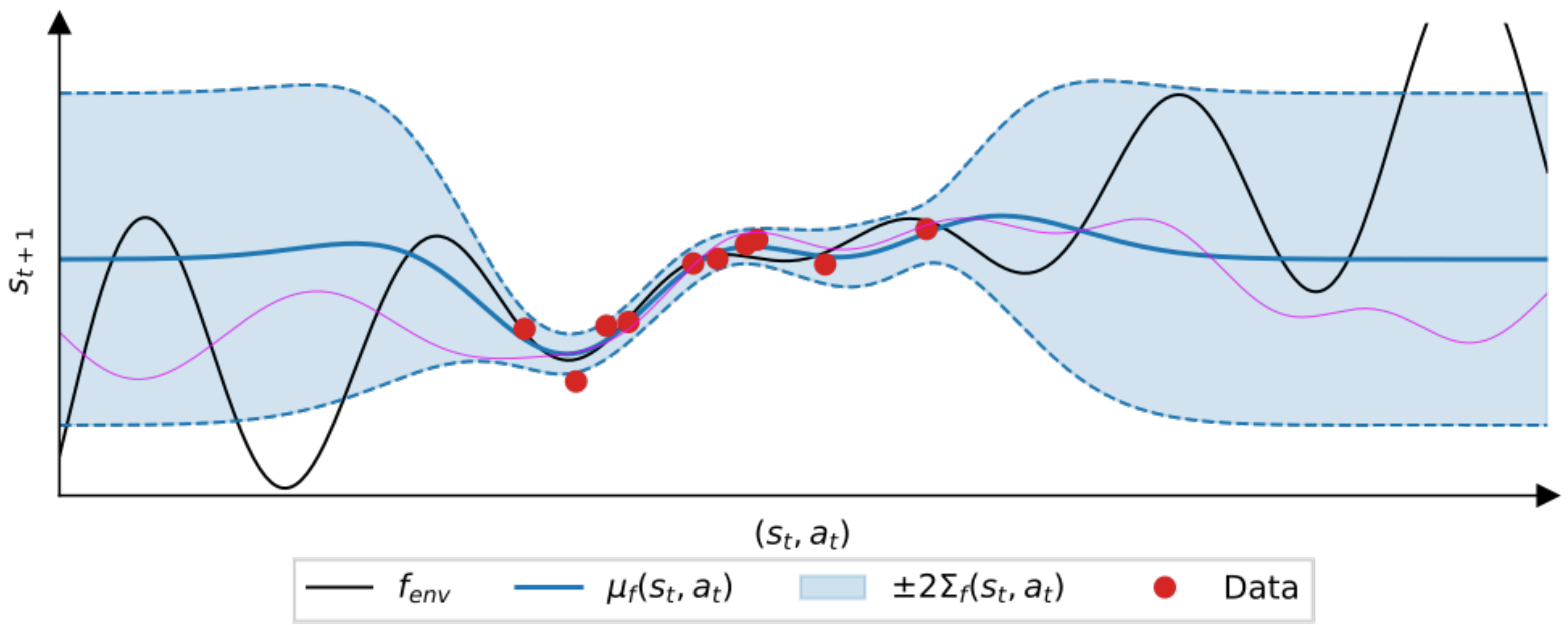

No extra hyperparamters

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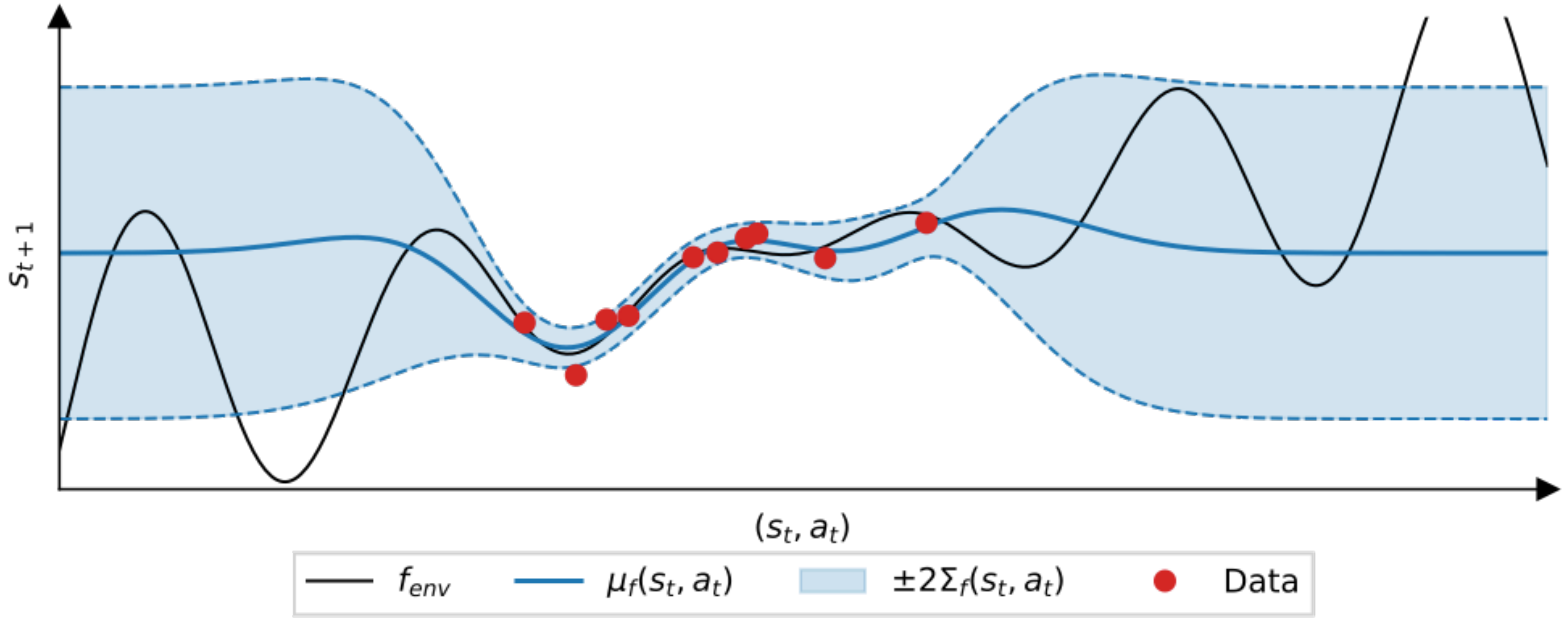

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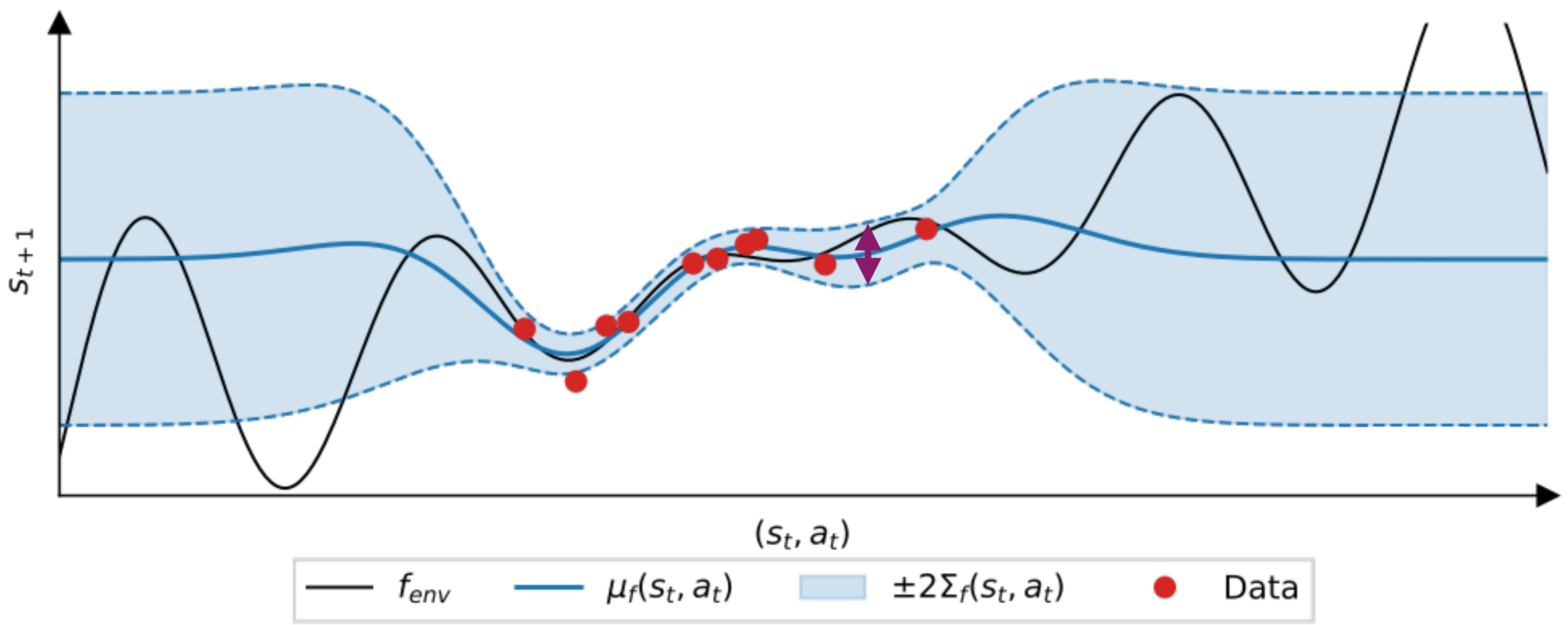

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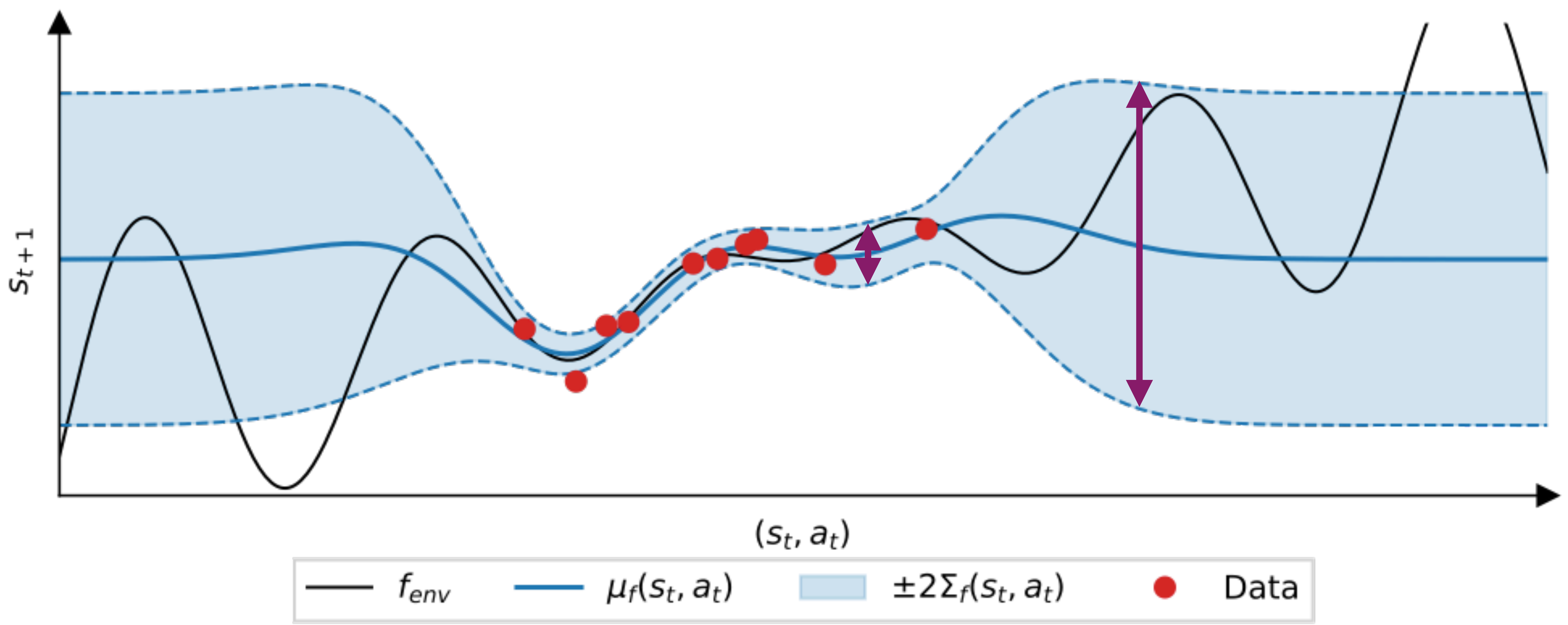

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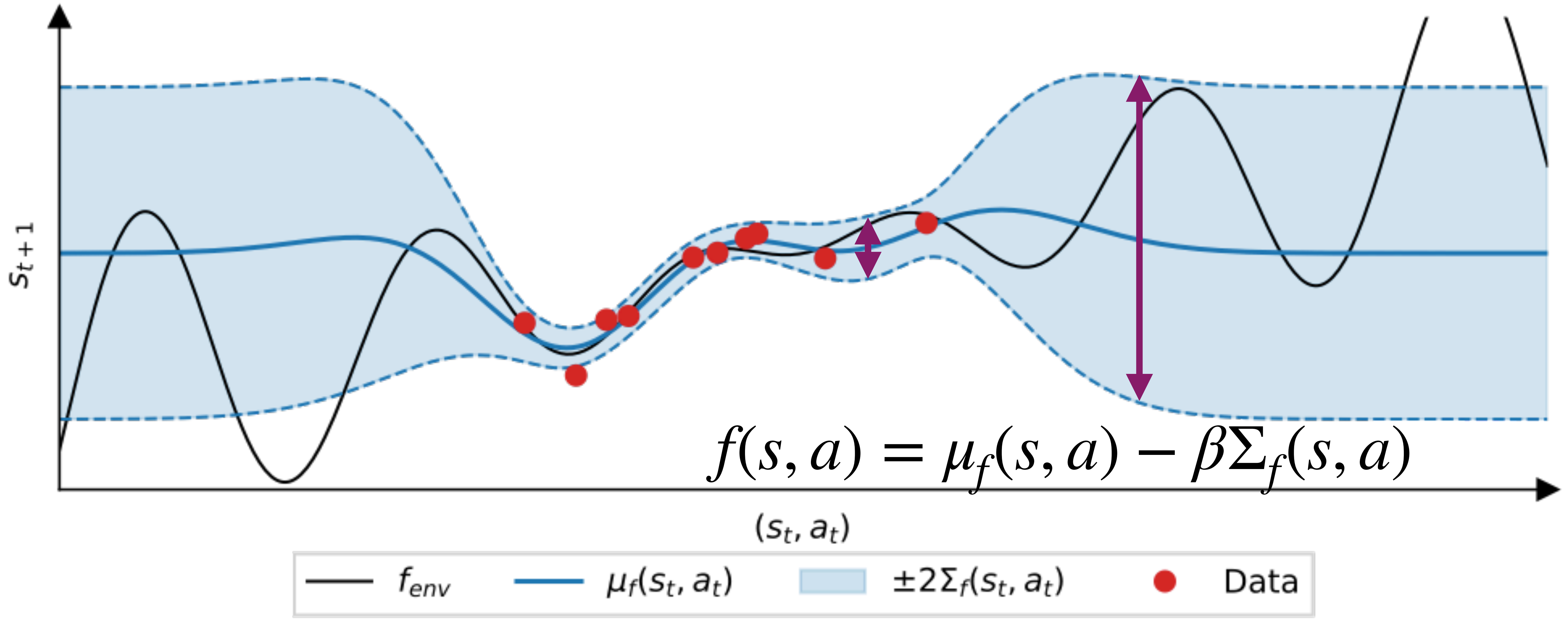

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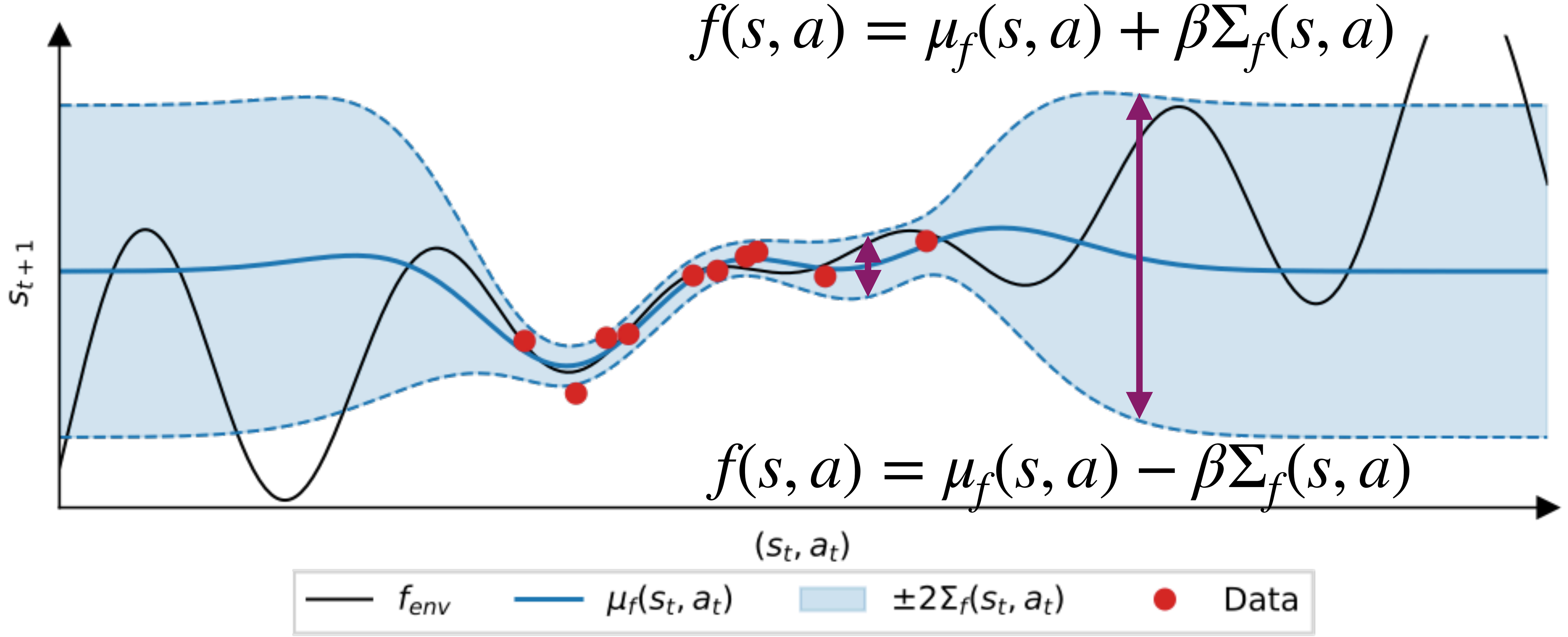

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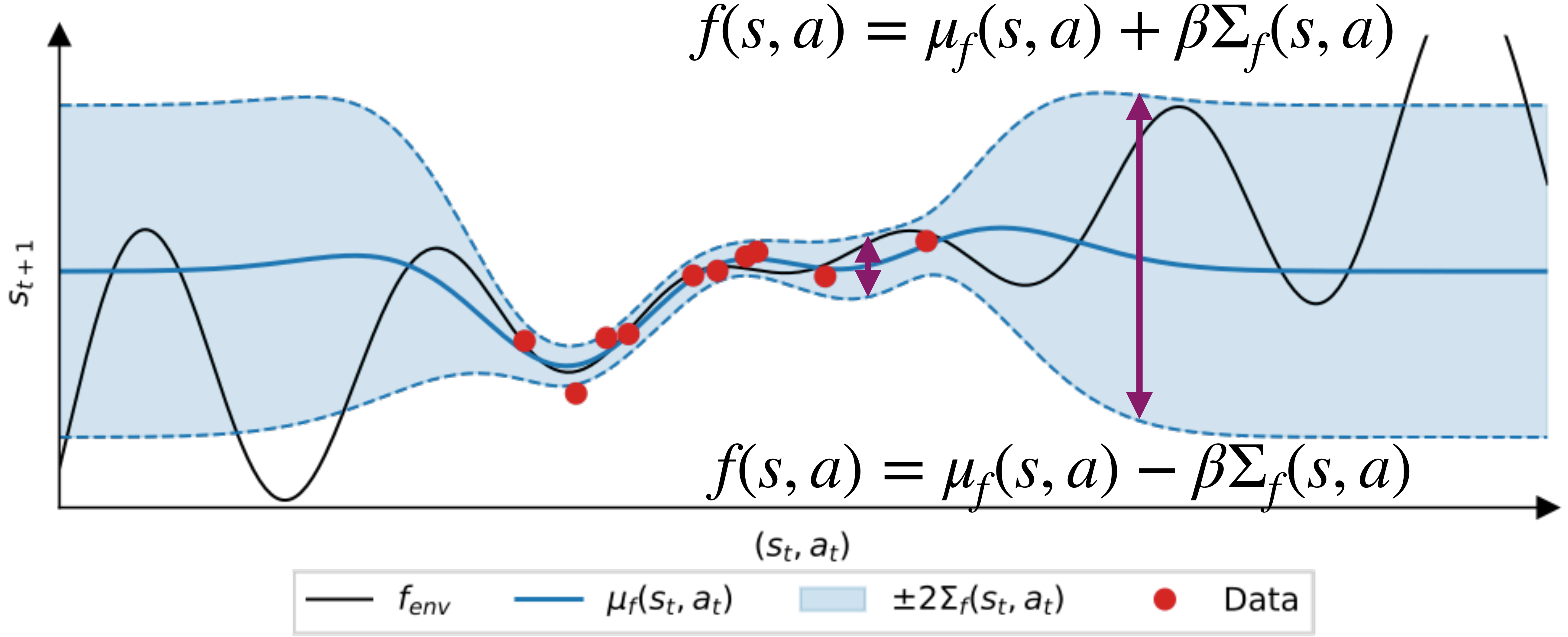
$$\mathcal{M} = \left\{ f \mid \|f(s,a) - \mu_f(s,a)\| \leq \beta \Sigma_f(s,a) \right\}$$



FCAI Curi et al. (2020). Efficient Model-Based Reinforcement Learning through Optimistic Policy Search and Planning. NeurIPS. fcai.fi

Exploration via Upper Confidence Bound

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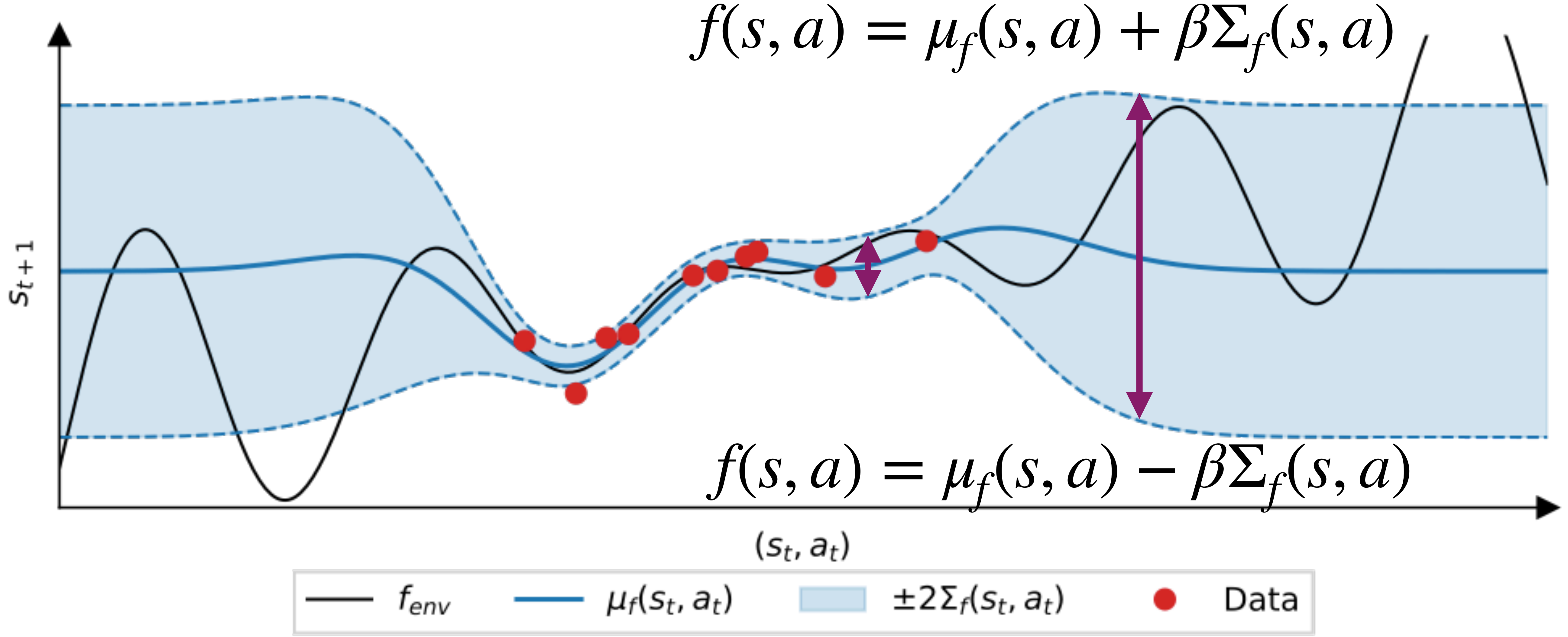
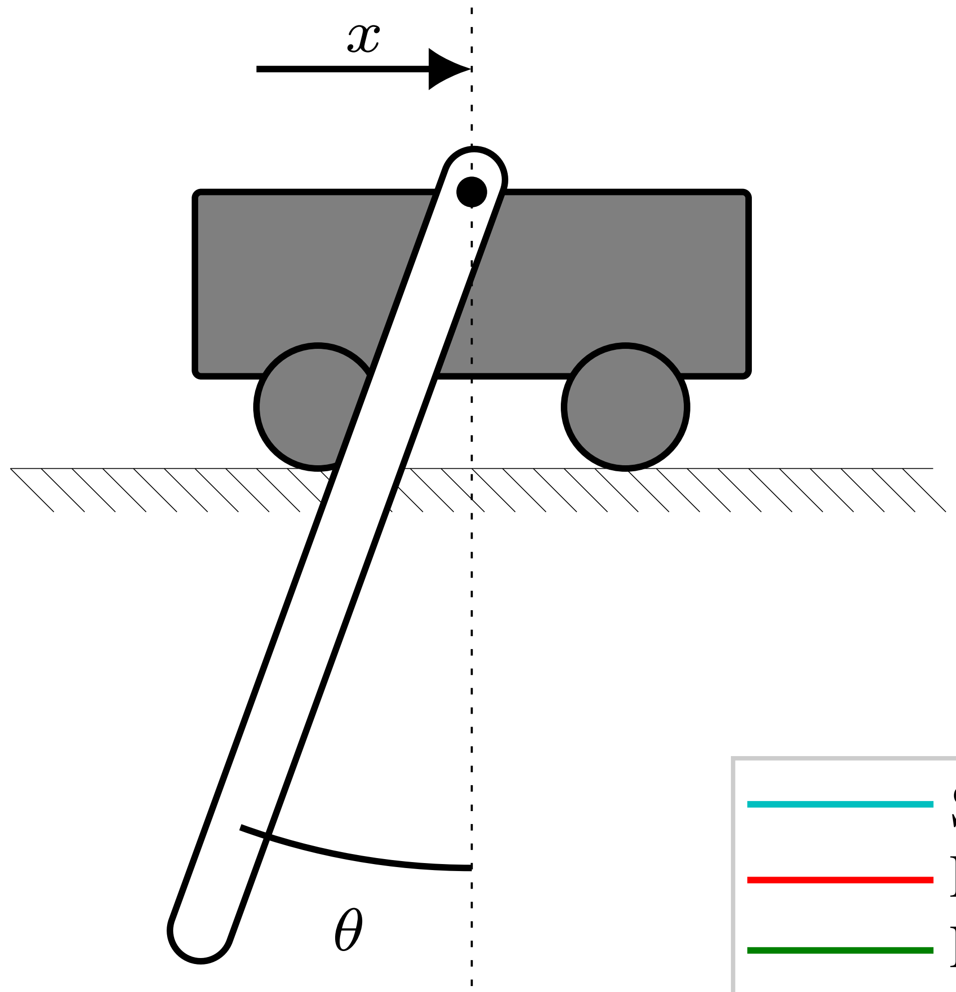
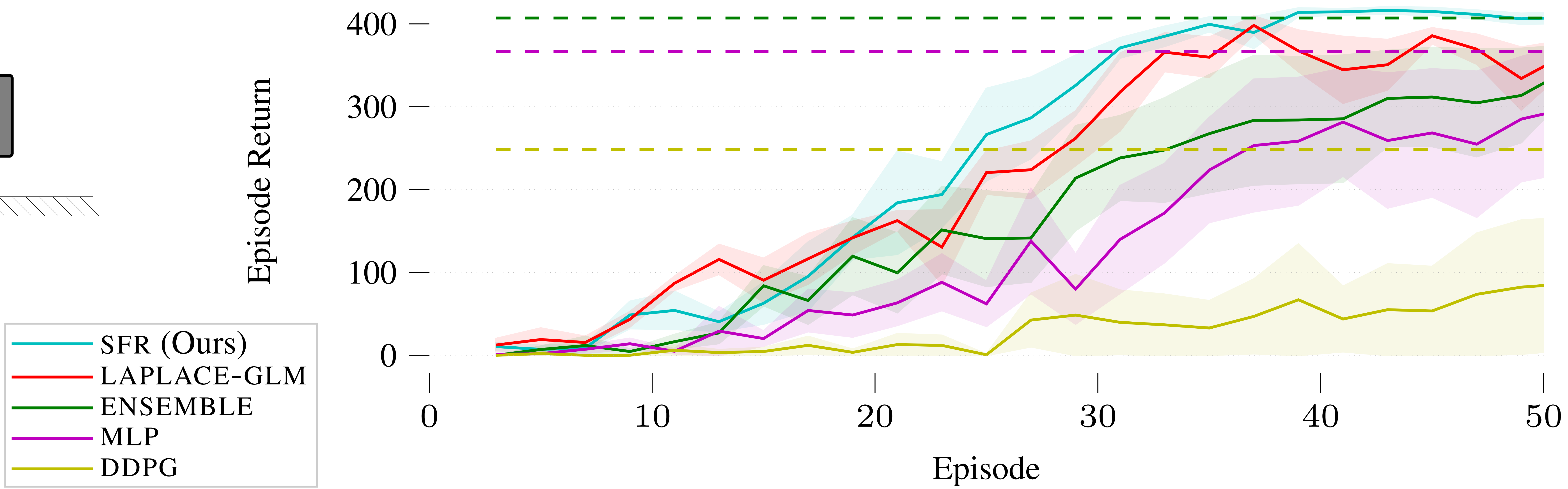


Extra hyperparamter β

 Curi et al. (2020). Efficient Model-Based Reinforcement Learning through Optimistic Policy Search and Planning. NeurIPS.

How to Quantify Uncertainty in Dynamics?





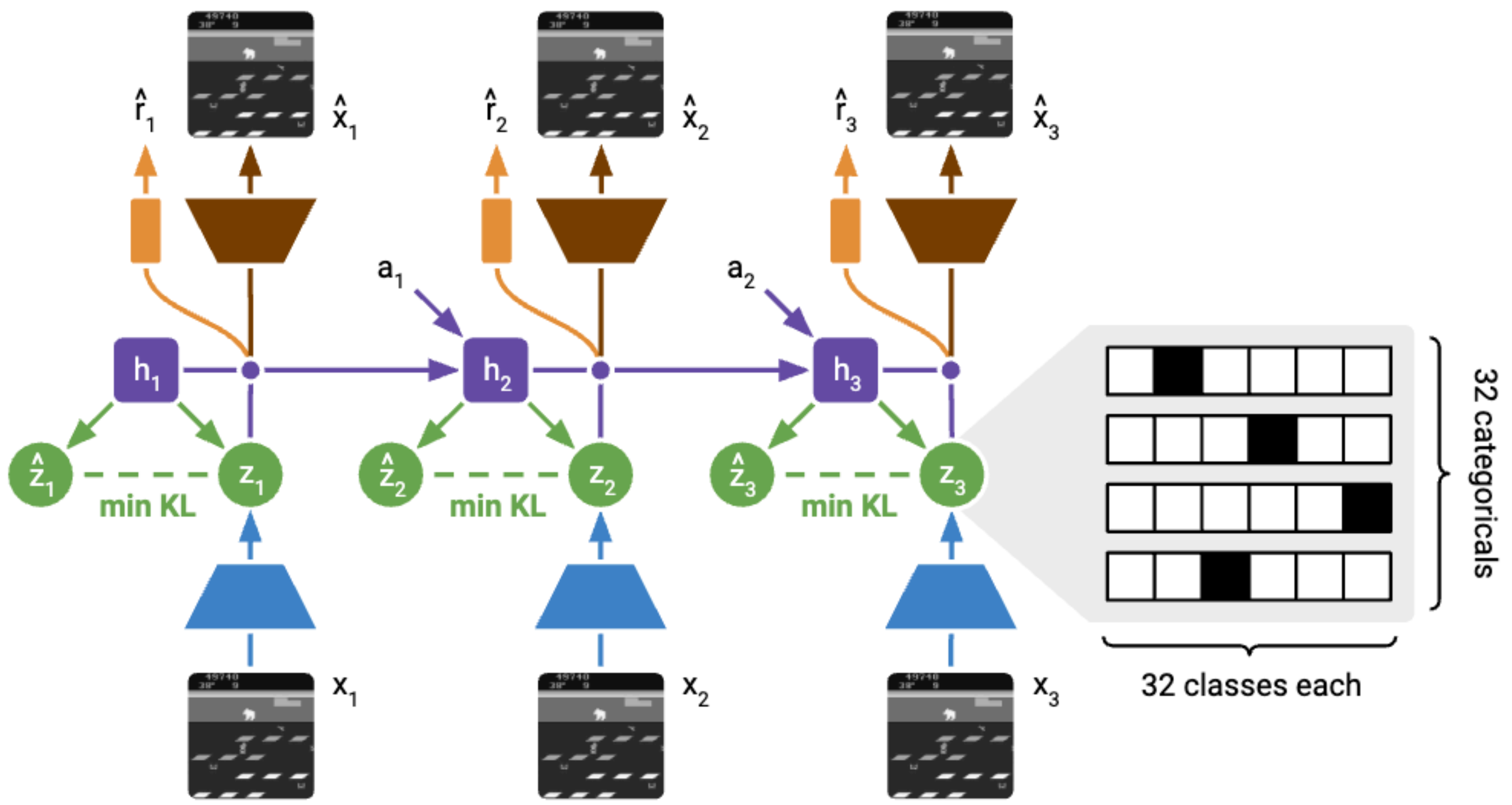
Scannell et al. (2024). Function-space Parameterisation of Neural Networks for Sequential Learning. ICLR.

Learning Objectives

Understand

1. ~~What a “model” is in model-based RL~~
2. ~~How a “model” can aid decision making~~
3. ~~Differences between background and decision-time planning~~
4. ~~Decision-time planning strategies for continuous actions~~
5. ~~Sources of uncertainty in model-based RL~~
6. ~~Rationale and insights for decision-making under uncertainty~~

Case Study: Dreamer (v2/v3)



World Model Learning

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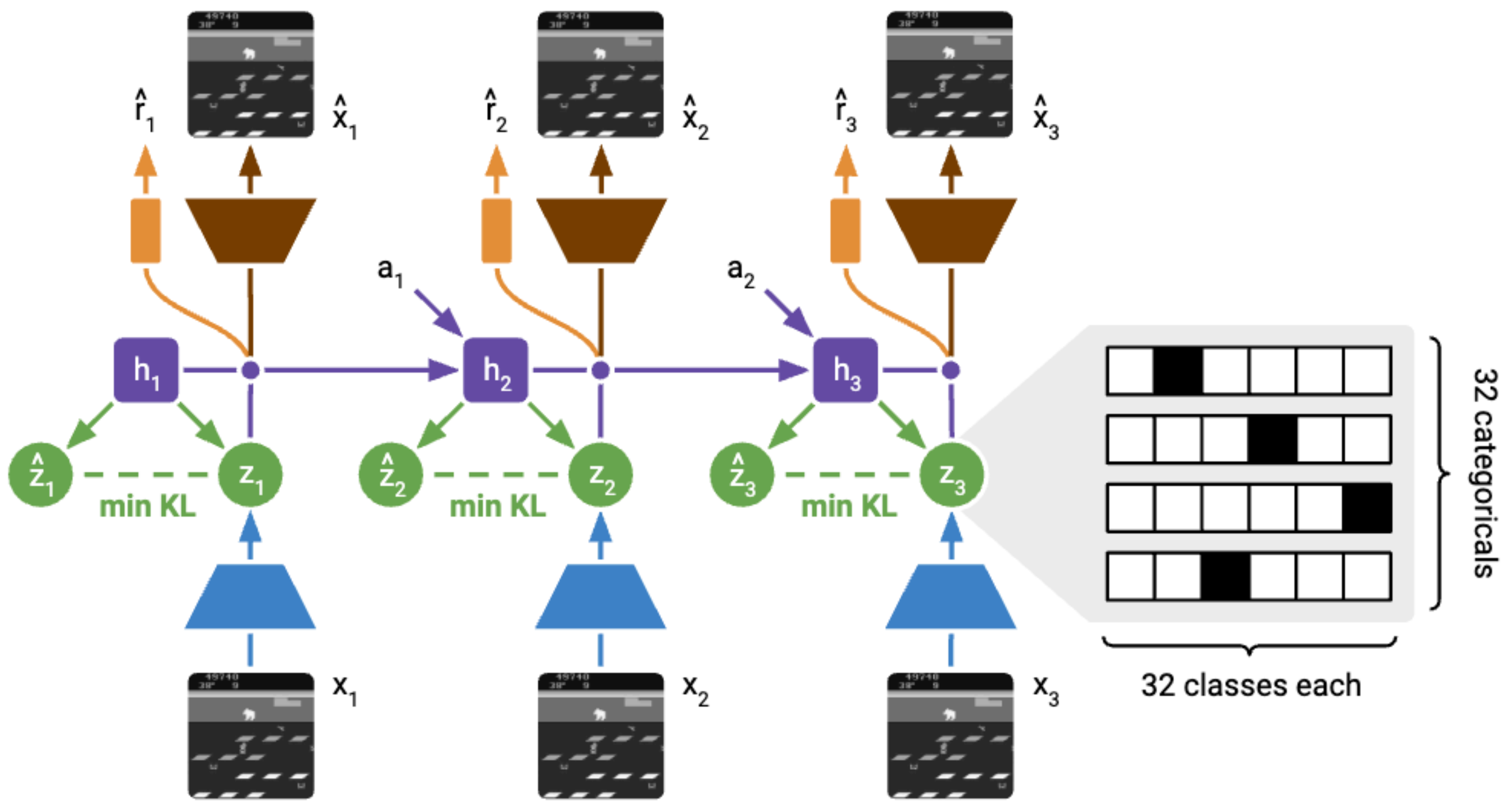
Hafner et al. (2021). Mastering Atari with Discrete World Models. ICLR.

Case Study: Dreamer (v2/v3)



World Model Learning

Latent dynamics with encoder/decoder

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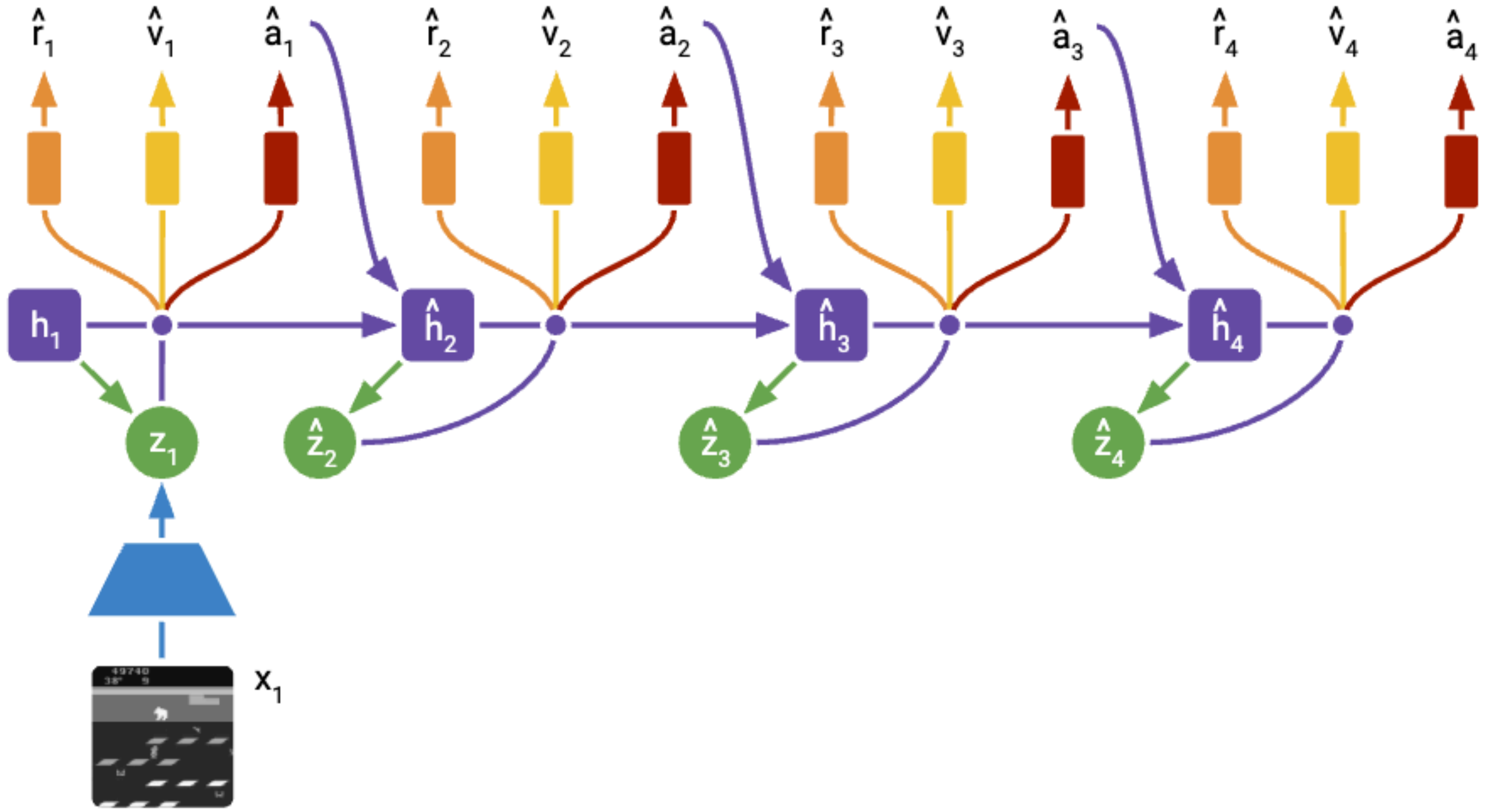
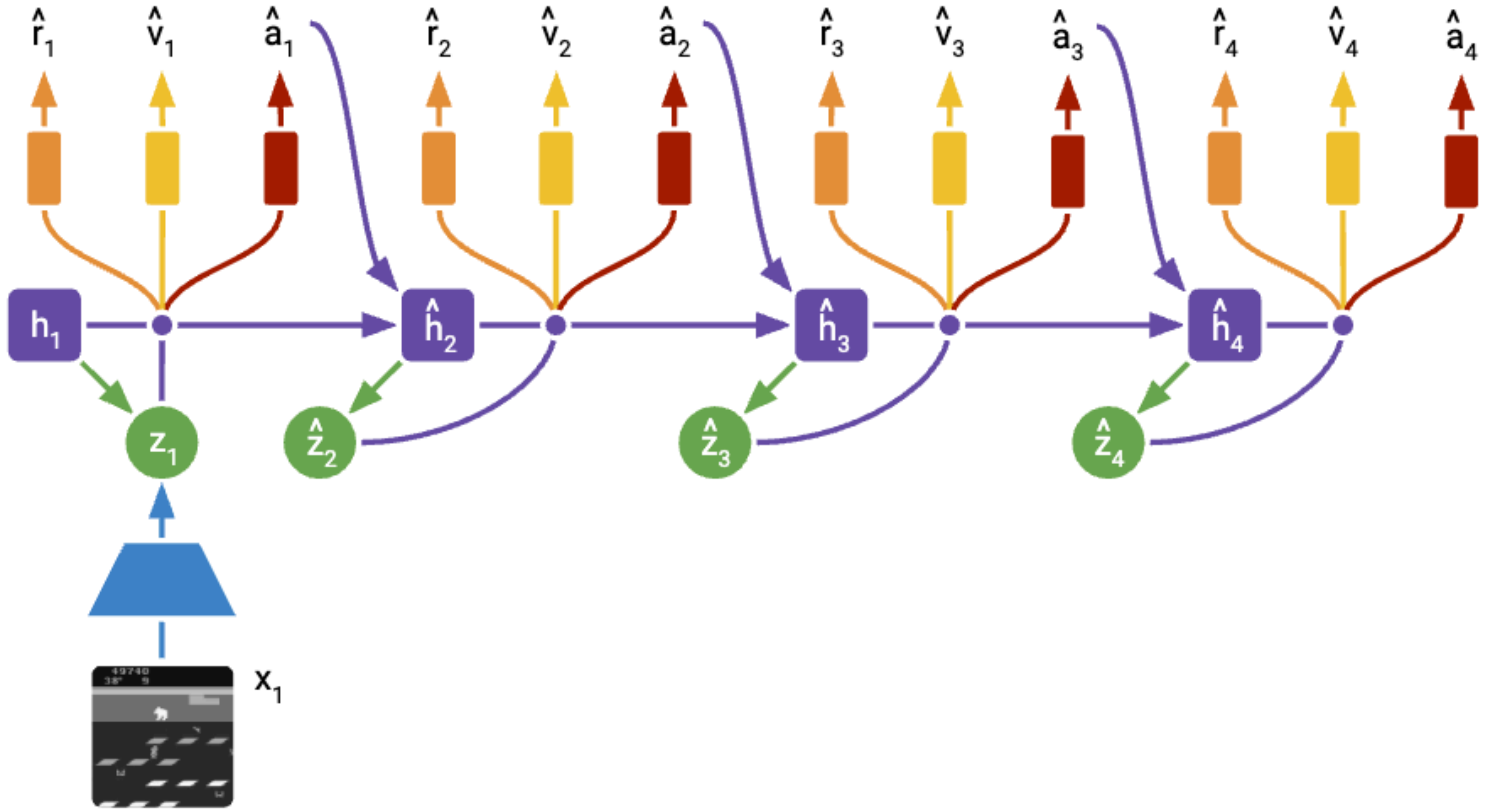


Actor Critic Learning

Latent dynamics with encoder/decoder

Case Study: Dreamer (v2/v3)



Actor Critic Learning

Latent dynamics with encoder/decoder

Actor $\pi_\theta(z)$ & critic $Q_\theta(z, a)$ in latent space

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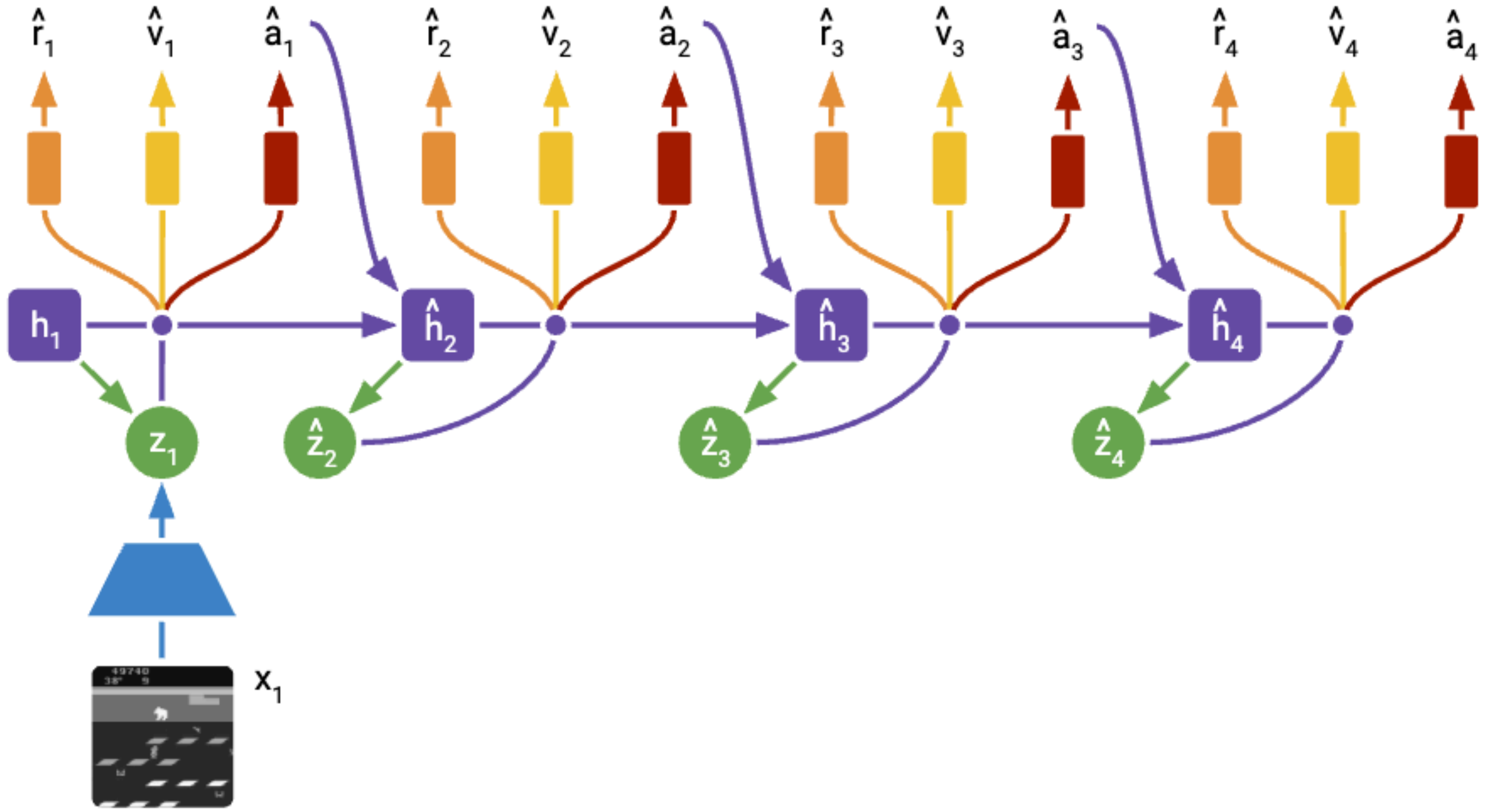

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Actor/critic leverage "imagined" outcomes

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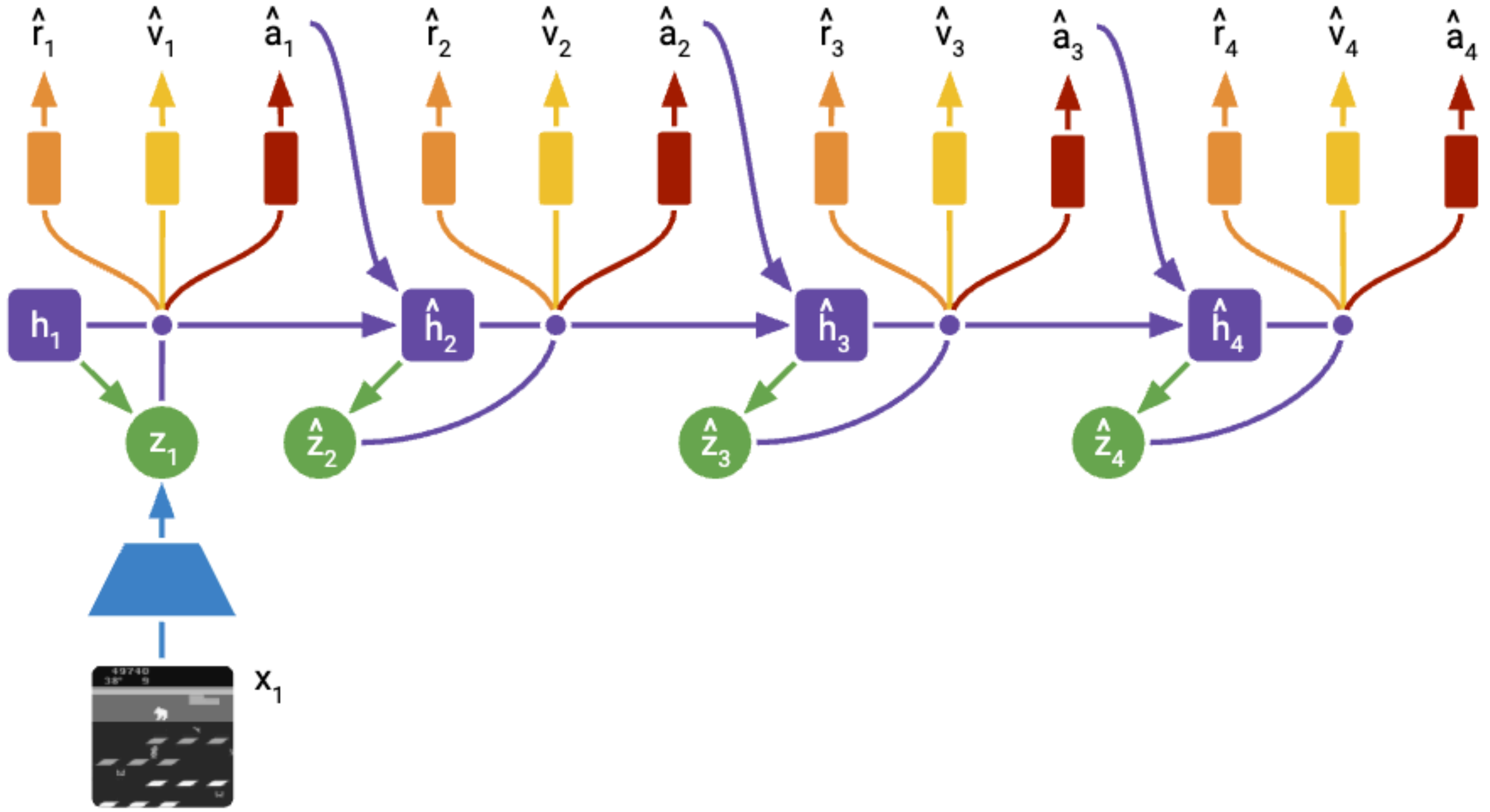


Actor Critic Learning

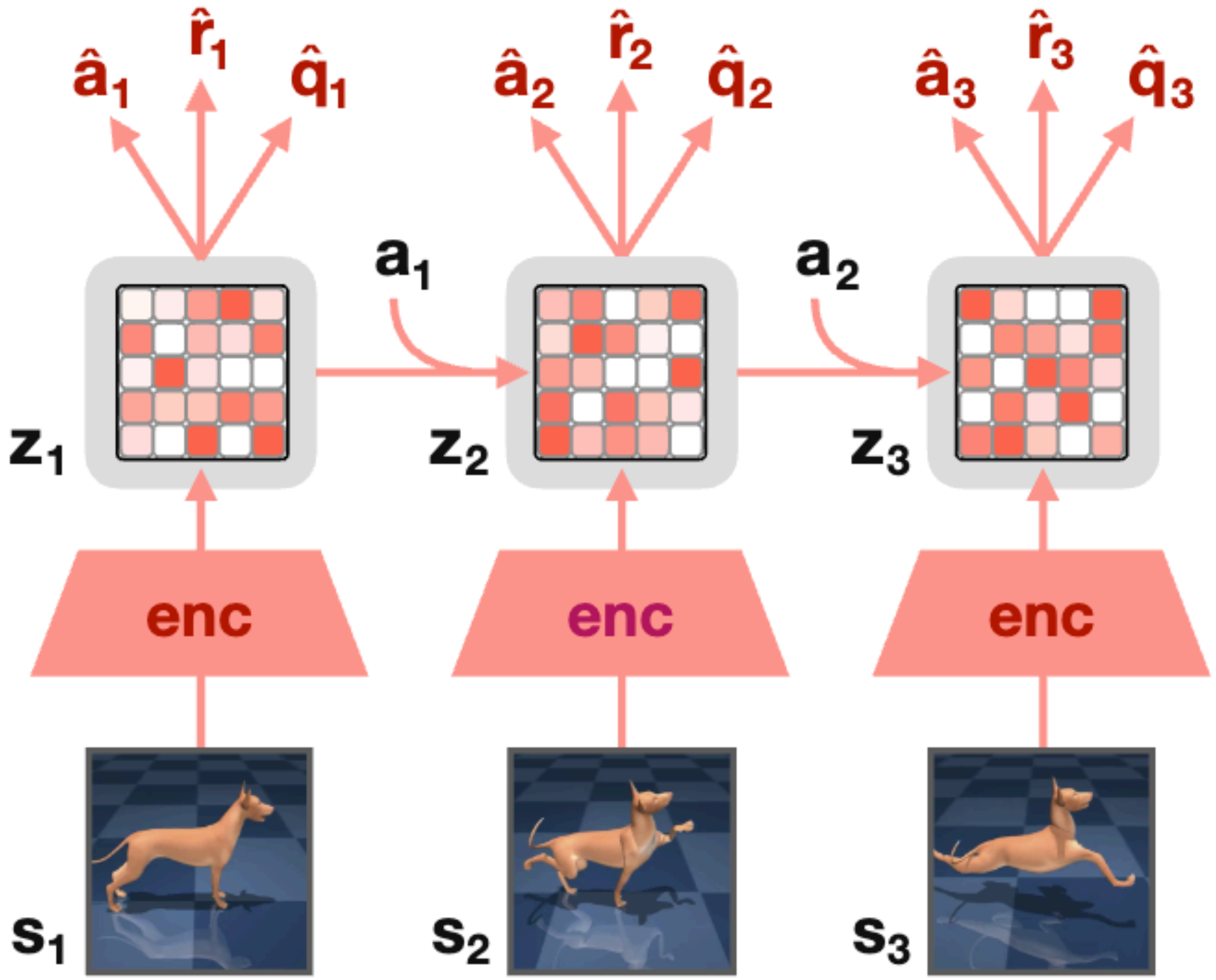
Latent dynamics with encoder/decoder

Actor $\pi_\theta(z)$ & critic $Q_\theta(z, a)$ in latent space

Actor/critic leverage “imagined” outcomes

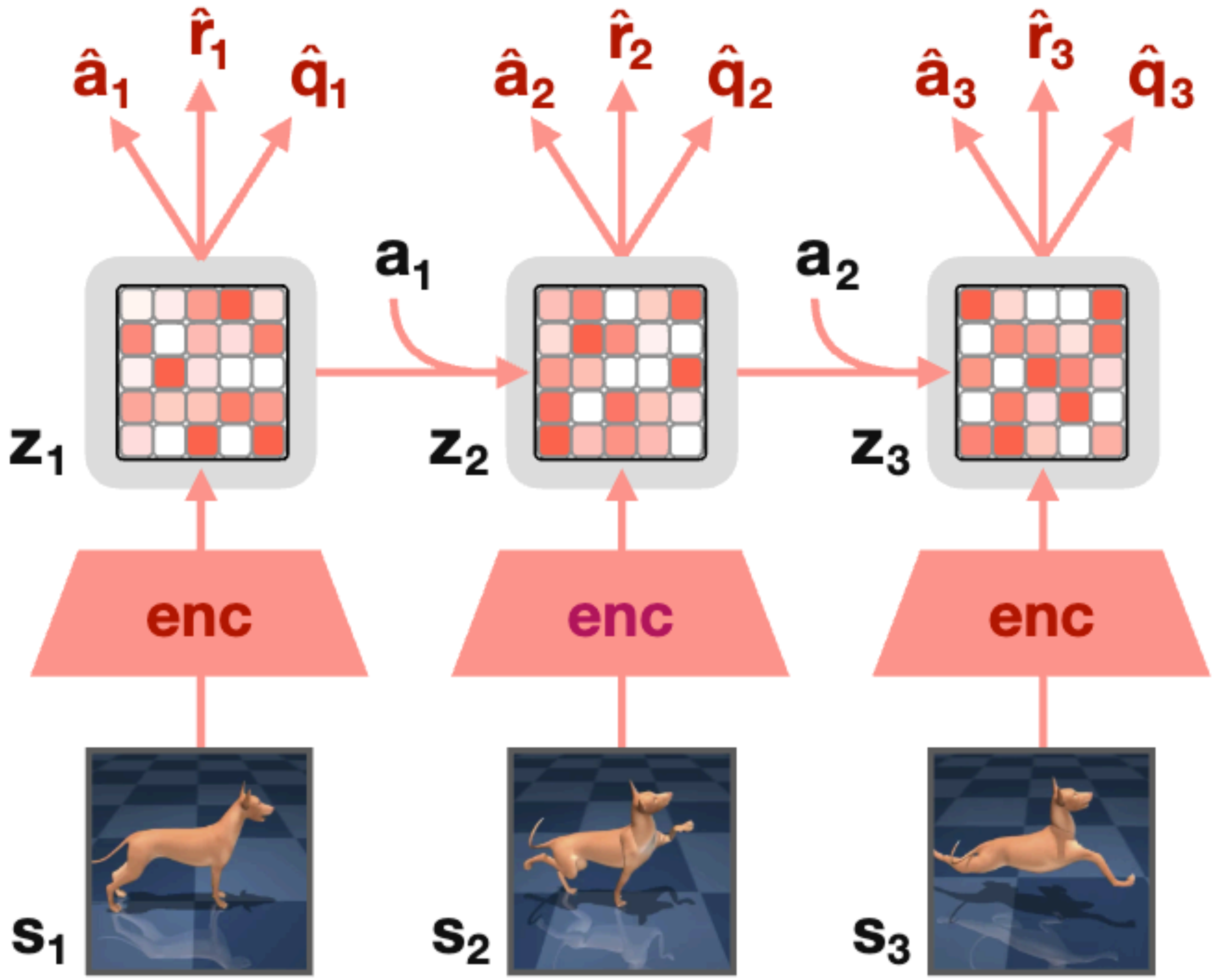
Background planning

Case Study: TD-MPC2



Hansen et al. (2024). TD-MPC2: Robust, Scalable World Models for Continuous Control. ICLR.

Case Study: TD-MPC2



Latent dynamics

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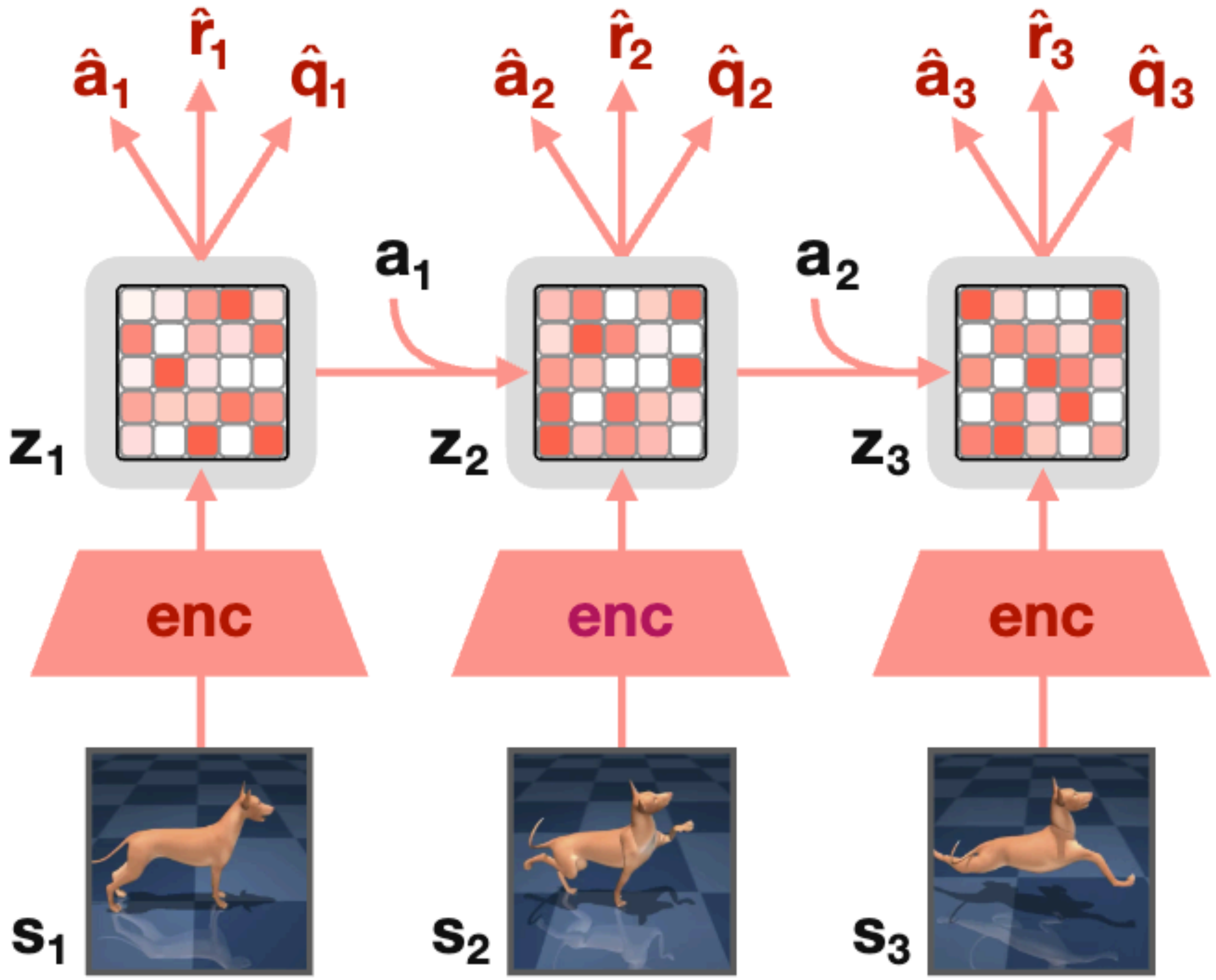

Latent dynamics

No decoder

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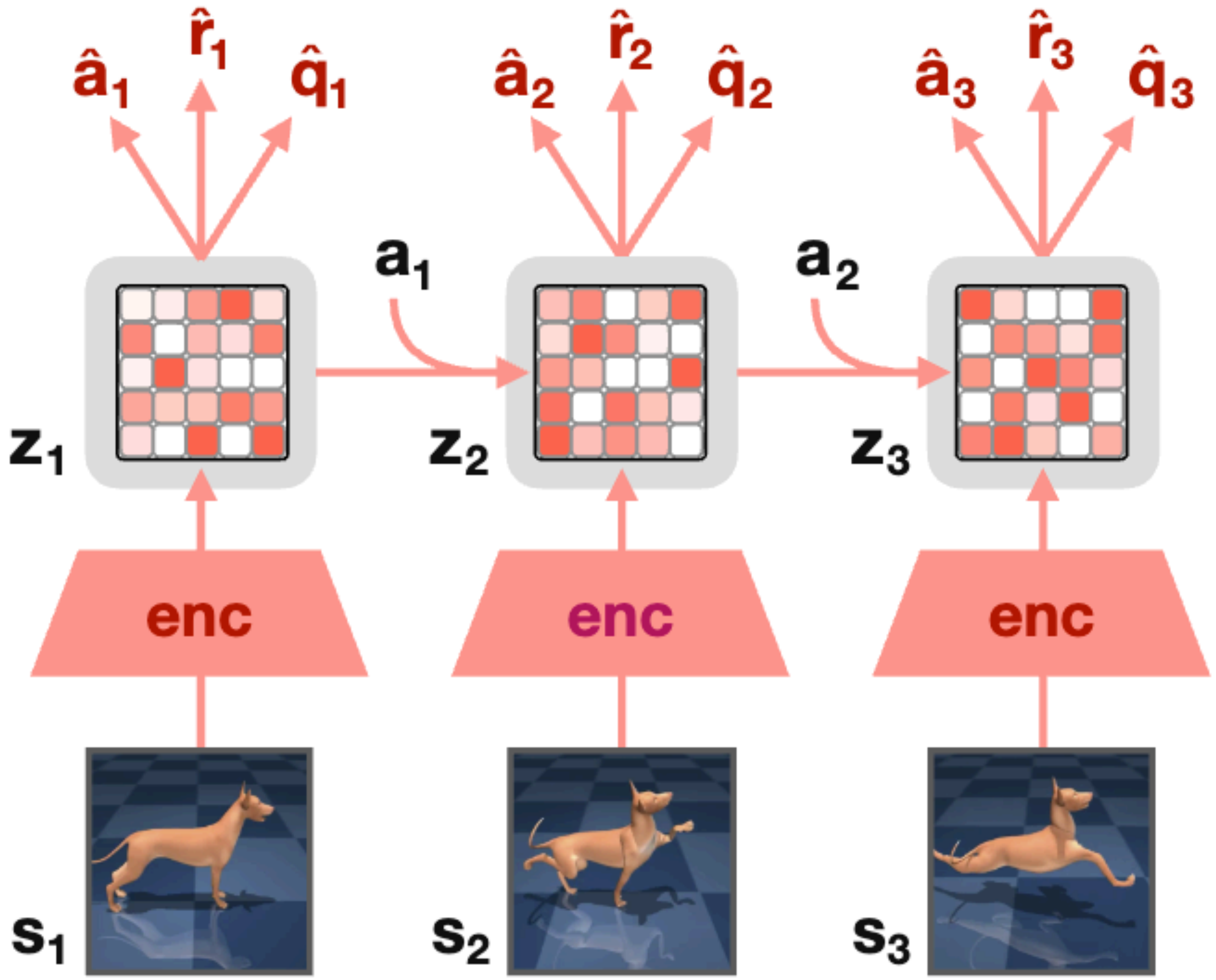
Case Study: TD-MPC2



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Actor $\pi_\theta(z)$ & critic $Q_\theta(z, a)$ in latent space

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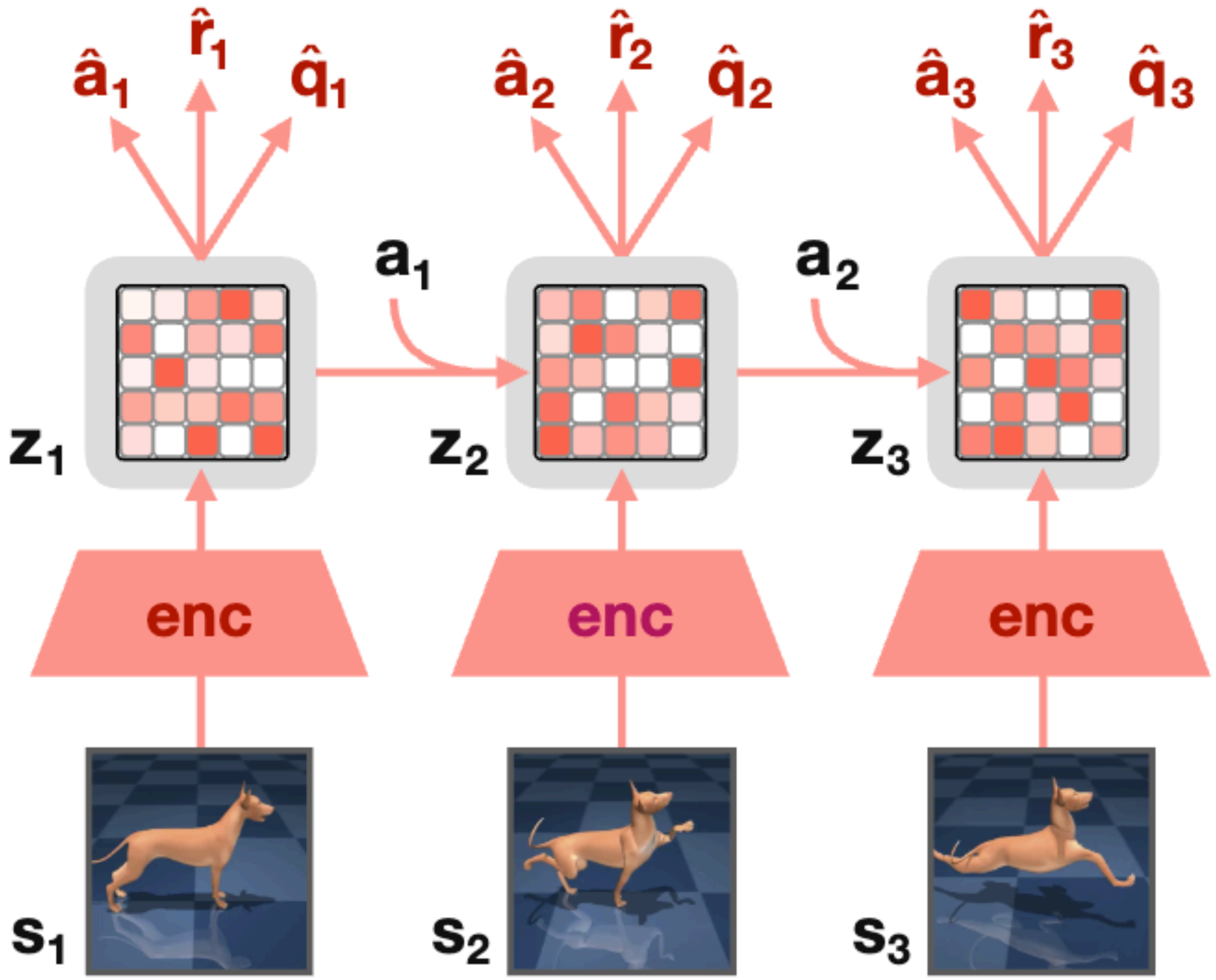

Latent dynamics

No decoder

Actor $\pi_\theta(z)$ & critic $Q_\theta(z, a)$ in latent space

Decision-time planning

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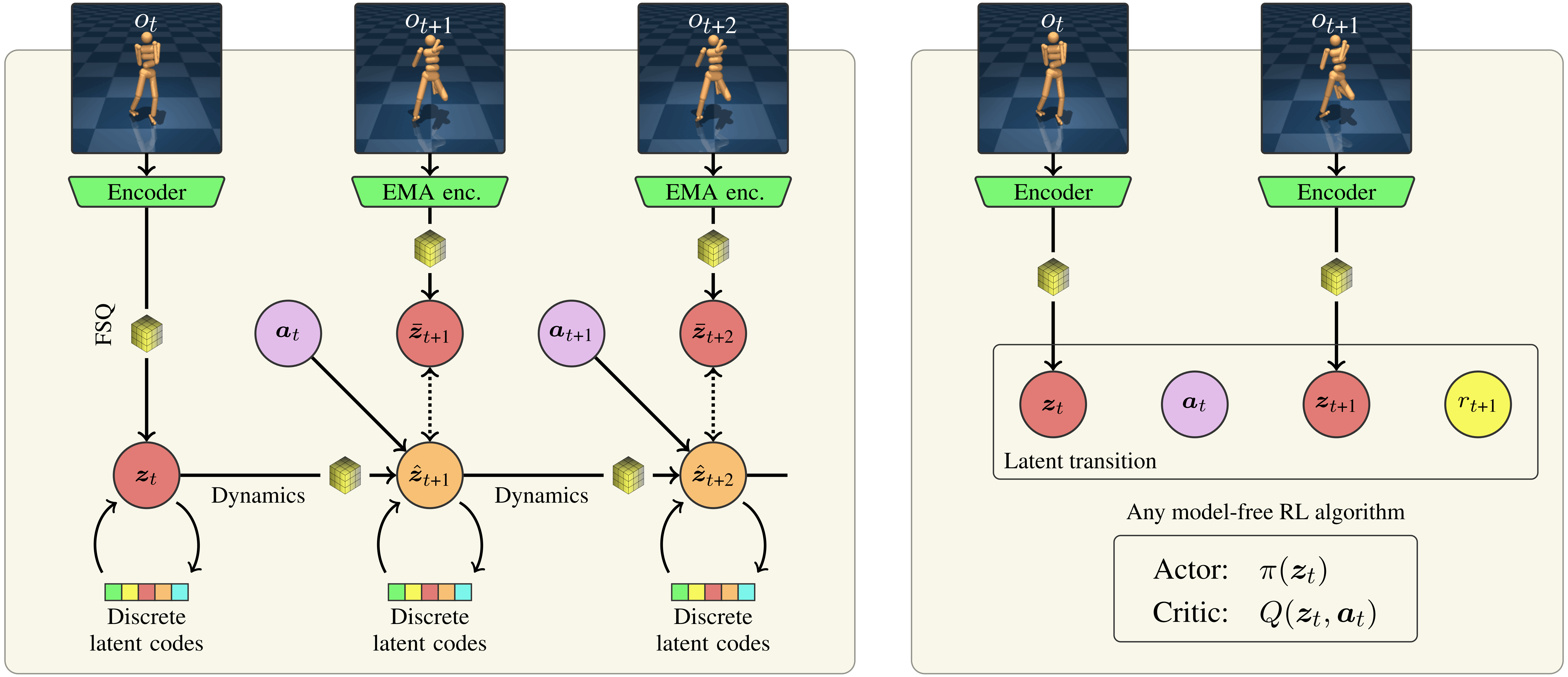
Case Study: iQRL

Dynamics Model but Not Model-based RL? 🤔



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Scannell et al. (2024). iQRL - Implicitly Quantized Representations for Sample-efficient RL. arXiv:2406.02696.

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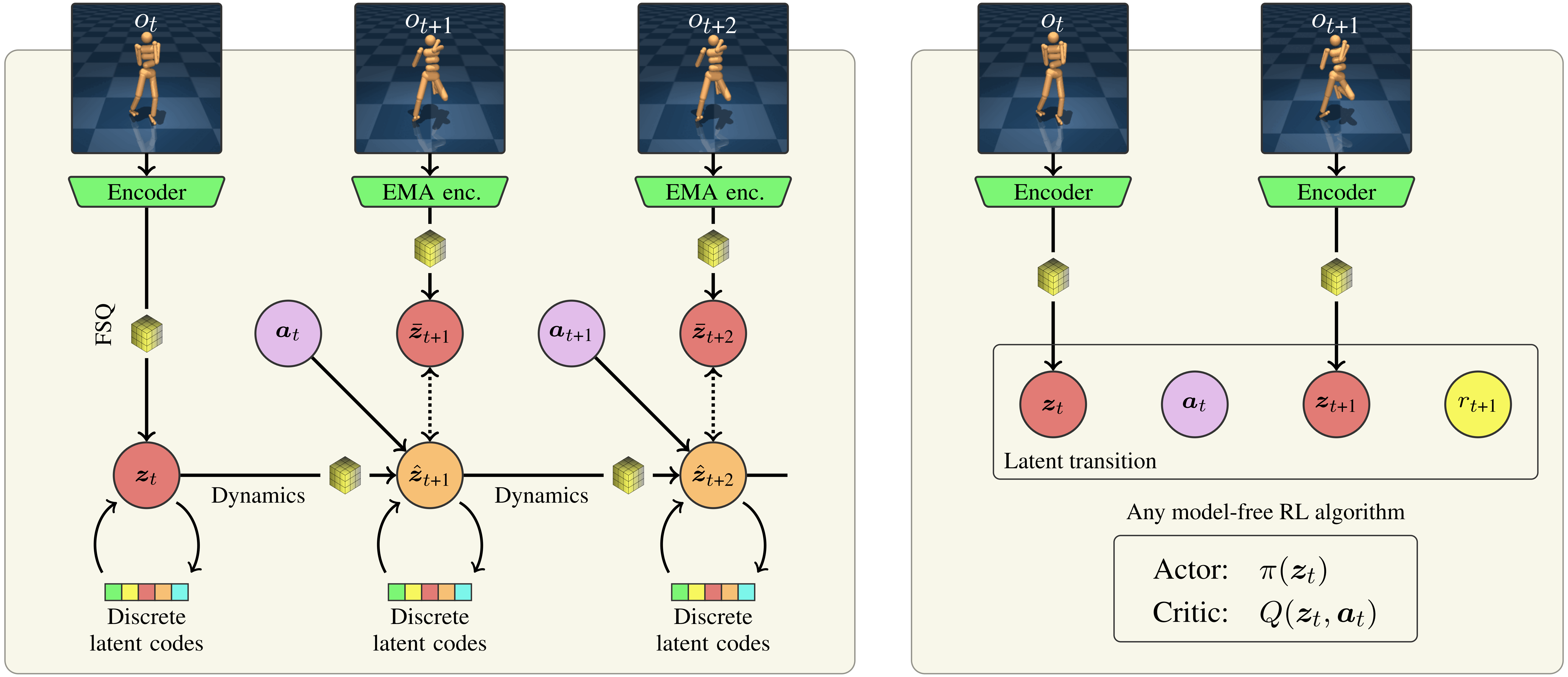
Case Study: iQRL

Dynamics Model but Not Model-based RL? 🤔

Use dynamics for representation learning



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Case Study: iQRL

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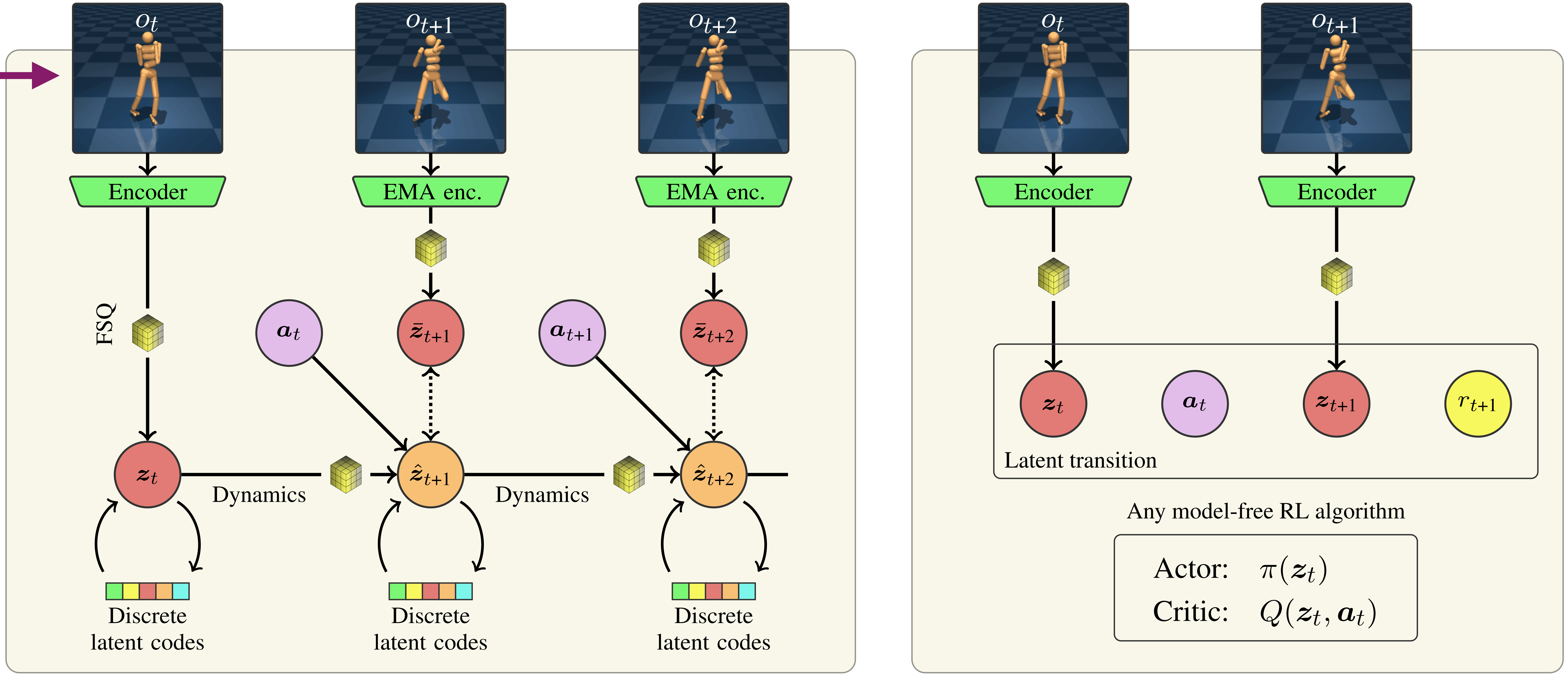
Use dynamics for representation learning

Observation



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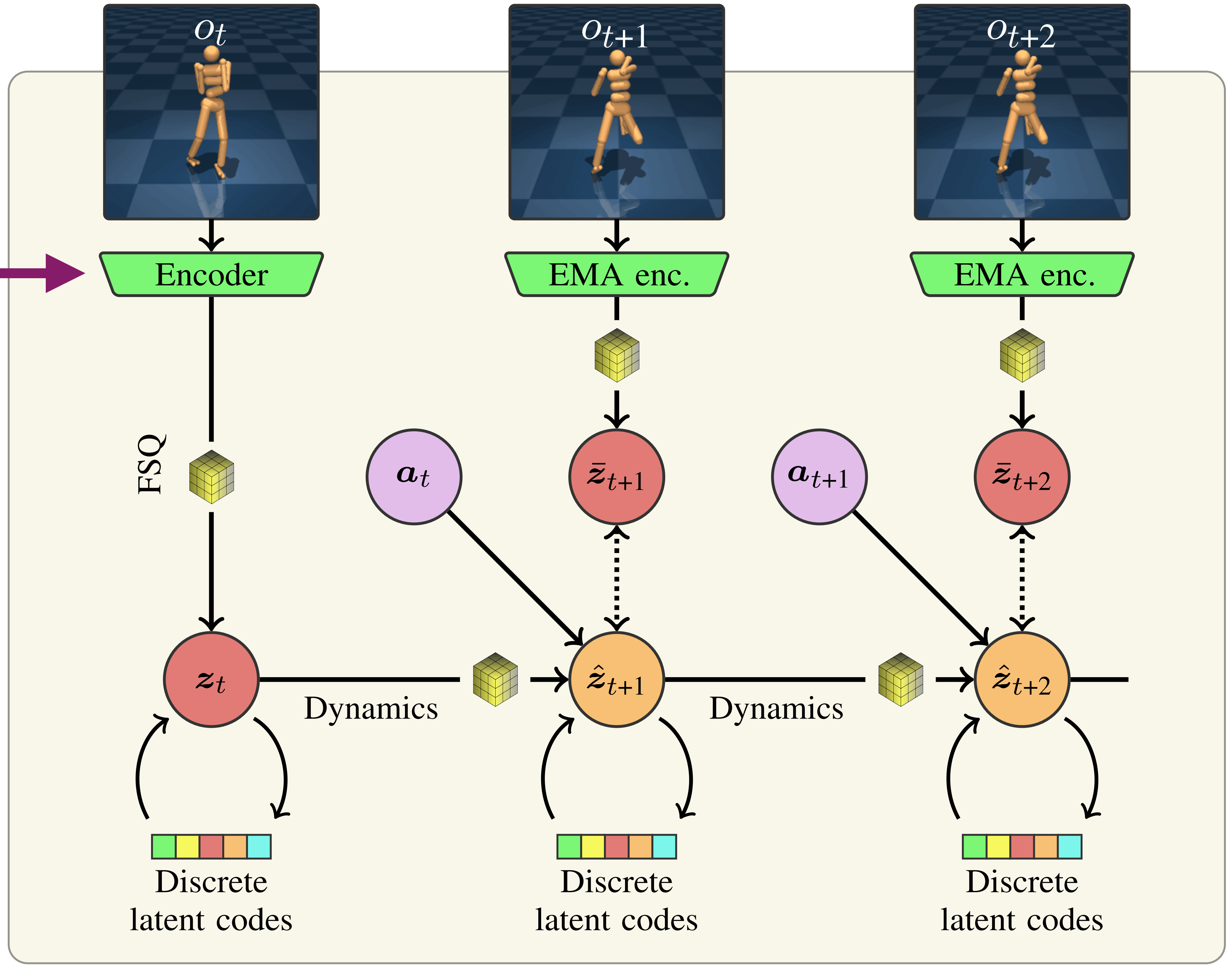
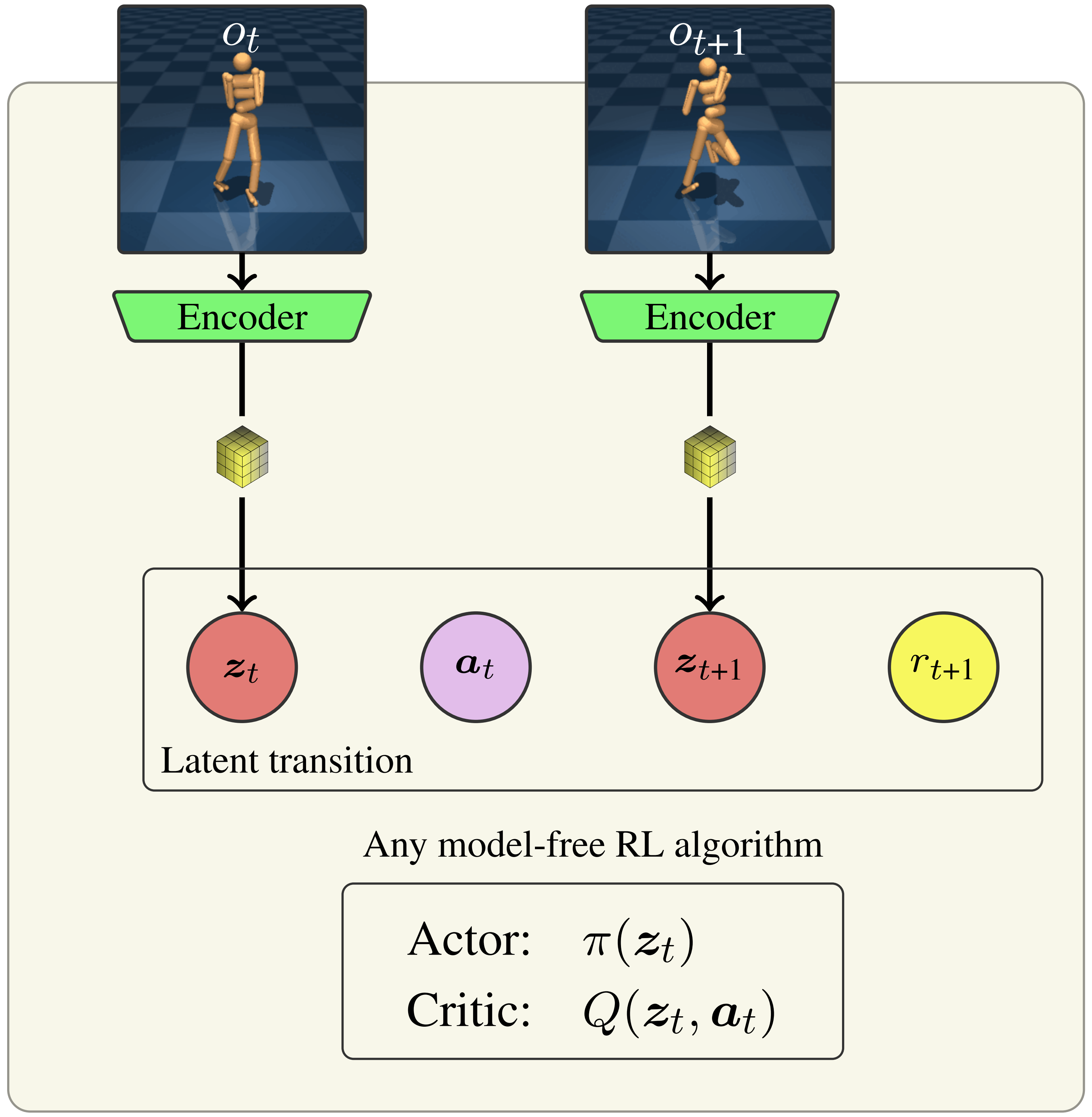
Encoder





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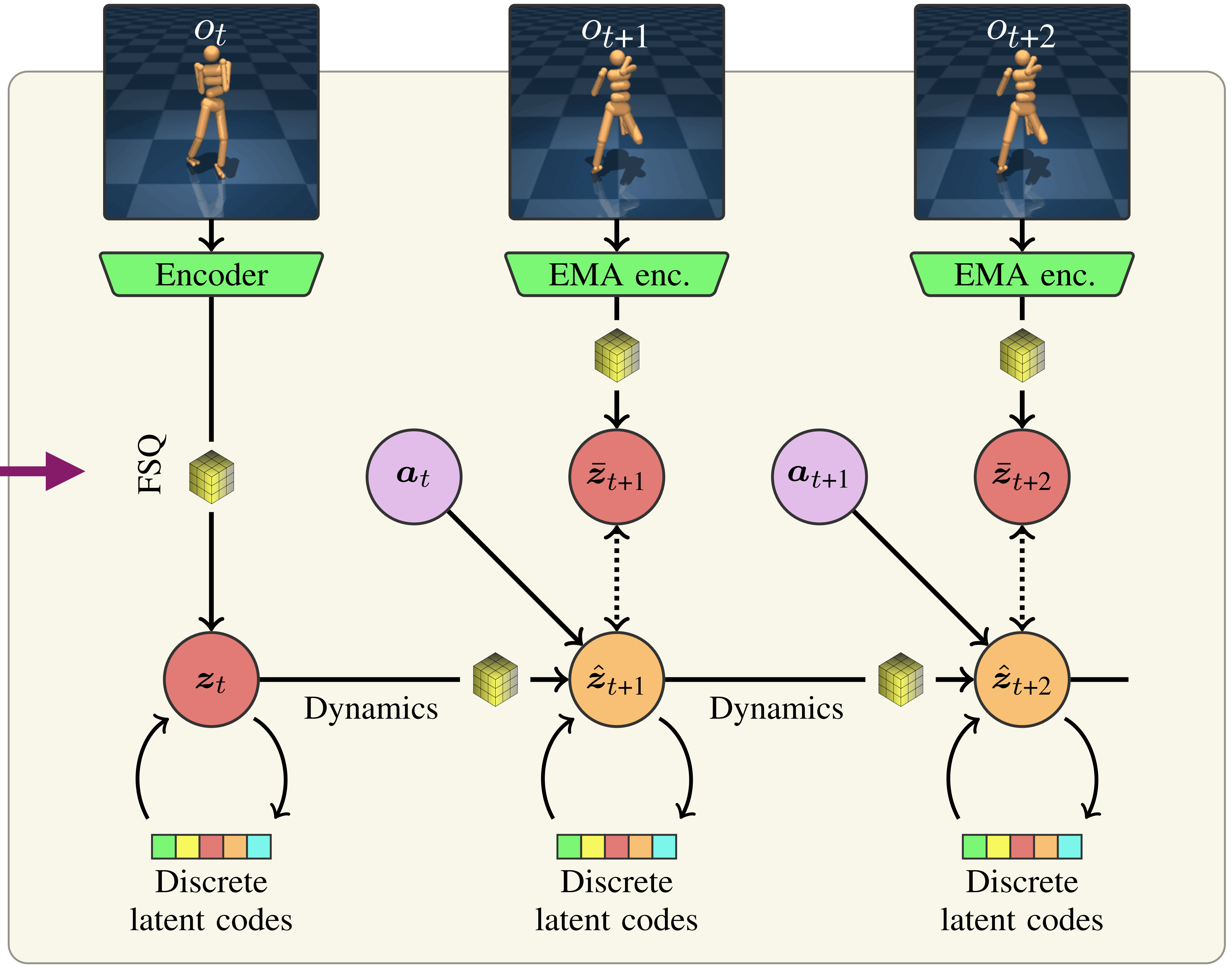
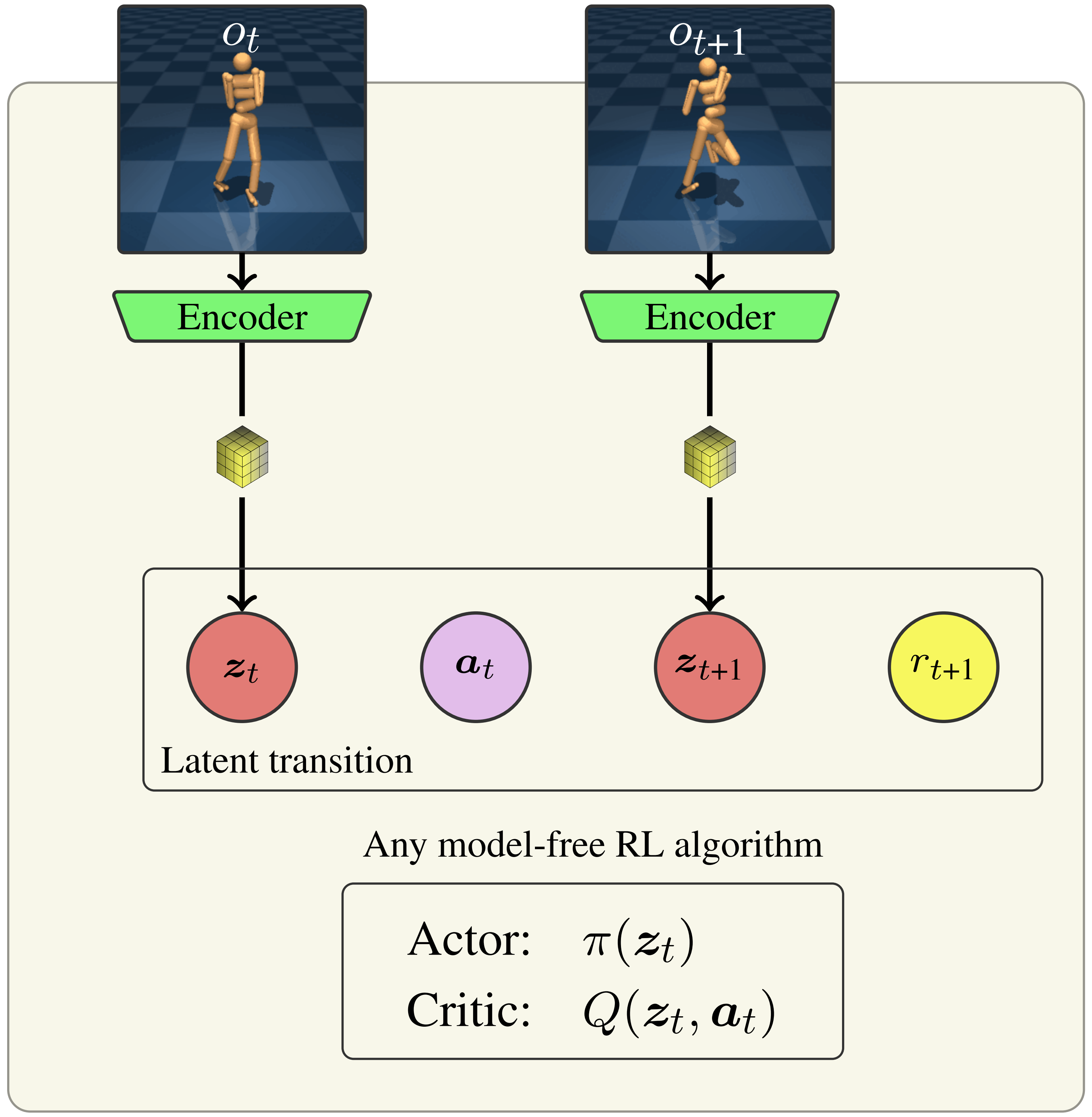
Quantization





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Case Study: iQRL

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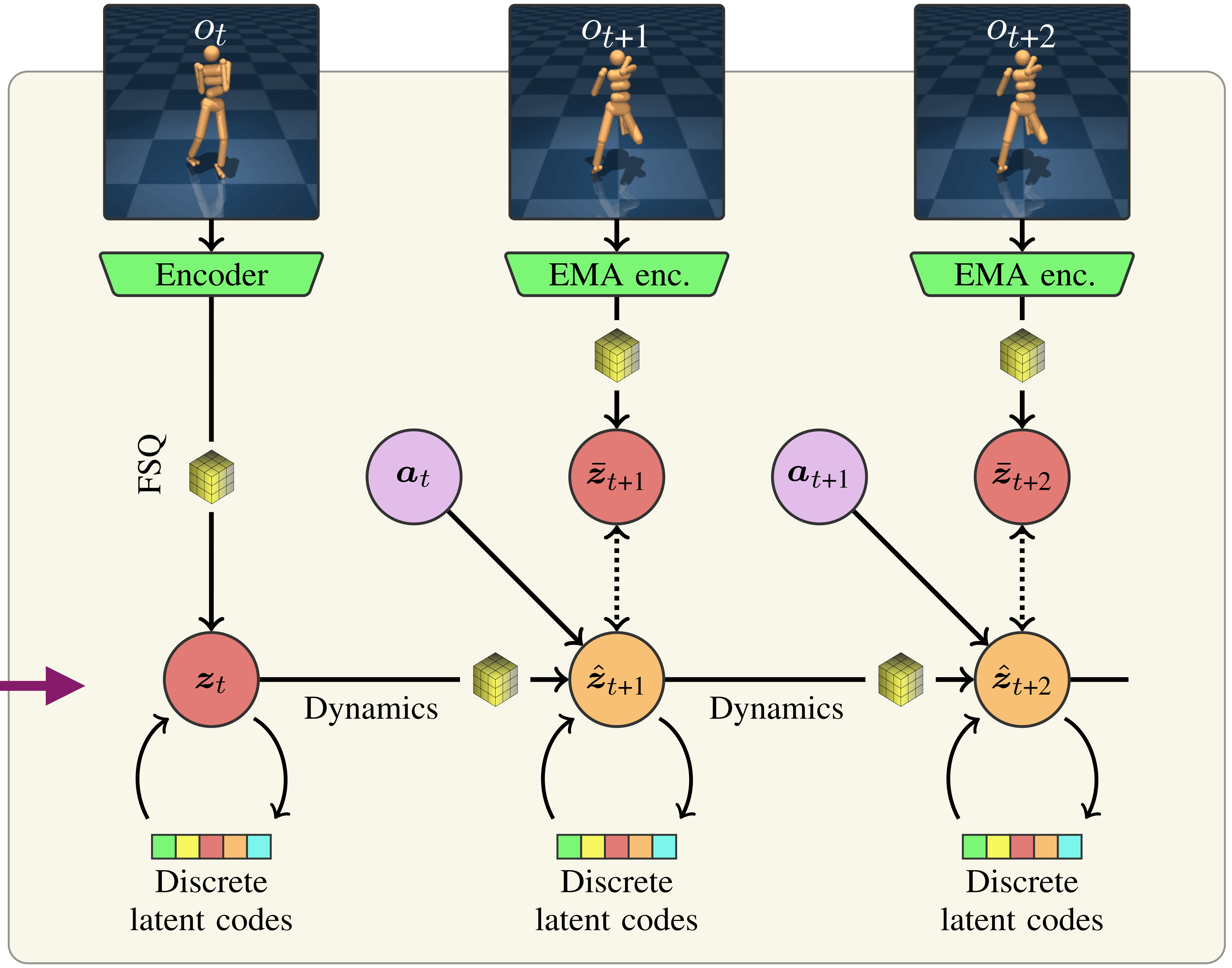
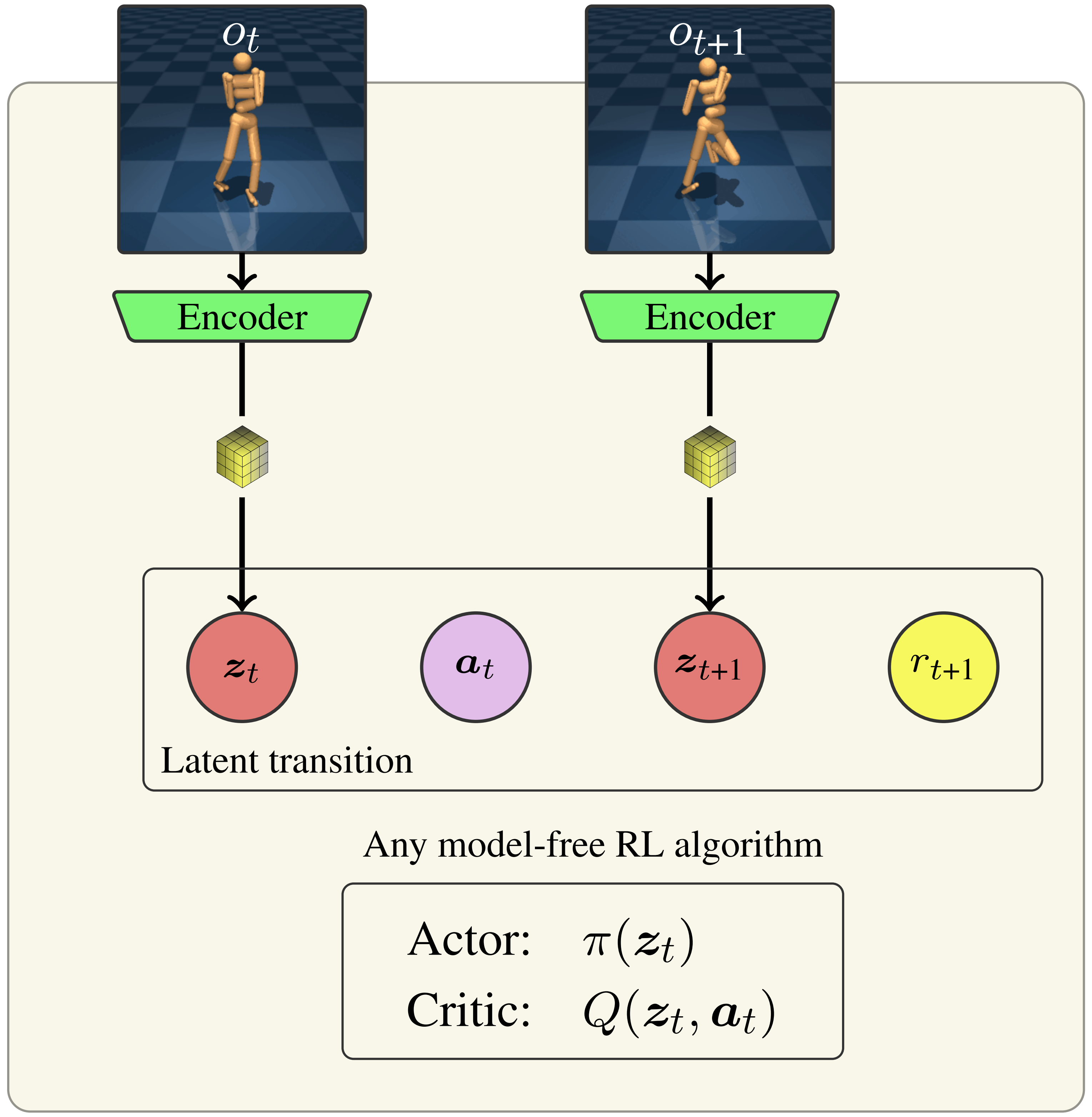
Use dynamics for representation learning

Latent dynamics





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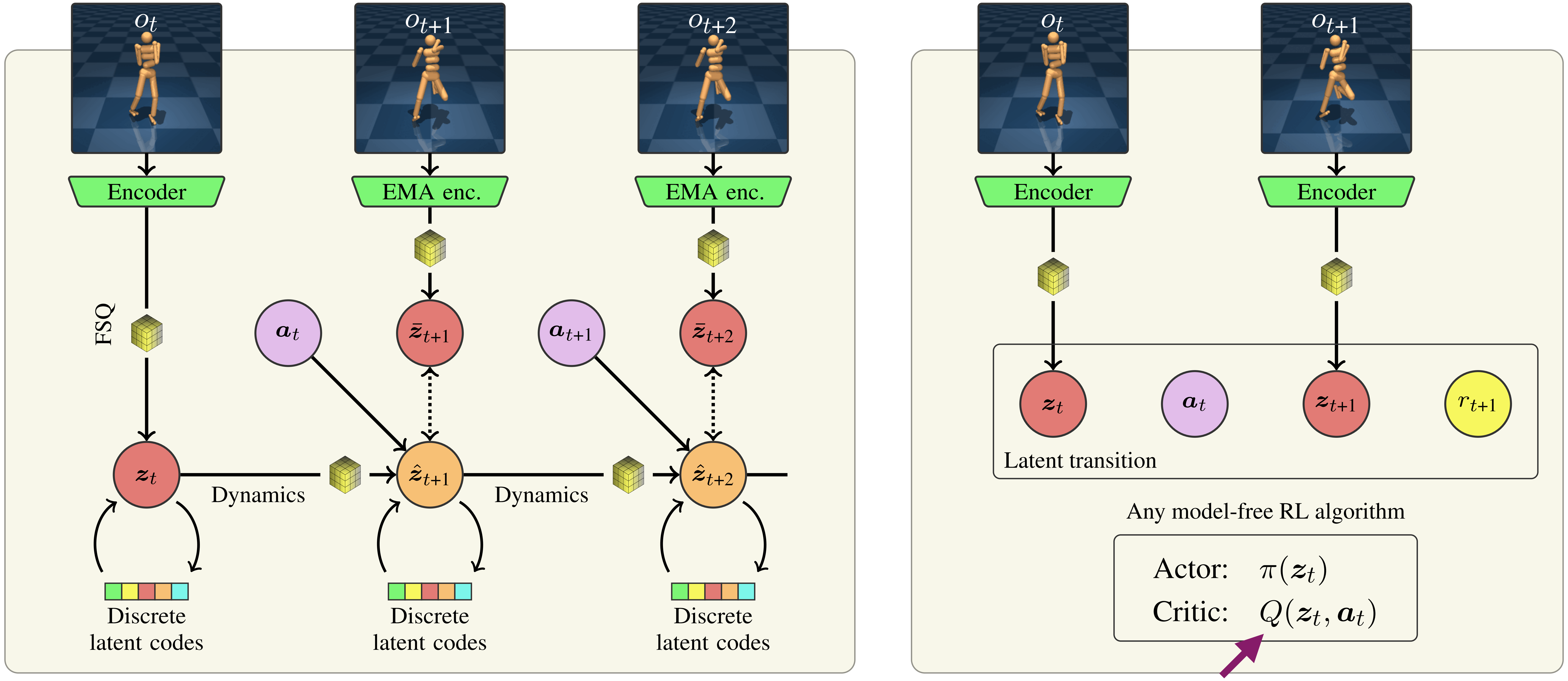

Model-free RL in latent space

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Outlook

Outlook

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1. Model-based RL is a powerful tool

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Outlook

1. Model-based RL is a powerful tool
2. Leveraging predictive models improves sample efficiency

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Outlook

1. Model-based RL is a powerful tool
2. Leveraging predictive models improves sample efficiency
3. Lots more exciting work to be done

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